DM204

SCHEDULING, TIMETABLING AND ROUTING

Lecture 1

Introduction to Scheduling: Terminology and Classification

Marco Chiarandini

Deptartment of Mathematics & Computer Science
University of Southern Denmark

Outline

Outline

Course Content

- Scheduling (Manufacturing)
  - Single and Parallel Machine Models
  - Flow Shops and Flexible Flow Shops
  - Job Shops
  - Resource-Constrained Project Scheduling
- Timetabling (Services)
  - Interval Scheduling, Reservations
  - Educational Timetabling
  - Crew, Workforce and Employee Timetabling
  - Transportation Timetabling
- Vehicle Routing
  - Capacited Vehicle Routing
  - Vehicle Routing with Time Windows
- General Optimization Methods
  - Mathematical Programming
  - Constraint Programming
  - Heuristics
  - Problem Specific Algorithms
    (Dynamic Programming, Branch and Bound, ...)

Course Overview

- Problem Introduction
  - Scheduling classification
  - Scheduling complexity
  - Timetabling
  - Vehicle Routing
- General Methods
  - Integer Programming
  - Constraint Programming
  - Heuristics
  - Dynamic Programming and Branch and Bound
- Scheduling
  - Single Machine
  - Parallel Machine and Flow Shop Models
  - Job Shop
  - Resource Constrained Project Scheduling Model
- Timetabling
  - Reservations and Education
  - University Timetabling
  - Crew Scheduling
  - Public Transports
- Vechicle Routing
  - Capacited Models
  - Time Windows models
  - Rich Models

Course Presentation

- Lecture plan and Schedule
- Communication tools
  - Course Public Web Site (WS) ⇔ Blackboard (Bb)
    (public web site: http://www.imada.sdu.dk/~marco/DM204/)
  - Announcements (Bb)
  - Discussion board or Blog (Bb)
  - Personal email (Bb)
  - My office in working hours (8-16).

Course Presentation

- Final Assessment (10 ECTS)
  - Oral exam: 30 minutes + 5 minutes defense project
    meant to assess the base knowledge
  - Project:
    Deliverables: program + report
    meant to assess the ability to apply
- Schedule:
  - Project start: end third quarter
  - Project hand in deadline: half June
  - Oral exam: June

Course Material

- Literature
  - B1 Pinedo, M. Planning and Scheduling in Manufacturing and Services Springer Verlag, 2005
    available online
  - B2 Pinedo, M. Scheduling: Theory, Algorithms, and Systems Springer New York, 2008
    available online
  - B3 Toth, P. & Vigo, D. (ed.) The Vehicle Routing Problem SIAM Monographs on Discrete Mathematics and Applications, 2002
    photocopies
  - B4 Comet Tutorial (see `doc/` in Comet Application)
- Articles and photocopies available from the web site
- Lecture slides
- Weekly assignments

General Guidelines

- Work at the weekly Assignments. Some imply coding
- The slides are a good reference for what has been done in class...
  but you will not find everything you need there:
  hence take notes!
- Revise and read the material before or after each lecture. Do not wait the end of the course!
- Use lectures as a place to pose questions and obtain feedback
- You will be asked for a mid-term evaluation of the course, take it seriously

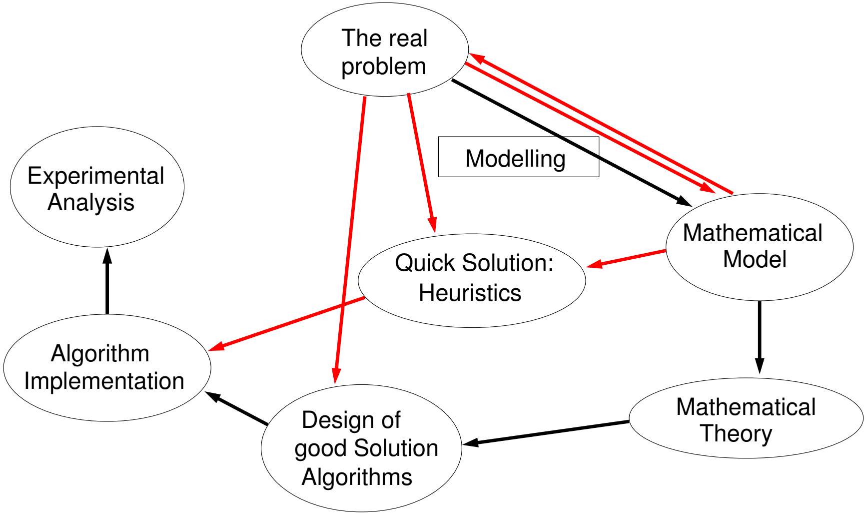
Course Goals

How to Tackle Real-life Optimization Problems:

- Formulate (mathematically) the problem
- Model the problem and recognize possible similar problems
- Search in the literature (or in the Internet) for:
  - complexity results (is the problem *NP*-hard?)
  - solution algorithms for original problem
  - solution algorithms for simplified problem
- Design solution algorithms and implement them
- Test experimentally with the goals of:
  - checking computational feasibility
  - configuring
  - comparing

Key ideas:
Decompose problems
Hybridize methods

The Problem Solving Cycle



Outline

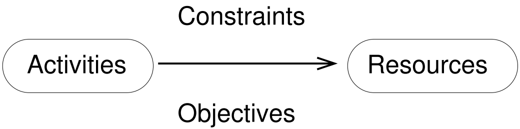
Scheduling

- Manufacturing
  - Project planning
  - Single, parallel machine and job shop systems
  - Flexible assembly systems
    Automated material handling (conveyor system)
  - Lot sizing
  - Supply chain planning

- Services
  - personnel/workforce scheduling
  - public transports

$\Rightarrow$ different models and algorithms

Problem Definition



Problem Definition

Given: a set of jobs $\mathcal{J} = \{J_1, \ldots, J_n\}$ to be processed by a set of machines $\mathcal{M} = \{M_1, \ldots, M_m\}$.
Let a schedule be a mapping of jobs to machines and processing times.

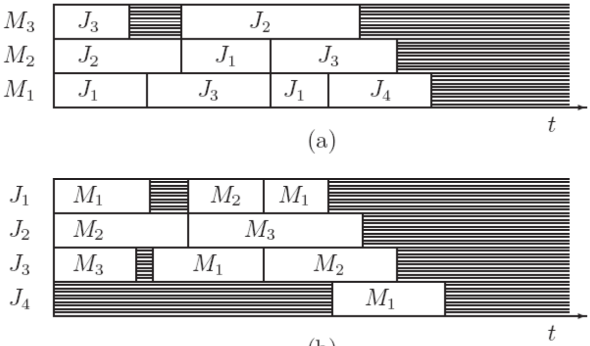
Task: Find a schedule that satisfies some constraints and is optimal w.r.t. some criteria.

Notation:
n, j, k jobs
m, i, h machines

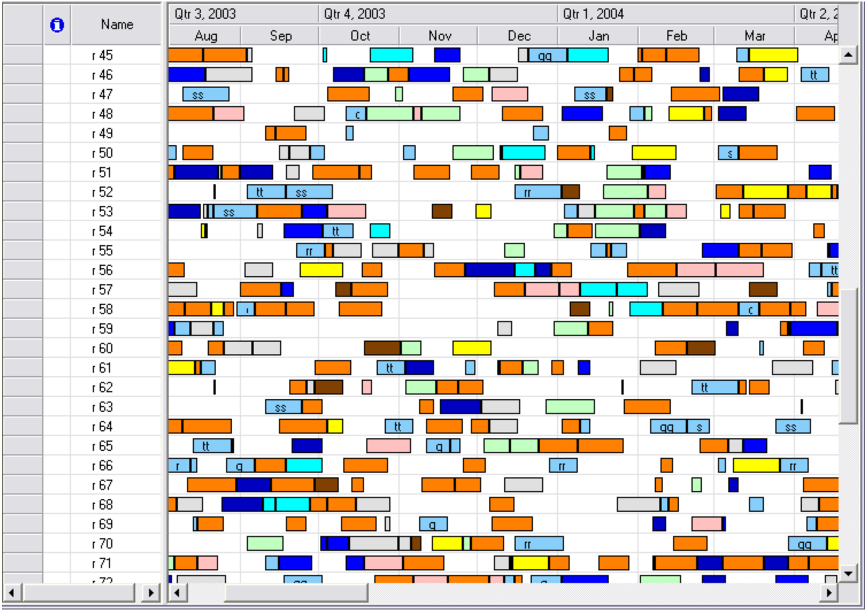
Visualization

Scheduling are represented by Gantt charts

(a) machine-oriented
(b) job-oriented



(a)
(b)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Data Associated to Jobs

- Processing time p_{ij}
- Release date r_j
- Due date d_j (called deadline, if strict)
- Weight w_j
- Cost function $h_j(t)$ measures cost of completing J_j at t
- A job J_j may also consist of a number n_j of operations $O_{j1}, O_{j2}, \dots, O_{jn_j}$ and data for each operation.
- A set of machines $\mu_{jl} \subseteq \mathcal{M}$ associated to each operation
  - $|\mu_{jl}| = 1$ dedicated machines
  - $\mu_{jl} = \mathcal{M}$ parallel machines
  - $\mu_{jl} \subseteq \mathcal{M}$ multipurpose machines

Data that depend on the schedule

- Starting times S_{ij}
- Completion time C_{ij}, C_j

Problem Classification

A scheduling problem is described by a triplet $\alpha \mid \beta \mid \gamma$.

- α machine environment (one or two entries)
- β job characteristics (none or multiple entry)
- γ objective to be minimized (one entry)

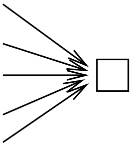
[R.L. Graham, E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan (1979): Optimization and approximation in deterministic sequencing and scheduling: a survey, Ann. Discrete Math. 4, 287-326.]

$\alpha \mid \beta \mid \gamma$ Classification Scheme

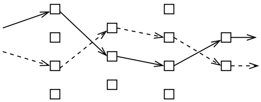
Machine Environment $\alpha_1\alpha_2 \mid \beta_1 \dots \beta_{13} \mid \gamma$

- single machine/multi-machine ($\alpha_1 = \alpha_2 = 1$ or $\alpha_2 = m$)
- parallel machines: identical ($\alpha_1 = P$), uniform p_j/v_i ($\alpha_1 = Q$), unrelated p_j/v_{ij} ($\alpha_1 = R$)
- multi operations models: Flow Shop ($\alpha_1 = F$), Open Shop ($\alpha_1 = O$), Job Shop ($\alpha_1 = J$), Mixed (or Group) Shop ($\alpha_1 = X$)

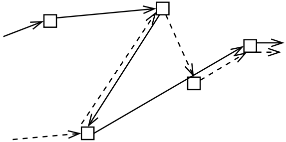
Single Machine



Flexible Flow Shop ($\alpha = FFc$)



Open, Job, Mixed Shop



$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics

$\alpha_1\alpha_2 \mid \boldsymbol{\beta_1 \ldots \beta_{13}} \mid \gamma$

- $\beta_1 = prmp$ presence of preemption (resume or repeat)
- β_2 precedence constraints between jobs acyclic digraph $G = (V, A)$
  - $\beta_2 = prec$ if G is arbitrary
  - $\beta_2 = \{chains, intree, outtree, tree, sp\text{-}graph\}$
- $\beta_3 = r_j$ presence of release dates
- $\beta_4 = p_j = p$ preprocessing times are equal
- ($\beta_5 = d_j$ presence of deadlines)
- $\beta_6 = \{s\text{-}batch, p\text{-}batch\}$ batching problem
- $\beta_7 = \{s_{jk}, s_{jik}\}$ sequence dependent setup times

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics (2) $\alpha_1\alpha_2 \mid \beta_1 \ldots \beta_{13} \mid \gamma$

- $\beta_8 = brkdwn$ machines breakdowns
- $\beta_9 = M_j$ machine eligibility restrictions (if $\alpha = Pm$)
- $\beta_{10} = prmu$ permutation flow shop
- $\beta_{11} = block$ presence of blocking in flow shop (limited buffer)
- $\beta_{12} = nwt$ no-wait in flow shop (limited buffer)
- $\beta_{13} = recrc$ recirculation in job shop

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective (always $f(C_j)$)

$\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Lateness $L_j = C_j - d_j$
- Tardiness $T_j = \max\{C_j - d_j, 0\} = \max\{L_j, 0\}$
- Earliness $E_j = \max\{d_j - C_j, 0\}$
- Unit penalty $U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective $\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Makespan: Maximum completion $C_{max} = \max\{C_1, \ldots, C_n\}$
  tends to max the use of machines
- Maximum lateness $L_{max} = \max\{L_1, \ldots, L_n\}$
- Total completion time $\sum C_j$ (flow time)
- Total weighted completion time $\sum w_j \cdot C_j$
  tends to min the av. num. of jobs in the system, ie, work in progress, or also the throughput time
- Discounted total weighted completion time $\sum w_j(1 - e^{-rC_j})$
- Total weighted tardiness $\sum w_j \cdot T_j$
- Weighted number of tardy jobs $\sum w_j U_j$

All regular functions (nondecreasing in $C_1, \ldots, C_n$) except E_i

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Other Objectives

$\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

Non regular objectives

- Min $\sum w_j' E_j + \sum w"_j T_j$ (just in time)
- Min waiting times
- Min set up times/costs
- Min transportation costs

Exercises

Gate Assignment at an Airport

- Airline terminal at a airport with dozes of gates and hundreds of arrivals each day.
- Gates and Airplanes have different characteristics
- Airplanes follow a certain schedule
- During the time the plane occupies a gate, it must go through a series of operations
- There is a scheduled departure time (due date)
- Performance measured in terms of on time departures.

Exercises

Scheduling Tasks in a Central Processing Unit (CPU)

- Multitasking operating system
- Schedule time that the CPU devotes to the different programs
- Exact processing time unknown but an expected value might be known
- Each program has a certain priority level
- Minimize expected time, ie, sum of the weighted completion times for all tasks
- Tasks are often sliced into little pieces. They are then rotated such that low priority tasks of short duration do not stay for ever in the system.

Exercises

Paper bag factory

- Basic raw material for such an operation are rolls of paper.
- Production process consists of three stages: (i) printing of the logo, (ii) gluing of the side of the bag, (iii) sewing of one end or both ends.
- Each stage consists of a number of machines which are not necessarily identical.
- Each production order indicates a given quantity of a specific bag that has to be produced and shipped by a committed shipping date or due date.
- Processing times for the different operations are proportional to the number of bags ordered.
- There are setup times when switching over different types of bags (colors, sizes) that depend on the similarities between the two consecutive orders
- A late delivery implies a penalty that depends on the importance of the order or the client and the tardiness of the delivery.

Solutions

Distinction between

- sequence
- schedule
- scheduling policy

Feasible schedule

A schedule is feasible if no two time intervals overlap on the same machine, and if it meets a number of problem specific constraints.

Optimal schedule

A schedule is optimal if it is feasible and it minimizes the given objective.

Classes of Schedules

Semi-active schedule

A feasible schedule is called semi-active if no operation can be completed earlier without changing the order of processing on any one of the machines. (local shift)

Active schedule

A feasible schedule is called active if it is not possible to construct another schedule by changing the order of processing on the machines and having at least one operation finishing earlier and no operation finishing later. (global shift without preemption)

Nondelay schedule

A feasible schedule is called nondelay if no machine is kept idle while an operation is waiting for processing. (global shift with preemption)

- There are optimal schedules that are nondelay for most models with regular objective function.
- There exists for $Jm||\gamma$ (γ regular) an optimal schedule that is active.
- nondelay $\Rightarrow$ active but active $\not\Rightarrow$ nondelay

Summary

- Scheduling Definitions (jobs, machines, Gantt charts)
- Classification
- Classes of schedules