Lecture 6

Logical Agents
First Order Logic

Marco Chiarandini

Deptartment of Mathematics & Computer Science
University of Southern Denmark

Slides by Stuart Russell and Peter Norvig

Course Overview

- ✔ Introduction
  - ✔ Artificial Intelligence
  - ✔ Intelligent Agents
- ✔ Search
  - ✔ Uninformed Search
  - ✔ Heuristic Search
- ✔ Adversarial Search
  - ✔ Minimax search
  - ✔ Alpha-beta pruning
- Knowledge representation and Reasoning
  - Propositional logic
  - First order logic
  - Inference
- Uncertain knoweldge and Reasoning
  - Probability and Bayesian approach
  - Bayesian Networks
  - Hidden Markov Chains
  - Kalman Filters
- Learning
  - Decision Trees
  - Maximum Likelihood
  - EM Algorithm
  - Learning Bayesian Networks
  - Neural Networks
  - Support vector machines

Last Time

- ◇ Knowledge-based agents
- ◇ Wumpus world
- ◇ Logic in general—models and entailment
- ◇ Propositional (Boolean) logic
- ◇ Equivalence, validity, satisfiability
- ◇ Inference rules and theorem proving
  - – resolution – forward chaining
  - – backward chaining
- ◇ Model checking

Outline

1. First Order Logic

Outline

◇ Why FOL?
◇ Syntax and semantics of FOL
◇ Fun with sentences
◇ Wumpus world in FOL

Pros and cons of propositional logic

☺ Propositional logic is **declarative**: pieces of syntax correspond to facts

☺ Propositional logic allows partial/disjunctive/negated information
(unlike most data structures and databases)

☺ Propositional logic is **compositional**:
meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

☺ Meaning in propositional logic is **context-independent**
(unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power
(unlike natural language)
E.g., cannot say "pits cause breezes in adjacent squares"
except by writing one sentence for each square

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries ...
- Relations/Predicates: red, round, bogus, prime, multistoried ..., brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, likes, friends, ...
- Functions: father of, best friend, successor, one more than, times, end of ...

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Functions	$Sqrt,\ Father \ldots$
Predicates	$BrotherOf,\ >, \ldots$
Connectives	$\land\ \lor\ \neg\ \implies\ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Note: constants, variables, predicates are distinguished typically by the case of the letters. Every system/book has diffrent conventions in this regard. PROLOG: costants in lower case and variables in upper case.

Atomic sentences

Atomic sentence $=$ $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $=$ $function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$

$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

But: E.g., $Plus(2, 3)$ is a function, not an atomic sentence.

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \land S_2, \quad S_1 \lor S_2, \quad S_1 \implies S_2, \quad S_1 \Leftrightarrow S_2$$

E.g. $Sibling(KingJohn, Richard) \implies Sibling(Richard, KingJohn)$

$>(1, 2) \lor \leq(1, 2)$

$>(1, 2) \land \neg{>}(1, 2)$

E.g., $Equal(Plus(2, 3), Seven))$

Semantics in first-order logic

Sentences are true with respect to an interpretation over a domain D.

DEFINITION

INTERPRETATION

Let the domain D be a nonempty set.

An *interpretation* over D is an assignment of the entities of D to each of the constant, variable, predicate, and function symbols of a predicate calculus expression, such that:

1. Each constant is assigned an element of D.
2. Each variable is assigned to a nonempty subset of D; these are the allowable substitutions for that variable.
3. Each function f of arity m is defined on m arguments of D and defines a mapping from D^m into D.
4. Each predicate p of arity n is defined on n arguments from D and defines a mapping from D^n into {T, F}.

Truth Value Assignment

Symbols in FOL are assigned values from the domain D as determined by the interpretation. Each precise assignment is a model

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$ in the interpretation
Example:
Consider the interpretation in which

$Richard \rightarrow$ Richard the Lionheart
$John \rightarrow$ the evil King John
$Brother \rightarrow$ the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true
just in case Richard the Lionheart and the evil King John
are in the brotherhood relation in the model (the assignment of values of the world to objects according to the interpretation)

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models
We **can** enumerate the FOL models for a given KB vocabulary.

But:

Sentences with quantifiers:
Eg. $\forall X(p(X) \vee q(Y)) \implies r(X))$
It requires checking truth by substituting all values that X can take in the subset of D assigned to X in the interpretation

Since the set maybe infinite predicate calculus is said to be undecidable

Existential quantifiers are not easier to check

Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \ \ At(x, Berkeley) \implies Smart(x)$

$\forall x \ \ P$ is true in a model iff P is true with x being
each possible object in the model
(**Roughly** speaking, equivalent to the conjunction of instantiations of P)

$$
\begin{array}{ll}
 & (At(KingJohn, Berkeley) \implies Smart(KingJohn)) \\
\wedge & (At(Richard, Berkeley) \implies Smart(Richard)) \\
\wedge & (At(Berkeley, Berkeley) \implies Smart(Berkeley)) \\
\wedge & \ldots
\end{array}
$$

Note: quantifiers are only on objects and variables, not on predicates and functions. This is done in higher order logic.
Eg.: $\forall(Likes)Likes(Geroge, Kate)$

A common mistake to avoid

Typically, $\implies$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$\forall x \ \ At(x, Berkeley) \wedge Smart(x)$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Someone at Stanford is smart:
$\exists x \ At(x, Stanford) \wedge Smart(x)$

$\exists x \ P$ is true in a model iff P is true with x being **some** possible object in the model
(**Roughly** speaking, equivalent to the disjunction of instantiations of P)

$$\begin{array}{ll} & (At(KingJohn, Stanford) \wedge Smart(KingJohn)) \\ \vee & (At(Richard, Stanford) \wedge Smart(Richard)) \\ \vee & (At(Stanford, Stanford) \wedge Smart(Stanford)) \\ \vee & \ldots \end{array}$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\implies$ as the main connective with $\exists$:

$$\exists x \ At(x, Stanford) \implies Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

- $\forall x \ \forall y$ is the same as $\forall y \ \forall x$
- $\exists x \ \exists y$ is the same as $\exists y \ \exists x$
- $\exists x \ \forall y$ is **not** the same as $\forall y \ \exists x$
- $\exists x \ \forall y \ Loves(x, y)$
  "There is a person who loves everyone in the world"
  $\forall y \ \exists x \ Loves(x, y)$
  "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other
  $\forall x \ Likes(x, IceCream) \qquad \neg\exists x \ \neg Likes(x, IceCream)$
  $\exists x \ Likes(x, Broccoli) \qquad \neg\forall x \ \neg Likes(x, Broccoli)$

Exercise

Translating natural language in FOL

Brothers are siblings
$\forall x, y \ Brother(x, y) \implies Sibling(x, y)$.

"Sibling" is symmetric
$\forall x, y \ Sibling(x, y) \Leftrightarrow Sibling(y, x)$.

One's mother is one's female parent
$\forall x, y \ Mother(x, y) \Leftrightarrow (Female(x) \wedge Parent(x, y))$.

A first cousin is a child of a parent's sibling
$\forall x, y \ FirstCousin(x, y) \Leftrightarrow \exists p, ps \ Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$

Note: there is not an unique way of translating
If it does not rain on Monday, Tom will go to the mountains
$\neg weather(rain, mountain) \implies go(tom, mountains)$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall x \;\; \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid

E.g., definition of (full) $Sibling$ in terms of $Parent$:
$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; [\neg(x = y) \wedge \exists m, f \;\; \neg(m = f) \wedge$
$Parent(m, x) \wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$

Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$Tell(KB, Percept([Smell, Breeze, None], 5))$
$Ask(KB, \exists a \;\; Action(a, 5))$

I.e., does KB entail any particular actions at $t = 5$?

Answer: $Yes, \; \{a/Shoot\}$ $\quad\leftarrow$ substitution (binding list)

Given a sentence S and a substitution σ,
$S\sigma$ denotes the result of plugging σ into S; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$
$Ask(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Deducing hidden properties

Properties of locations:
$\forall x, t \;\; At(Agent, x, t) \wedge Smelt(t) \implies Smelly(x)$
$\forall x, t \;\; At(Agent, x, t) \wedge Breeze(t) \implies Breezy(x)$

Squares are breezy near a pit:
Diagnostic rule—infer cause from effect
$\forall y \;\; Breezy(y) \implies \exists x \;\; Pit(x) \wedge Adjacent(x, y)$
Causal rule—infer effect from cause
$\forall x, y \;\; Pit(x) \wedge Adjacent(x, y) \implies Breezy(y)$
Neither of these is complete—e.g., the causal rule doesn't say whether
squares far away from pits can be breezy

Definition for the $Breezy$ predicate:
$\forall y \;\; Breezy(y) \;\Leftrightarrow\; [\exists x \;\; Pit(x) \wedge Adjacent(x, y)]$

Knowledge base for the wumpus world

"Perception"
$\forall b, g, t \;\; Percept([Smell, b, g], t) \implies Smelt(t)$
$\forall s, b, t \;\; Percept([s, b, Glitter], t) \implies AtGold(t)$

Reflex: $\forall t \;\; AtGold(t) \implies Action(Grab, t)$

Reflex with internal state: do we have the gold already?
$\forall t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \implies Action(Grab, t)$

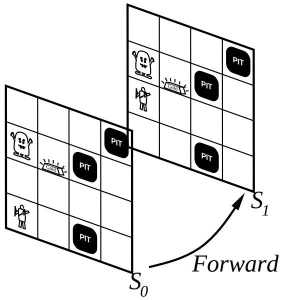
$Holding(Gold, t)$ cannot be observed
$\Rightarrow$ keeping track of change is essential

Keeping track of change

Facts hold in situations, rather than eternally
E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

Situation calculus is one way to represent change in FOL:
Adds a situation argument to each non-eternal predicate
E.g., Now in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing a in s



Describing actions I

- "Effect" axiom—describe changes due to action
  $\forall\, s \;\; AtGold(s) \implies Holding(Gold, Result(Grab, s))$
- "Frame" axiom—describe **non-changes** due to action
  $\forall\, s \;\; HaveArrow(s) \implies HaveArrow(Result(Grab, s))$

Frame problem: find an elegant way to handle non-change
(a) representation—avoid frame axioms
(b) inference—avoid repeated "copy-overs" to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .

Describing actions II

Successor-state axioms solve the representational frame problem
Each axiom is "about" a **predicate** (not an action per se):

P true afterwards	$\Leftrightarrow$	[an action made P true
	$\vee$	P true already and no action made P false]

For holding the gold:

$$\forall\, a, s \;\; Holding(Gold, Result(a, s)) \Leftrightarrow \\ [(a = Grab \wedge AtGold(s)) \\ \vee (Holding(Gold, s) \wedge a \neq Release)]$$

Making plans

Initial condition in KB:
$At(Agent, [1, 1], S_0)$
$At(Gold, [1, 2], S_0)$

Query: $Ask(KB, \exists\, s \;\; Holding(Gold, s))$
i.e., in what situation will I be holding the gold?

Answer: $\{s/Result(Grab, Result(Forward, S_0))\}$
i.e., go forward and then grab the gold

This assumes that the agent is interested in plans starting at S_0 and that S_0 is the only situation described in the KB

Making plans: A better way

Represent plans as action sequences $p = [a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing p in s

Then the query $Ask(KB, \exists\, p\;\; Holding(Gold, PlanResult(p, S_0)))$
has the solution $\{p/[Forward, Grab]\}$

Definition of $PlanResult$ in terms of $Result$:
$\forall\, s\;\; PlanResult([\,], s) = s$
$\forall\, a, p, s\;\; PlanResult([a|p], s) = PlanResult(p, Result(a, s))$

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner

Knowledge Engineer

The one just saw is called knowledge engineer process.
It is the production of special-purpose knowledge systems, aka expert systems (eg, in medical diagnosis)

- Identify the task
- Assemble the relevant knowledge
- Decide on a vocabulary of predicates, functions, and constants
- Encode general knowledge about the domain
- Encode a description of the specific problem instance (input data)
  decide what is a constant, a predicate, a function
  leads to definition of the ontology of the domain (what kind of things exist)
- Pose queries to the inference procedure and get answers
- Debug the knowledge base

Summary

First-order logic:
– objects and relations are semantic primitives
– syntax: constants, functions, predicates, equality, quantifiers
Increased expressive power: sufficient to define wumpus world

Situation calculus:
– conventions for describing actions and change in FOL
– can formulate planning as inference on a situation calculus KB

Outline

◇ Reducing first-order inference to propositional inference
◇ Unification
◇ Generalized Modus Ponens
◇ Forward and backward chaining
◇ Logic programming
◇ Resolution