DMP204
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 16
Job Shop
The Alternative Graph Model

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Outline

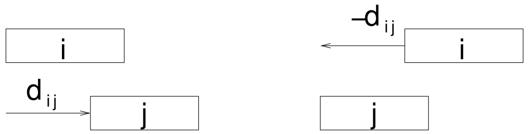
Resume

Job Shop:

- Definition
- Starting times and m-tuple permutation representation
- Disjunctive graph representation [Roy and Sussman, 1964]
- Shifting Bottleneck Heuristic [Adams, Balas and Zawack, 1988]

Outline

Generalizations: Time Lags



Generalized time constraints

They can be used to model:

- Release time:

$$S_0 + r_i \leq S_i \qquad \Longleftrightarrow \qquad d_{0i} = r_i$$

- Deadlines:

$$S_i + p_i - d_i \leq S_0 \qquad \Longleftrightarrow \qquad d_{i0} = p_i - d_i$$

- Modelling

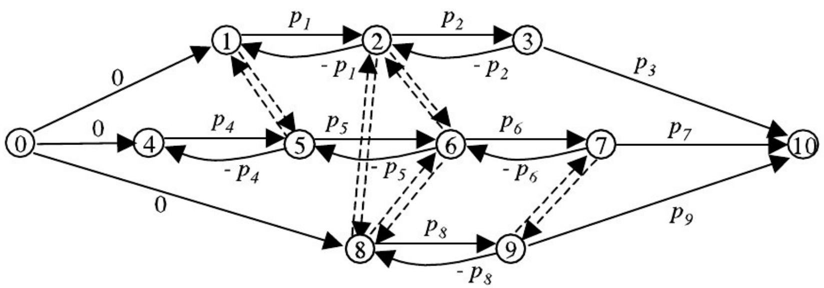
$$\begin{array}{lll} \min & C_{max} & \\ s.t. & x_{ij} + d_{ij} \leq C_{max} & \forall\, O_{ij} \in N \\ & x_{ij} + d_{ij} \leq x_{lj} & \forall\, (O_{ij}, O_{lj}) \in A \\ & x_{ij} + d_{ij} \leq x_{ik} \vee x_{ij} + d_{ij} \leq x_{ik} & \forall\, (O_{ij}, O_{ik}) \in E \\ & x_{ij} \geq 0 & \forall\, i = 1, \ldots, m \;\; j = 1, \ldots, N \end{array}$$

- In the disjunctive graph, d_{ij} become the lengths of arcs

- Exact relative timing (perishability constraints):
  if operation j must start l_{ij} after operation i:

  $$S_i + p_i + l_{ij} \leq S_j \qquad \text{and} \qquad S_j - (p_i + l_{ij}) \leq S_i$$

  ($l_{ij} = 0$ if no-wait constraint)



- Set up times:

$$S_i + p_i + s_{ij} \leq S_j \qquad \text{or} \qquad S_j + p_j + s_{ji} \leq S_i$$

- Machine unavailabilities:
  - Machine M_k unavailable in $[a_1, b_1]$, $[a_2, b_2]$, $\ldots$, $[a_v, b_v]$
  - Introduce v artificial operations with $\lambda = 1, \ldots, v$ with $\mu_\lambda = M_k$ and:
    $p_\lambda = b_\lambda - a_\lambda$
    $r_\lambda = a_\lambda$
    $d_\lambda = b_\lambda$

- Minimum lateness objectives:

$$L_{max} = \max_{j=1}^{N} \{C_j - d_j\} \qquad \Longleftrightarrow \qquad d_{n_j, n+1} = p_{n_j} - d_j$$

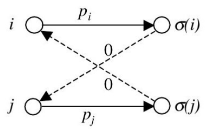
Arises with limited buffers:
after processing, a job remains on the machine until the next machine is freed

- Needed a generalization of the disjunctive graph model
  $\Longrightarrow$ Alternative graph model $G = (N, E, A)$ [Mascis, Pacciarelli, 2002]

1. two non-blocking operations to be processed on the same machine

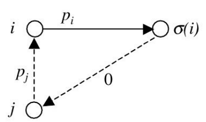
$$S_i + p_i \leq S_j \quad \text{or} \quad S_j + p_j \leq S_i$$

2. Two blocking operations i, j to be processed on the same machine $\mu(i) = \mu(j)$



$$S_{\sigma(j)} \leq S_i \quad \text{or} \quad S_{\sigma(i)} \leq S_j$$

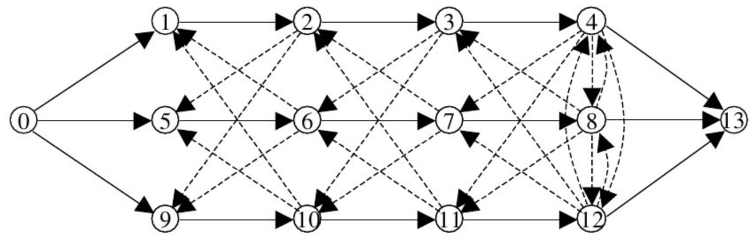
3. i is blocking, j is non-blocking (ideal) and i, j to be processed on the same machine $\mu(i) = \mu(j)$.



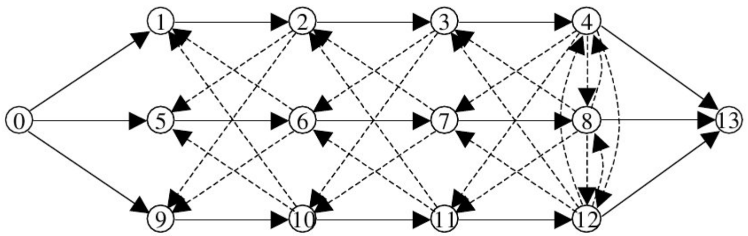
$$S_i + p_i \leq S_j \quad \text{or} \quad S_{\sigma(j)} \leq S_i$$

Example

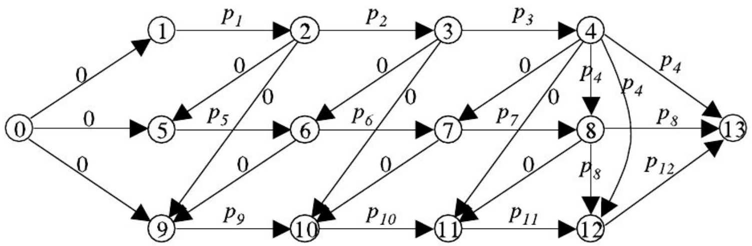
- $O_0, O_1, \ldots, O_{13}$
- $M(O_1) = M(O_5) = M(O_9)$
  $M(O_2) = M(O_6) = M(O_{10})$
  $M(O_3) = M(O_7) = M(O_{11})$



- Length of arcs can be negative
- Multiple occurrences possible: $((i, j), (u, v)) \in A$ and $((i, j), (h, k)) \in A$
- The last operation of a job j is always non-blocking.



- A complete selection S is consistent if it chooses alternatives from each pair such that the resulting graph does not contain positive cycles.



Example:

- $p_a = 4$
- $p_b = 2$
- $p_c = 1$

- b must start at least 9 days after a has started
- c must start at least 8 days after b is finished
- c must finish within 16 days after a has started

$$\begin{aligned} S_a + 9 &\leq S_b \\ S_b + 10 &\leq S_c \\ S_c - 15 &\leq S_a \end{aligned}$$

This leads to an absurd.
In the alternative graph the cycle is positive.

- The Makespan still corresponds to the longest path in the graph with the arc selection $G(S)$.
- Problem: now the digraph may contain cycles. Longest path with simple cyclic paths is NP-complete. However, here we have to care only of non-positive cycles.
- If there are no cycles of length strictly positive it can still be computed efficiently in $O(|N||E \cup A|)$ by Bellman-Ford (1958) algorithm.
- The algorithm iteratively considers all edges in a certain order and updates an array of longest path lengths for each vertex. It stops if a loop over all edges does not yield any update or after $|N|$ iterations over all edges (in which case we know there is a positive cycle).
- Possible to maintain incremental updates when changing the selection [Demetrescu, Frangioni, Marchetti-Spaccamela, Nanni, 2000].

Heuristic Methods

- The search space is highly constrained + detecting positive cycles is costly
- Hence local search methods not very successful
- Rely on the construction paradigm
- Rollout algorithm [Meloni, Pacciarelli, Pranzo, 2004]

Rollout

- Master process: grows a partial selection S^k:
  decides the next element to fix based on an heuristic function (selects the one with minimal value)
- Slave process: evaluates heuristically the alternative choices. Completes the selection by keeping fixed what passed by the master process and fixing one alternative at a time.

- Slave heuristics
  - *Avoid Maximum Current Completion time*
    find an arc (h, k) that if selected would increase most the length of the longest path in $G(S^k)$ and select its alternative
    $$\max_{(uv)\in A} \{l(0,u) + a_{uv} + l(u,n)\}$$
  - *Select Most Critical Pair*
    find the pair that, in the worst case, would increase least the length of the longest path in $G(S^k)$ and select the best alternative
    $$\max_{((ij),(hk))\in A} \min\{l(0,u) + a_{hk} + l(k,n), l(0,i) + a_{ij} + l(j,n)\}$$
  - *Select Max Sum Pair*
    finds the pair with greatest potential effect on the length of the longest path in $G(S^k)$ and select the best alternative
    $$\max_{((ij),(hk))\in A} |l(0,u) + a_{hk} + l(k,n) + l(0,i) + a_{ij} + l(j,n)|$$

Trade off quality *vs* keeping feasibility
Results depend on the characteristics of the instance.

Implementation details of the slave heuristics

- Once an arc is added we need to update all $L(0, u)$ and $L(u, n)$. Backward and forward visit $O(|F| + |A|)$
- When adding arc a_{ij}, we detect positive cycles if $L(i, j) + a_{ij} > 0$. This happens only if we updated $L(0, i)$ or $L(j, n)$ in the previous point and hence it comes for free.
- Overall complexity $O(|A|(|F| + |A|))$

Speed up of Rollout:

- Stop if partial solution overtakes upper bound
- limit evaluation to say 20% of arcs in A