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February 16, 2009
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DM204 - Assignment N. 1

Sport Scheduling by Integer Programming[1]

Part 1 The schedule of games of real leagues in sports is affected by a vast array of requirements, preferences, and idiosyncrasies. For instance, for Major League Baseball in USA, the schedule balances the number of weekend games each team plays in its home stadium, balances monthly home game counts, balances summer home game counts, limits number of consecutive games on the road, meets television requests, and handles many other constraints. There are even teams that do not want to play at home on the starting day of hunting season! Meeting all of these requirements is far beyond human capability, but is an ideal task for integer programming.

Let's begin with an initial "generalized round-robin" problem. We have ten teams, named 1 through 10. In this schedule, teams play in pairs and we are not concerned with "home games" or "away games". Every team plays once per week.

Teams 1, 2, 3, 4 and 5 are in one division and teams 6, 7, 8, 9 and 10 are in another. We will begin with every team playing every other team once, but will then create schedules where teams play X games against opponents in their own division and Y games against those outside their division for $(X, Y) = (2, 1), (3, 1), (3, 2)$ and $(4, 1)$.

As an objective, the league would rather play divisional games later in the season. We can model this by assigning a value to divisional games depending on the week in which they occur. For Week 1 (the first week), we will have a value of 0, Week 2 has a value of 1, Week 3 has value 2, Week 4 has a value of 4, and so on, so Week k has value $2^(k-2)$ for a divisional game in that week. What is the maximum value schedule possible?

Model this problem, write the model in ZIMPL and solve it with SCIP. You may use Table 1 to get inspiration and to check the solution for the case $(1, 1)$. Did the formulation embedded what required? Does the resulting "optimal solution" match our intuition as to what an answer should look like? What is the most divisional games a week can have?

Part 2 Include in the model the following extensions.

- Prohibited games (e.g. 1 and 6 can't play in week 3)
- Required games (3 and 5 must play in week 4)
- Ordering constraints (2 and 4 must play in a week after that for 2 and 3)
- Conflict constraints(4 and 5 can't play in the same week as 2 and 3)
- Attendance objective (given a projected attendance for each game and each week, maximize the total projected attendance)
- Divisional prohibitions (no divisional games in week 3)

[1] Trick M. A. (2004), "Using Sports Scheduling to Teach Integer Programming," INFORMS Transactions on Education, Vol. 5, No 1

VARIABLES															
Team i	1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
Team j	2	3	4	5	6	3	4	5	6	4	5	6	5	6	6
Week															
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CONSTRAINTS															
Correct Number of Games Played															
Count	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Goal	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
One game per slot															
	1	2	3	4	5	6									
1	0	0	0	0	0	0									
2	0	0	0	0	0	0									
3	0	0	0	0	0	0									
4	0	0	0	0	0	0									
5	0	0	0	0	0	0									
OBJECTIVE															
Objective	0														
Coefficients															
1	0	0				0							0	0	0
2	1	1				1							1	1	1
3	2	2				2							2	2	2
4	4	4				4							4	4	4
5	8	8				8							8	8	8

Table 1: A schematic view in the basic problem

Part 3 Find a (X, Y) which is hard to solve and modify the model in order to help finding a solution faster. In particular, consider the branch and bound algorithm and the linear relaxation that has to be solved. Try to strengthen your formulation by introducing "cuts", that is additional constrains that satisfy two properties:

1. They are valid (every feasible integer solution satisfies the cut), and
2. They are not satisfied by some fractional solution of the original model.