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DM811 - Heuristics for Combinatorial Optimization

Assignment Sheet 3, Fall 2009

Prepare for class discussion an answer to the following exercises. Work possibly in group. Due date: September 22.

Exercise 1

(Posed in class) In a 3-opt local search algorithm for the TSP how many possible ways are there to add three new edges once three edges have been removed in order to re-obtain an Hamiltonian tour? Justify your answer.

Exercise 2

(Posed in class) A possible application of local search to the GCP defines the following:

- solution representation: k-variable, complete unproper coloring;
- neighborhood: one-exchange;
- evaluation function: $-\sum_{i=1}^{k} |C_i|^2 + \sum_{i=1}^{k} 2|C_i||E_i|$

Show that any local optimum of this function corresponds to a feasible coloring. Does a global optimum correspond to a coloring that use the minimal possible number of colors?

Exercise 3

Make a randomized version of the construction heuristic for the set covering problem that was presented in class.

Exercise 4

Make a Venn diagram (set diagram) representing the relation between the following classes of algorithms: construction heuristics, metaheuristics, local search, stochastic local search, (best) iterative improvement, (first) iterative improvement, uninformed random walk.

Exercise 5

We defined iterative improvement such that a neighbor is only accepted if it has strictly better cost. Give a reason not to accept a neighbor that has equal cost.

Exercise 6

The Steiner tree problem is a generalization of the minimum spanning tree problem in that it asks for a spanning tree covering the vertices of a set U. Extra intermediate vertices and edges may be added to the graph in order to reduce the length of the spanning tree. These new vertices introduced to decrease the total length of the connection are called Steiner[1] vertices.

Definition 1 Steiner Tree Problem [2]
Input: *A graph $G = (V, E)$, a weight function $\omega : E \mapsto \mathbf{N}$, and a subset $U \subseteq V$.*
Task: *Find a Steiner tree, that is, a subtree $T = (V_T, E_T)$ of G that includes all the vertices of U and such that the sum of the weights of the edges in the subtree is minimal.*

The example in Figure 1 is an instance of the Euclidean Steiner problem showing that the use of Steiner vertices may help to obtain cheaper subtrees including all vertices from U.

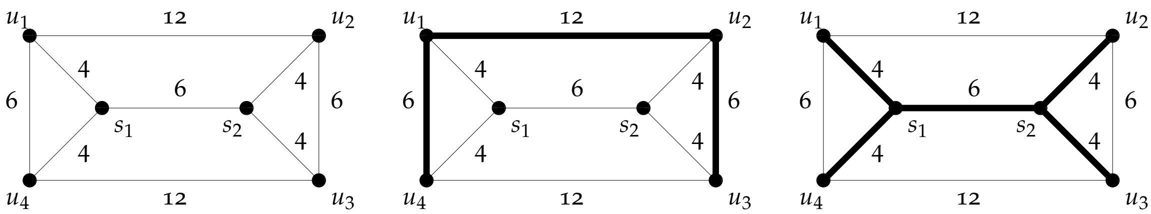

Figure 1: Vertices u_1, u_2, u_3, u_4 belong to the set U of special vertices to be covered and vertices s_1, s_2 belong to the set S of Steiner vertices. The Steiner tree in the second graph has cost 24 while the one in the third graph has cost 22.

1. Design one or more local search algorithms for the Steiner tree problem. In particular, define the solution representation and the neighborhood function.
2. Provide an analysis of the computational cost of the basic operations in the local search algorithms designed at the previous point. In particular, consider the size of the neighborhood, and the cost of evaluating a neighbor.

Exercise 7

Definition 2 Total Weighted Completion Time on Unrelated Parallel Machines Problem
Input: *A set of jobs J to be processed on a set of parallel machines M. Each job $j \in J$ has a weight w_j and processing time p_{ij} that depends on the machine $i \in M$ on which it is processed.*
Task: *Find a schedule of the jobs on the machines such that the sum of weighted completion time of the jobs is minimal.*

Design a local search algorithm for it.

[1] Jakob Steiner (18 March 1796 April 1, 1863) was a Swiss mathematician.
[2] It is recommendable to search information on the problems posed, above all about the proof of their hardness. However, to maximize the positive effect of the exercises, it should be preferable to search information after you understood the problem and answered the questions.

Exercise 8

Definition 3 BIN PACKING PROBLEM
Input: *A finite set U of items, a size $s(u) \in Z^+$ for each $u \in U$, and a positive integer bin capacity B.*
Task: *Find the minimal number of bins K for which there exits a partition of U into disjoint sets $U_1, U_2, \ldots, U_k$ and the sum of the sizes of the items in each U_i is B or less.*

Definition 4 TWO-DIMENSIONAL BIN PACKING
Input: *A finite set U of rectangular items, each with a width $w_u \in Z^+$ and a height $h_u \in Z^+$, $u \in U$, and an unlimited number of identical rectangular bins of width $W \in Z^+$ and height $H \in Z^+$.*
Task: *Allocate all the items into a minimum number of bins, such that the bin widths and heights are not exceeded and the original orientation is respected (no rotation of the items is allowed).*

1. Design construction heuristics for the two problems.
2. Design local search algorithms for the two problems focusing on solution representation and neighborhood function.

Exercise 9

Definition 5 P-MEDIAN PROBLEM
Input: *A set U of locations for n* users, *a set F of locations for m* facilities *and a distance matrix $D = [d_{ij}] \in \mathbf{R}^{n \times m}$.*
Task: *Select a set $J \subseteq F$ of p locations where to install facilities such that the sum of the distances of each user to its closest installed facility is minimized, i.e.,*

$$\min \left\{ \sum_{i \in U} \min_{j \in J} d_{ij} \mid J \subseteq F \text{ and } |J| = p \right\}$$

Design a simple construction heuristic and a simple local search algorithm.

Exercise 10

In an iterative improvement algorithm for GCP that defines:

- solution representation: k-fixed, complete unproper coloring;
- neighborhood: one-exchange;
- evaluation function: number of edges causing a violation;

provide a computational analysis for the cost of examining the neighborhood and select the best neighbor.