Department of Mathematics and Computer Science
University of Southern Denmark, Odense

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Marco Chiarandini

DM533 - Introduction to Artificial Intelligence

Assignment 2, Fall 2009

The submission deadline for this assignment is **11.00 of Monday, December 14**.

The following assignment does not include programming, henc eit can be hand written. Unless differently notified in the next days, for submission instructions, still refer to Assignment 0 and use only the `doc` directory where you can put your answers scanned. Failure to submit properly could result in a non-passed assignment. It is not required in order to pass the assignment that all exercises are carried out correctly but do an honest effort at most of the questions, possibly all.

Exercises that do not involve programming, are examples of exercises that might appear at the exam. This assignment assumes knowledge from chapter 13, 14, 15 of the course book.

Exercises

1. Given the full joint distribution in Figure 13.3 of the course book, calculate the following:

   - $P(toothache)$
   - $P(Cavity)$
   - $P(Toothache|cavity)$
   - $P(Cavity|toothache \vee catch)$

2. Show that the statement

   $$P(A, B|C) = P(A|C)P(B|C)$$

   is equivalent to either of the statements

   $$P(A|B, C) = P(A|C) \qquad \text{and} \qquad P(B|A, C) = P(B|C)$$

3. Often we need to carry out reasoning over some pair of variables X, Y conditioned on the value of other variable E.

   a Using the definitions of conditional probabilities, prove the conditionalized version of the product rule:

   $$P(x, y|e) = P(x|y, e)P(y|e)$$

   b Prove the conditionalized version of Bayes' rule:

   $$P(y|x, e) = P(x|y, e)P(y|e)/P(x|e)$$

4. Following the example on medical diagnosis of page 480 of the text book, give the probability $P(M|s)$ by completing the normalization calculation. Assume 0.05 as a reasonable value for $P(s|\neg m)$ and ignore the value for $P(s)$.

5. Three prisoners A, B, C are in their cells. They are told that one of them will be executed the next day and the others will be pardoned. Only the gorvenor knows who will be executed. Prisoner A asks the guard a favor. "Please ask the governor who will be executed, and then tell either prisoner B or C that they will be pardoned." The guard does as was asked and then comes back and tells prisoner A that he has told prisoner B that he (B) will be pardoned. What are prisoner A's chances of being executed, given this message? Is there more information than before his request to the guard? [This problem adapted from Pearl 1988, is also a variant of a rather famous puzzle you're given the choice of three doors. Behind one door is a car; behind the others, goats. They are placed randomly. After you have chosen a door, the door remains closed. Another person, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. You are now posed with the question, whether you want to stay with your first choice or to switch to the last remaining door. Hint: try using Bayesian inference.]

6. Suppose that you go out to purchase an automobile. The probability that you will go to dealer 1, d_1 is 0.2. The probability of going to dealer 2, d_2 is 0.4. There are only three dealers you are considering and the probability that you go to the third, d_3 is 0.4. At d_1 the probability of purchasing a particular automobile, a_1, is 0.2.; at dealer d_2 the probability of purchasing a automobile a_1 is 0.4. Finally, at dealer d_3, the probability of purchasing a_1 is 0.3. Suppose you purchase automobile a_1. What is the probability that you purchased it at dealer d_2?

7. Exercise 14.3 of the text book.

8. Exercise 14.11 of the text book.

9. Exercise 15.1 of the text book.

10. This example is a slightly modified version of one taken from Wikipedia. Consider two friends, Alice and Bob, who live far apart from each other and who talk together daily over the telephone about what they did that day. Bob is only interested in two activities: walking in the park or staying at home and reading. The choice of what to do is determined exclusively by the weather on a given day. Alice has no definite information about the weather where Bob lives, but she knows general trends. Based on what Bob tells her he did each day, Alice tries to guess what the weather must have been like.

    Alice believes that the weather of a day depends only on the weather of the previous day. There are three states, "Rainy", "Sunny" and "Foggy", but she cannot observe them directly. On each day, there is a certain chance that Bob will perform one of the following activities, depending on the weather: "walk" or "read". Bob tells Alice about his activities.

```
states = ('Rainy', 'Sunny')

observations = ('walk', 'read')
```

```
transition_probability = {
   'Rainy' : {'Rainy': 0.6, 'Sunny': 0.2, 'Foggy':0.2},
   'Sunny' : {'Rainy': 0.05, 'Sunny': 0.8, 'Foggy':0.15},
   'Foggy' : {'Rainy': 0.3, 'Sunny': 0.2, 'Foggy':0.5},
   }

emission_probability = {
   'Sunny' : {'walk': 0.9, 'read': 0.1},
   'Rainy' : {'walk': 0.2, 'read': 0.8},
   'Foggy' : {'walk': 0.7, 'read': 0.3},
   }
```

Derive manually the computations for the answering the following questions. (If you wish to ascertain the correctness of the numerical result you may try to modify the Viterbi algorithm whose python code is available from Wikipedia.)

a Suppose that where Bob lives one day the weather was sunny. The next day, Bob tells you that he read. Assuming that the prior probability that he goes walking or reading each day is 0.5, what is the probability that the next day it was rainy?

b Suppose that where Bob lives one day it is foggy; Bob tells you that the successive day (day 2) he read, but that he went walking on day 3. Again, assuming that the prior probability of Bob reading or walking is 0.5, what is the probability that it is foggy on day 3?