DMP204
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 17

Resource Constrained Project Scheduling Reservations

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Outline

Outline

RCPS Model

Resource Constrained Project Scheduling Model

Given:

- activities (jobs) $j = 1, \ldots, n$
- renewable resources $i = 1, \ldots, m$
- amount of resources available R_i
- processing times p_j
- amount of resource used r_{ij}
- precedence constraints $j \rightarrow k$

Further generalizations

- Time dependent resource profile $R_i(t)$ given by (t_i^μ, R_i^μ)
  where $0 = t_i^1 < t_i^2 < \ldots < t_i^{m_i} = T$
- Multiple modes for an activity j
  processing time and use of resource depends on its mode m: p_{jm}, r_{jkm}.

Modeling

Case 1

- A contractor has to complete n activities.
- The duration of activity j is p_j
- each activity requires a crew of size W_j.
- The activities are not subject to precedence constraints.
- The contractor has W workers at his disposal
- his objective is to complete all n activities in minimum time.

Modeling

Case 2

- Exams in a college may have different duration.
- The exams have to be held in a gym with W seats.
- The enrollment in course j is W_j and
- all W_j students have to take the exam at the same time.
- The goal is to develop a timetable that schedules all n exams in minimum time.
- Consider both the cases in which each student has to attend a single exam as well as the situation in which a student can attend more than one exam.

Modeling

Case 3

- A set of jobs $J_1, \ldots, J_g$ are to be processed by auditors $A_1, \ldots, A_m$.
- Job J_l consists of n_l tasks ($l = 1, \ldots, g$).
- There are precedence constraints $i_1 \to i_2$ between tasks i_1, i_2 of the same job.
- Each job J_l has a release time r_l, a due date d_l and a weight w_l.
- Each task must be processed by exactly one auditor. If task i is processed by auditor A_k, then its processing time is p_{ik}.
- Auditor A_k is available during disjoint time intervals $[s_k^\nu, l_k^\nu]$ ($\nu = 1, \ldots, m$) with $l_k^\nu < s_k^\nu$ for $\nu = 1, \ldots, m_k - 1$.
- Furthermore, the total working time of A_k is bounded from below by H_k^- and from above by H_k^+ with $H_k^- \leq H_k^+$ $(k = 1, \ldots, m)$.
- We have to find an assignment $\alpha(i)$ for each task $i = 1, \ldots, n := \sum_{l=1}^{g} n_l$ to an auditor $A_{\alpha(i)}$ such that
  - each task is processed without preemption in a time window of the assigned auditor
  - the total workload of A_k is bounded by H_k^- and H_k^k for $k = 1, \ldots, m$.
  - the precedence constraints are satisfied,
  - all tasks of J_l do not start before time r_l, and
  - the total weighted tardiness $\sum_{l=1}^{g} w_l T_l$ is minimized.

Preprocessing: Temporal Analysis

- Precedence network must be acyclic
- Heads r_j and Tails q_j $\Leftarrow$ Longest paths $\Leftarrow$ Topological ordering (deadlines d_j can be obtained as $UB - q_j$)

Preprocessing: constraint propagation

1. conjunctions $i \to j$ $\qquad S_i + p_i \leq S_j$
   [precedence constrains]
2. parallelity constraints $i \,||\, j$ $\qquad S_i + p_i \geq S_j$ and $S_j + p_j \geq S_i$
   [time windows $[r_j, d_j]$,$[r_l, d_l]$ and $p_l + p_j > \max\{d_l, d_j\} - \min\{r_l, r_j\}$]
3. disjunctions $i - j$ $\qquad S_i + p_i \leq S_j$ or $S_j + p_j \leq S_i$
   [resource constraints: $r_{jk} + r_{lk} > R_k$]

N. Strengthenings: symmetric triples, etc.

Solutions

Task: Find a schedule indicating the starting time of each activity

- All solution methods restrict the search to feasible schedules, S, S'
- Types of schedules
  - Local left shift (LLS): $S \to S'$ with $S'_j < S_j$ and $S'_l = S_l$ for all $l \neq j$.
  - Global left shift (GLS): LLS passing through infeasible schedule
  - Semi active schedule: no LLS possible
  - Active schedule: no GLS possible
  - Non-delay schedule: no GLS and LLS possible even with preemption
- If regular objectives $\Longrightarrow$ exists an optimum which is active

Hence:

- Schedule not given by start times S_i
  - space too large $O(T^n)$
  - difficult to check feasibility
- Sequence (list, permutation) of activities $\pi = (j_1, \ldots, j_n)$
- π determines the order of activities to be passed to a schedule generation scheme

Schedule Generation Schemes

Given a sequence of activity, SGS determine the starting times of each activity

Serial schedule generation scheme (SSGS)

n stages, S_λ scheduled jobs, E_λ eligible jobs

Step 1 Select next from E_λ and schedule at earliest.

Step 2 Update E_λ and $R_k(\tau)$.
If E_λ is empty then STOP,
else go to Step 1.

Parallel schedule generation scheme (PSGS)

(Time sweep)

stage λ at time t_λ

S_λ (finished activities), A_λ (activities not yet finished), E_λ (eligible activities)

Step 1 In each stage select maximal resource-feasible subset of eligible activities in E_λ and schedule it at t_λ.

Step 2 Update E_λ, A_λ and $R_k(\tau)$.
If E_λ is empty then STOP,
else move to $t_{\lambda+1} = \min \left\{ \min_{j \in A_\lambda} C_j, \min_{\substack{k=1,\ldots,r \\ i \in m_k}} t_i^\mu \right\}$
and go to Step 1.

- If constant resource, it generates non-delay schedules

Possible uses:

- Forward
- Backward
- Bidirectional
- Forward-backward improvement (justification techniques)
  [V. Valls, F. Ballestín and S. Quintanill, EJOR, 2005]

Dispatching Rules

Determines the sequence of activities to pass to the schedule generation scheme

- activity based
- network based
- path based
- resource based

Static vs Dynamic

Local Search

All typical neighborhood operators can be used:

- Swap
- Interchange
- Insert

reduced to only those moves compatible with precedence constraints

Genetic Algorithms

Recombination operator:

- One point crossover
- Two point crossover
- Uniform crossover

Implementations compatible with precedence constraints

Outline

Reservations without slack

Interval Scheduling

Given:

- m parallel machines (resources)
- n activities
- r_j starting times (integers),
  d_j termination (integers),
  w_j or w_{ij} weight,
  M_j eligibility
- without slack $p_j = d_j - r_j$

Task: Maximize weight of assigned activities

Examples: Hotel room reservation, Car rental

Polynomially solvable cases

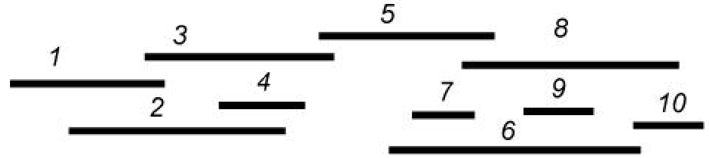
1. $p_j = 1$

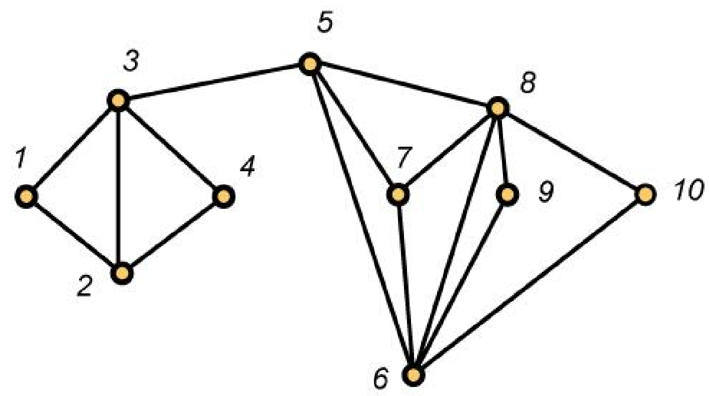
Solve an assignment problem at each time slot

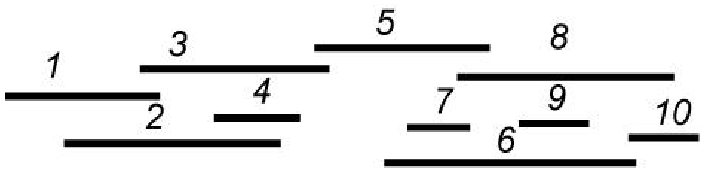
2. $w_j = 1$, $M_j = M$, Obj. minimize resources used

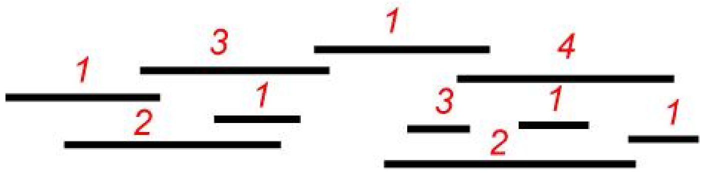
- Corresponds to coloring interval graphs with minimal number of colors
- Optimal greedy algorithm (First Fit):

  order $r_1 \leq r_2 \leq \ldots \leq r_n$
  Step 1 assign resource 1 to activity 1
  Step 2 **for** j from 2 to n **do**
  Assume k resources have been used.
  Assign activity j to the resource with minimum feasible value from $\{1, \ldots, k+1\}$









3. $w_j = 1$, $M_j = M$, Obj. maximize activities assigned

- Corresponds to coloring max # of vertices in interval graphs with k colors
- Optimal k-coloring of interval graphs:

order $r_1 \leq r_2 \leq \ldots \leq r_n$
$J = \emptyset$, $j = 1$

Step 1 if a resource is available at time r_j then assign activity j to that resource;
include j in J; go to Step 3

Step 2 Else, select j^* such that $C_{j^*} = \max_{j \in J} C_j$
if $C_j = r_j + p_j > C_{j^*}$ go to Step 3
else remove j^* from J, assign j in J

Step 3 **if** $j = n$ STOP **else** $j = j + 1$ go to Step 1

Outline

Reservations with Slack

Given:

- m parallel machines (resources)
- n activities
- r_j starting times (integers),
  d_j termination (integers),
  w_j or w_{ij} weight,
  M_j eligibility
- with slack $p_j \leq d_j - r_j$

Task: Maximize weight of assigned activities

Heuristics

Most constrained variable, least constraining value heuristic

$|M_j|$ indicates how much constrained an activity is

ν_{it}: # activities that can be assigned to i in $[t-1, t]$

Select activity j with smallest $I_j = f\left(\frac{w_j}{p_j}, |M_j|\right)$

Select resource i with smallest $g(\nu_{i,t+1}, \dots, \nu_{i,t+p_j})$ (or discard j if no place free for j)

Examples for f and g:

$$f\left(\frac{w_j}{p_j}, |M_j|\right) = \frac{|M_j|}{w_j/p_j}$$

$$g(\nu_{i,t+1}, \dots, \nu_{i,t+p_j}) = \max(\nu_{i,t+1}, \dots, \nu_{i,t+p_j})$$

$$g(\nu_{i,t+1}, \dots, \nu_{i,t+p_j}) = \sum_{l=1}^{p_j} \frac{\nu_{i,t+l}}{p_j}$$

Outline

Timetabling with one Operator

There is only one type of operator that processes all the activities

Example:

- A contractor has to complete n activities.
- The duration of activity j is p_j
- Each activity requires a crew of size W_j.
- The activities are not subject to precedence constraints.
- The contractor has W workers at his disposal
- His objective is to complete all n activities in minimum time.

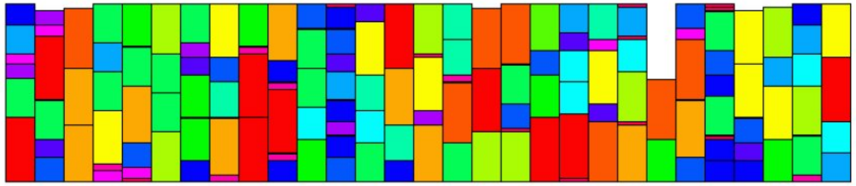
- RCPSP Model
- If p_j all the same ➜ Bin Packing Problem (still NP-hard)

Example: Exam scheduling

- Exams in a college with same duration.
- The exams have to be held in a gym with W seats.
- The enrollment in course j is W_j and
- all W_j students have to take the exam at the same time.
- The goal is to develop a timetable that schedules all n exams in minimum time.
- Each student has to attend a single exam.

- Bin Packing model
- In the more general (and realistic) case it is a RCPSP

Heuristics for Bin Packing



- Construction Heuristics
  - Best Fit Decreasing (BFD)
  - First Fit Decreasing (FFD) $C_{max}(FFD) \leq \frac{11}{9} C_{max}(OPT) + \frac{6}{9}$
- Local Search: [Alvim and Aloise and Glover and Ribeiro, 1999]
  - Step 1: remove one bin and redistribute items by BFD
  - Step 2: if infeasible, re-make feasible by redistributing items for pairs of bins, such that their total weights becomes equal (number partitioning problem)

[Levine and Ducatelle, 2004]

The solution before local search (the bin capacity is 10):
The bins: | 3 3 3 | 6 2 1 | 5 2 | 4 3 | 7 2 | 5 4 |

Open the two smallest bins:
Remaining: | 3 3 3 | 6 2 1 | 7 2 | 5 4 |
Free items: 5, 4, 3, 2

Try to replace 2 current items by 2 free items, 2 current by 1 free or 1 current by 1 free:
First bin: 3 3 3 → 3 5 2 new free: 4, 3, 3, 3
Second bin: 6 2 1 → 6 4 new free: 3, 3, 3, 2, 1
Third bin: 7 2 → 7 3 new free: 3, 3, 2, 2, 1
Fourth bin: 5 4 stays the same

Reinsert the free items using FFD:
Fourth bin: 5 4 → 5 4 1
Make new bin: 3 3 2 2
Final solution: | 3 5 2 | 6 4 | 7 3 | 5 4 1 | 3 3 2 2 |

Repeat the procedure: no further improvement possible