DM87
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 20

Vehicle Routing
Rich Models

Marco Chiarandini

Outline

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Performance of exact methods

Current limits of exact methods [Ropke, Pisinger (2007)]:

CVRP: up to 135 customers by branch and cut and price

VRPTW: 50 customers (but 1000 customers can be solved if the instance has some structure)

CP can handle easily side constraints but hardly solve VRPs with more than 30 customers.

Large Neighborhood Search

Other approach with CP: [Shaw, 1998]

- Use an over all local search scheme
- Moves change a large portion of the solution
- CP system is used in the exploration of such moves.
- CP used to check the validity of moves and determine the values of constrained variables
- As a part of checking, constraint propagation takes place. Later, iterative improvement can take advantage of the reduced domains to speed up search by performing fast legality checks.

Solution representation:

- Handled by local search:
  Next pointers: A variable n_i for every customer i representing the next visit performed by the same vehicle

$$n_i \in N \cup S \cup E$$

  where $S = \bigcup S_k$ and $E = \bigcup E_k$ are additional visits for each vehicle k marking the start and the end of the route for vehicle k
- Handled by the CP system: time and capacity variables.

In the literature, the overall heuristic idea received different names:

- Removal and reinsertion
- Ruin and repair
- Iterated greedy
- Fix and re-optimize

Remove

Remove some related customers
(their re-insertion is likely to change something)

Relatedness measure r_{ij}

- geographical

$$r_{ij} = \frac{1}{D}\left(d'(i,j) + d'(i,j+n) + d'(i+n,j) + d'(i+n,j+n)\right)$$

- temporal and load based

$$d'(u,v) = |T_{p_i} - T_{p_j}| + |T_{d_i} - T_{d_j}| + |l_i - l_j|$$

- cluster removal
- history based: neighborhood graph removal

Dispersion sub-problem:
choose q customers to remove with minimal r_{ij}

Heuristic stochastic procedure:

- choose a pair randomly;
- select an already removed i and find j that minimizes r_{ij}

Insertion

by CP:

- constraint propagation rules: time windows, load and bound considerations
- search heuristic most constrained variable + least constrained valued (for each v find cheapest insertion and choose v with largest such cost)
- Complete search: ok for 15 visits (25 for VRPTW) but with heavy tails
- Limited discrepancy search

[Shaw, 1998]

```
Reinsert(RoutingPlan plan, VisitSet visits, integer discrep)
    if |visits| = 0 then
        if Cost(plan) < Cost(bestplan) then
            bestplan := plan
        end if
    else
        Visit v := ChooseFarthestVisit(visits)
        integer i := 0
        for p in rankedPositions(v) and i ≤ discrep do
            Store(plan) // Preserve plan on stack
            InsertVisit(plan, v, p)
            Reinsert(plan, visits - v, discrep - i)
            Restore(plan) // Restore plan from stack
            i := i + 1
        end for
    end if
end Reinsert
```


Other insertion procedures:

- Greedy (cheapest insertion)
- Max regret:

  Δf_i^q due to insert i into its best position in its q^{th} best route

  $i = \arg\max(\Delta f_i^2 - \Delta f_i^1)$

Advantages of removal-reinsert procedure with many side constraints:

- the search space in local search may become disconnected
- it is easier to implement feasibility checks
- no need of computing delta functions in the objective function

Further ideas

- Adaptive removal: start by removing 1 pair and increase after a certain number of iterations
- use of roulette wheel to decide which removal and reinsertion heuristic to use

$$p_i = \frac{\pi_i}{\sum \pi_i} \qquad \text{for each heuristic } i$$

- SA as accepting criterion after each reconstruction

Outline

Stochastic VRP (SVRP)

Stochastic VRP (SVRP) are VRPs where one or several components of the problem are random.
Three different kinds of SVRP are:

- Stochastic customers: Each customer i is present with probability p_i and absent with probability $1 - p_i$.
- Stochastic demands: The demand d_i of each customer is a random variable.
- Stochastic times: Service times δ_i and travel times t_{ij} are random variables.

Current Research Directions

Optimization under uncertainty: some problem parameters are unknown.

- Stochastic optimization: If probability distributions governing the data are known or can be estimated

  In stochastic optimization the goal is to find some policy that is feasible for all (or almost all) the possible data instances and maximizes the expectation of some function of the decisions and the random variables.

- Robust optimization: If the parameters are known only within certain bounds

  In robust optimization the goal is to find a solution which is feasible for all such data and optimal in some sense.

Current Research Directions

Multistage stochastic optimization:

- Requests arrive dynamically
- Decisions on which requests to serve and how must be taken with a certain frequency
- Previous decisions can be changed to accommodate the new requests at best.
- large scale instances

➜ needed fast solvers that account for possible incoming data