DMP204
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 2

Outline

1. Complexity Hierarchies

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1. Complexity Hierarchies

Polynomial time solvable problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid r_j, p_j = 1, prec \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid L_{\max}$	$O2 \mid\mid C_{\max}$
$1 \mid r_j, prmp \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid \sum C_j$	
$1 \mid tree \mid \sum w_j C_j$		$Om \mid r_j, prmp \mid L_{\max}$
	$Pm \mid p_j = 1, tree \mid C_{\max}$	
$1 \mid prec \mid L_{\max}$	$Pm \mid prmp, tree \mid C_{\max}$	$F2 \mid block \mid C_{\max}$
$1 \mid r_j, prmp, prec \mid L_{\max}$	$Pm \mid p_j = 1, outtree \mid \sum C_j$	$F2 \mid nwt \mid C_{\max}$
	$Pm \mid p_j = 1, intree \mid L_{\max}$	
$1 \mid\mid \sum U_j$	$Pm \mid prmp, intree \mid L_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum C_j$
$1 \mid r_j, prmp \mid \sum U_j$		$Fm \mid p_{ij} = p_j \mid L_{\max}$
$1 \mid r_j, p_j = 1 \mid \sum w_j U_j$	$Q2 \mid prmp, prec \mid C_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum U_j$
	$Q2 \mid r_j, prmp, prec \mid L_{\max}$	
$1 \mid r_j, p_j = 1 \mid \sum w_j T_j$		$J2 \mid\mid C_{\max}$
	$Qm \mid r_j, p_j = 1 \mid C_{\max}$	
	$Qm \mid p_j = 1, M_j \mid C_{\max}$	
	$Qm \mid r_j, p_j = 1 \mid \sum C_j$	
	$Qm \mid prmp \mid \sum C_j$	
	$Qm \mid p_j = 1 \mid \sum w_j C_j$	
	$Qm \mid p_j = 1 \mid L_{\max}$	
	$Qm \mid prmp \mid \sum U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j T_j$	
	$Rm \mid\mid \sum C_j$	
	$Rm \mid r_j, prmp \mid L_{\max}$	

NP-hard problems in the ordinary sense

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid\mid \sum w_j U_j \quad (*)$ $1 \mid r_j, prmp \mid \sum w_j U_j \quad (*)$ $1 \mid\mid \sum T_j \quad (*)$	$P2 \mid\mid C_{\max} \quad (*)$ $P2 \mid r_j, prmp \mid \sum C_j$ $P2 \mid\mid \sum w_j C_j \quad (*)$ $P2 \mid r_j, prmp \mid \sum U_j$ $Pm \mid prmp \mid \sum w_j C_j$ $Qm \mid\mid \sum w_j C_j \quad (*)$ $Rm \mid r_j \mid C_{\max} \quad (*)$ $Rm \mid\mid \sum w_j U_j \quad (*)$ $Rm \mid prmp \mid \sum w_j U_j$	$O2 \mid prmp \mid \sum C_j$ $O3 \mid\mid C_{\max}$ $O3 \mid prmp \mid \sum w_j U_j$

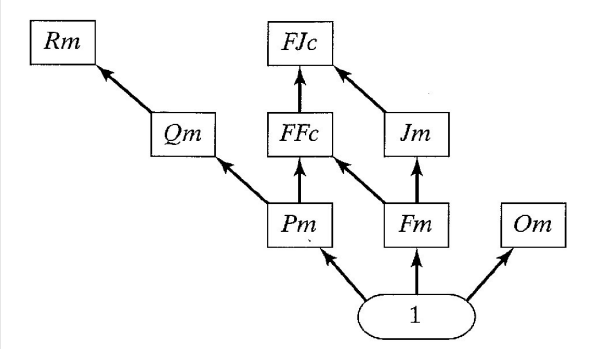
Strongly NP-hard problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid s_{jk} \mid C_{\max}$ $1 \mid r_j \mid \sum C_j$ $1 \mid prec \mid \sum C_j$ $1 \mid r_j, prmp, tree \mid \sum C_j$ $1 \mid r_j, prmp \mid \sum w_j C_j$ $1 \mid r_j, p_j = 1, tree \mid \sum w_j C_j$ $1 \mid p_j = 1, prec \mid \sum w_j C_j$ $1 \mid r_j \mid L_{\max}$ $1 \mid r_j \mid \sum U_j$ $1 \mid p_j = 1, chains \mid \sum U_j$ $1 \mid r_j \mid \sum T_j$ $1 \mid p_j = 1, chains \mid \sum T_j$ $1 \mid\mid \sum w_j T_j$	$P2 \mid chains \mid C_{\max}$ $P2 \mid chains \mid \sum C_j$ $P2 \mid prmp, chains \mid \sum C_j$ $P2 \mid p_j = 1, tree \mid \sum w_j C_j$ $R2 \mid prmp, chains \mid C_{\max}$	$F2 \mid r_j \mid C_{\max}$ $F2 \mid r_j, prmp \mid C_{\max}$ $F2 \mid\mid \sum C_j$ $F2 \mid prmp \mid \sum C_j$ $F2 \mid\mid L_{\max}$ $F2 \mid prmp \mid L_{\max}$ $F3 \mid\mid C_{\max}$ $F3 \mid prmp \mid C_{\max}$ $F3 \mid nwt \mid C_{\max}$ $O2 \mid r_j \mid C_{\max}$ $O2 \mid\mid \sum C_j$ $O2 \mid prmp \mid \sum w_j C_j$ $O2 \mid\mid L_{\max}$ $O3 \mid prmp \mid \sum C_j$ $J2 \mid rcrc \mid C_{\max}$ $J3 \mid p_{ij} = 1, rcrc \mid C_{\max}$

http://www.mathematik.uni-osnabrueck.de/research/OR/class/

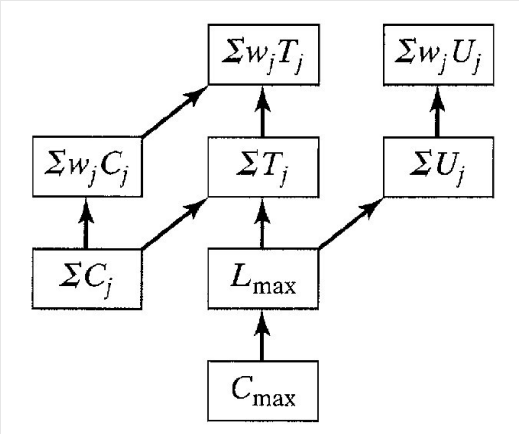
Complexity Hierarchy

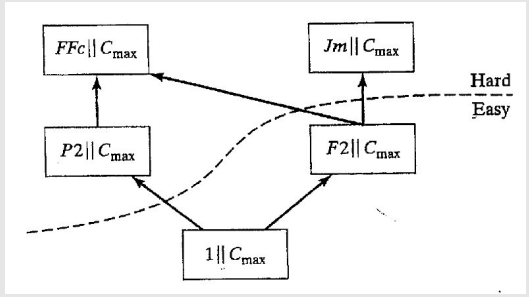
Elementary reductions for machine environment

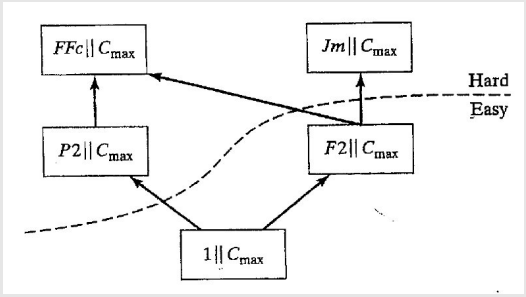


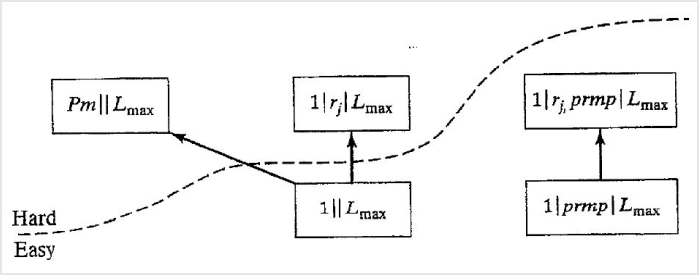
Complexity Hierarchy

Elementary reductions for regular objective functions




$FFc||C_{max}$
$Jm||C_{max}$
Hard
Easy
$P2||C_{max}$
$F2||C_{max}$
$1||C_{max}$


$FFc||C_{max}$
$Jm||C_{max}$
Hard
Easy
$P2||C_{max}$
$F2||C_{max}$
$1||C_{max}$


$Pm||L_{max}$
$1|r_j|L_{max}$
$1|r_j\,prmp|L_{max}$
$1||L_{max}$
$1|prmp|L_{max}$
Hard
Easy