DMP204
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 15

Flow Shop and Job Shop Models

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Outline

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Job Shop

General Shop Scheduling:

- $J = \{1, \ldots, N\}$ set of jobs; $M = \{1, 2, \ldots, m\}$ set of machines
- $J_j = \{O_{ij} \mid i = 1, \ldots, n_j\}$ set of operations for each job
- p_{ij} processing times of operations O_{ij}
- $\mu_{ij} \subseteq M$ machine eligibilities for each operation
- precedence constraints among the operations
- one job processed per machine at a time,
  one machine processing each job at a time
- C_j completion time of job j

➨ Find feasible schedule that minimize some regular function of C_j

Job shop

- $\mu_{ij} = l, l = 1, \ldots, n_j$ and $\mu_{ij} \neq \mu_{i+1,j}$ (one machine per operation)
- $O_{1j} \to O_{2j} \to \ldots \to O_{n_j,j}$ precedences (without loss of generality)
- without repetition and with unlimited buffers

Task:

- Find a schedule $S = (S_{ij})$, indicating the starting times of O_{ij}, such that:

  it is feasible, that is,

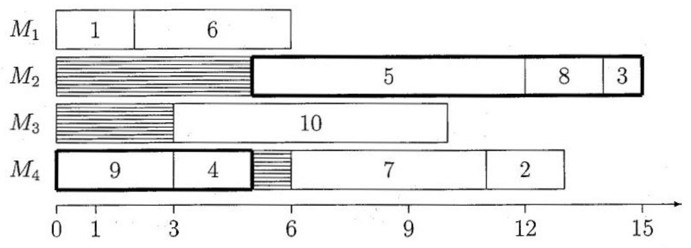
  - $S_{ij} + p_{ij} \leq S_{i+1,j}$ for all $O_{ij} \to O_{i+1,j}$
  - $S_{ij} + p_{ij} \leq S_{uv}$ or $S_{uv} + p_{uv} \leq S_{ij}$ for all operations with $\mu_{ij} = \mu_{uv}$.

  and has minimum makespan: $\min\{\max_{j \in J}(S_{n_j,j} + p_{n_j,j})\}$.

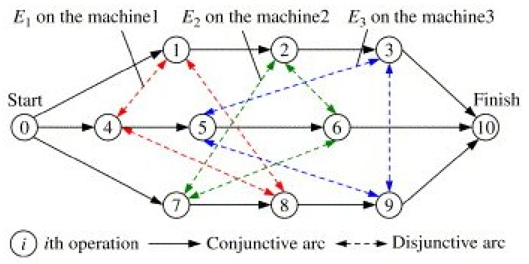
A schedule can also be represented by an m-tuple $\pi = (\pi^1, \pi^2, \ldots, \pi^m)$ where π^i defines the processing order on machine i.

There is always an optimal schedule that is semi-active.

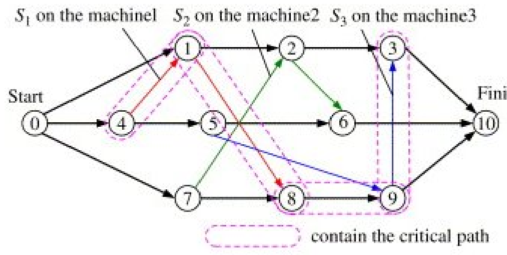
(semi-active schedule: for each machine, start each operation at the earliest feasible time.)



- Often simplified notation: $N = \{1, \ldots, n\}$ denotes the set of operations
- Disjunctive graph representation: $G = (N, A, E)$
  - vertices N: operations with two dummy operations 0 and $n+1$ denoting "start" and "finish".
  - directed arcs A, conjunctions
  - undirected arcs E, disjunctions
  - length of (i, j) in A is p_i



- A complete selection corresponds to choosing one direction for each arc of E.
- A complete selection that makes D acyclic corresponds to a feasible schedule and is called consistent.
- Complete, consistent selection $\Leftrightarrow$ semi-active schedule (feasible earliest start schedule).
- Length of longest path 0–$(n+1)$ in D corresponds to the makespan



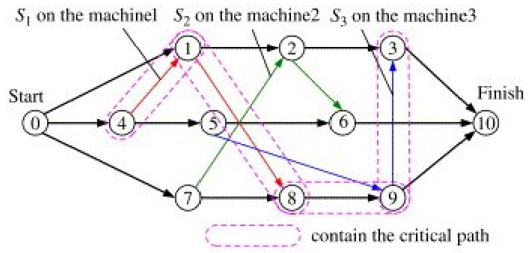
Longest path computation

In an acyclic digraph:

- construct topological ordering ($i < j$ for all $i \to j \in A$)
- recursion:

$$r_0 = 0$$
$$r_l = \max_{\{j \mid j \to l \in A\}} \{r_j + p_j\} \qquad for l = 1, \dots, n+1$$

- A block is a maximal sequence of adjacent critical operations processed on the same machine.
- In the Fig. below: $B_1 = \{4, 1, 8\}$ and $B_2 = \{9, 3\}$



- Any operation, u, has two immediate predecessors and successors:
  - its job predecessor $JP(u)$ and successor $JS(u)$
  - its machine predecessor $MP(u)$ and successor $MS(u)$

Exact methods

- Disjunctive programming

$$\begin{array}{lll} \min & C_{max} & \\ s.t. & x_{ij} + p_{ij} \leq C_{max} & \forall\, O_{ij} \in N \\ & x_{ij} + p_{ij} \leq x_{lj} & \forall\, (O_{ij}, O_{lj}) \in A \\ & x_{ij} + p_{ij} \leq x_{ik} \vee x_{ij} + p_{ij} \leq x_{ik} & \forall\, (O_{ij}, O_{ik}) \in E \\ & x_{ij} \leq 0 & \forall\, i = 1, \dots, m \;\; j = 1, \dots, N \end{array}$$

- Constraint Programming
- Branch and Bound [Carlier and Pinson, 1983]

Typically unable to schedule optimally more than 10 jobs on 10 machines.
Best result is around 250 operations.

Branch and Bound [Carlier and Pinson, 1983] [B2, p. 179]

Let Ω contain the first operation of each job;
Let $r_{ij} = 0$ for all $O_{ij} \in \Omega$

Machine Selection Compute for the current partial schedule

$$t(\Omega) = \min_{ij \in \Omega} \{r_{ij} + p_{ij}\}$$

and let i^* denote the machine on which the minimum is achieved

Branching Let Ω' denote the set of all operations O_{i^*j} on machine i^* such that

$$r_{i^*j} < t(\Omega) \qquad \text{(i.e. eliminate } r_{i^*j} \geq t(\Omega))$$

For each operation in Ω', consider an (extended)partial schedule with that operation as the next one on machine i^*.
For each such (extended) partial schedule, delete the operations from Ω, include its immediate follower in Ω and return to Machine Selection.

Lower Bounding:

- longest path in partially selected disjunctive digraph
- solve $1|r_{ij}|L_{max}$ on each machine i like if all other machines could process at the same time (see later shifting bottleneck heuristic) + longest path.

Efficient local search for job shop

Solution representation:
m-tuple $\pi = (\pi^1, \pi^2, \ldots, \pi^m) \Longleftrightarrow$ oriented digraph $D_\pi = (N, A, E_\pi)$

Neighborhoods
Change the orientation of certain disjunctive arcs of the current complete selection

Issues:

1. Can it be decided easily if the new digraph $D_{\pi'}$ is acyclic?
2. Can the neighborhood selection S' improve the makespan?
3. Is the neighborhood connected?

Swap Neighborhood [Novicki, Smutnicki]
Reverse one oriented disjunctive arc (i, j) on some critical path.

Theorem
All neighbors are consistent selections.

Note: If the neighborhood is empty then there are no disjunctive arcs, nothing can be improved and the schedule is already optimal.

Theorem
The swap neighborhood is connected.

Insertion Neighborhood [Balas, Vazacopoulos, 1998]
For some nodes u, v in the critical path:

- move u right after v (forward insert)
- move v right before u (backward insert)

Theorem: If a critical path containing u and v also contains $JS(v)$ and

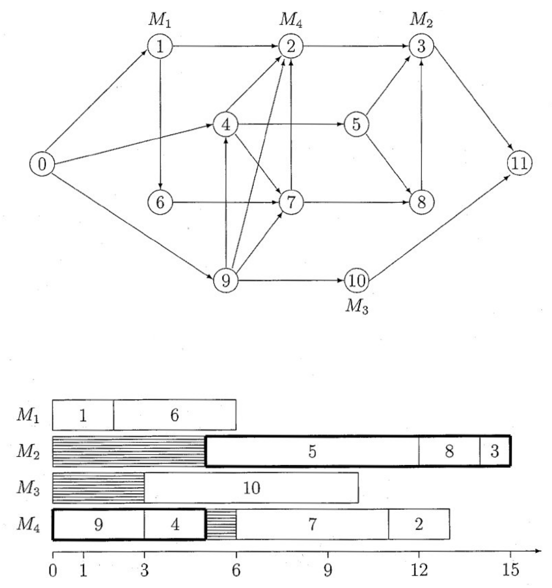
$$L(v, n) \geq L(JS(u), n)$$

then a forward insert of u after v yields an acyclic complete selection.

Theorem: If a critical path containing u and v also contains $JS(v)$ and

$$L(0, u) + p_u \geq L(0, JP(v)) + p_{JP(v)}$$

then a backward insert of v before v yields an acyclic complete selection.



Theorem: (Elimination criterion) If $C_{max}(S') < C_{max}(S)$ then at least one operation of a machine block B on the critical path has to be processed before the first or after the last operation of B.

- Swap neighborhood can be restricted to first and last operations in the block
- Insert neighborhood can be restricted to moves similar to those saw for the flow shop. [Grabowski, Wodecki]

Tabu Search requires a best improvement strategy hence the neighborhood must be search very fast.

Neighbor evaluation:

- exact recomputation of the makespan $O(n)$
- approximate evaluation (rather involved procedure but much faster and effective in practice)

The implementation of Tabu Search follows the one saw for flow shop.

Shifting Bottleneck Heuristic

- A complete selection is made by the union of selections S_k for each clique E_k that corresponds to machines.
- **Idea**: use a priority rule for ordering the machines.
  chose each time the bottleneck machine and schedule jobs on that machine.
- Measure bottleneck quality of a machine k by finding optimal schedule to a certain single machine problem.
- Critical machine, if at least one of its arcs is on the critical path.

- $M_0 \subset M$ set of machines already sequenced.
- $k \in M \setminus M_0$
- $P(k, M_0)$ is problem $1 \mid r_j \mid L_{max}$ obtained by:
  - the selections in M_0
  - removing any disjunctive arc in $p \in M \setminus M_0$
- $v(k, M_0)$ is the optimum of $P(k, M_0)$
- bottleneck $m = \arg \max\limits_{k \in M \setminus M_0} \{v(k, M_0)\}$
- $M_0 = \emptyset$

Step 1: Identify bottleneck m among $k \in M \setminus M_0$ and sequence it optimally. Set $M_0 \leftarrow M_0 \cup \{m\}$

Step 2: Reoptimize the sequence of each critical machine $k \in M_0$ in turn: set $M_o' = M_0 - \{k\}$ and solve $P(k, M_0')$.
Stop if $M_0 = M$ otherwise Step 1.

- Local Reoptimization Procedure

Construction of $P(k, M_0)$

$1 \mid r_j \mid L_{max}$:

- $r_j = L(0, j)$
- $d_j = L(0, n) - L(j, n) + p_j$

$L(i, j)$ length of longest path in G: Computable in $O(n)$

An acyclic complete directed graph is the transitive closure of its unique directed Hamilton path.

Hence, only predecessors and successor are to be checked.
The graph is not constructed explicitly, but by maintaining a list of jobs per machines and a list machines per jobs.

$1 \mid r_j \mid L_{max}$ can be solved optimally very efficiently.
Results reported up to 1000 jobs.

$1 \mid r_j \mid L_{max}$ From Lecture 9

[Maximum lateness with release dates]

- Strongly NP-hard (reduction from 3-partition)
- might have optimal schedule which is not non-delay
- Branch and bound algorithm (valid also for $1 \mid r_j, prec \mid L_{max}$)
  - **Branching**:
    schedule from the beginning (level k, $n!/(k-1)!$ nodes)
    elimination criterion: do not consider job j_k if:

    $$r_j > \min_{l \in J} \{\max(t, r_l) + p_l\} \qquad J \text{ jobs to schedule, } t \text{ current time}$$

  - **Lower bounding**: relaxation to preemptive case for which EDD is optimal