**DM204**, 2010
SCHEDULING, TIMETABLING AND ROUTING

Lecture 19
# Flow Shop

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## Outline







## Course Overview

- ✔ Problem Introduction
  - ✔ Scheduling classification
  - ✔ Scheduling complexity
  - ✔ RCPSP
- ✔ General Methods
  - ✔ Integer Programming
  - ✔ Constraint Programming
  - ✔ Heuristics
  - ✔ Dynamic Programming
  - Branch and Bound
- Scheduling
  - Single Machine
  - Parallel Machine and Flow Shop Models
  - Job Shop
  - Resource Constrained Project Scheduling Model
- Timetabling
  - Reservations and Education
  - University Timetabling
  - Crew Scheduling
  - Public Transports
- Vechicle Routing
  - Capacited Models
  - Time Windows models
  - Rich Models





## Outline

# Flow Shop

General Shop Scheduling:

- $J = \{1, \ldots, N\}$ set of jobs; $M = \{1, 2, \ldots, m\}$ set of machines
- $J_j = \{O_{ij} \mid i = 1, \ldots, n_j\}$ set of operations for each job
- $p_{ij}$ processing times of operations $O_{ij}$
- $\mu_{ij} \subseteq M$ machine eligibilities for each operation
- precedence constraints among the operations
- one job processed per machine at a time, one machine processing each job at a time
- $C_j$ completion time of job $j$

➨ Find feasible schedule that minimize some regular function of $C_j$

Flow Shop Scheduling:

- $\mu_{ij} = l$, $l = 1, 2, \ldots, m$
- precedence constraints: $O_{ij} \rightarrow O_{i+1,j}$, $i = 1, 2, \ldots, n$ for all jobs

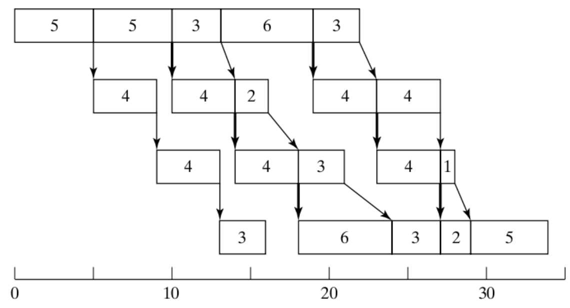




# Example

| *jobs* | $j_1$ | $j_2$ | $j_3$ | $j_4$ | $j_5$ |
|---|---|---|---|---|---|
| $p_{1,j_k}$ | 5 | 5 | 3 | 6 | 3 |
| $p_{2,j_k}$ | 4 | 4 | 2 | 4 | 4 |
| $p_{3,j_k}$ | 4 | 4 | 3 | 4 | 1 |
| $p_{4,j_k}$ | 3 | 6 | 3 | 2 | 5 |

Gantt chart




schedule representation

$\pi_1, \pi_2, \pi_3, \pi_4$:

$\pi_1 : O_{11}, O_{12}, O_{13}, O_{14}$
$\pi_2 : O_{21}, O_{22}, O_{23}, O_{24}$
$\pi_3 : O_{31}, O_{32}, O_{33}, O_{34}$
$\pi_4 : O_{41}, O_{42}, O_{43}, O_{44}$

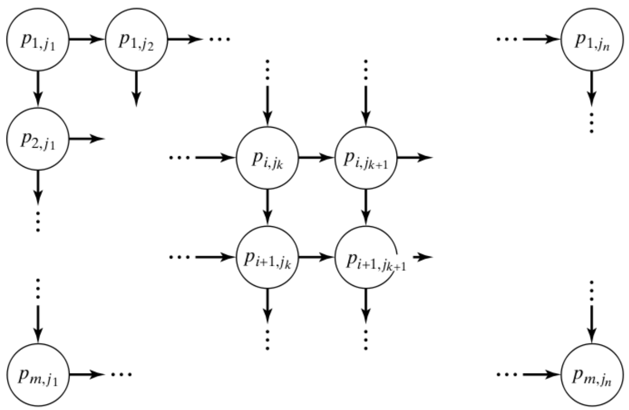
- we assume unlimited buffer
- if same job sequence on each machine ➨ permutation flow shop





# Directed Graph Representation

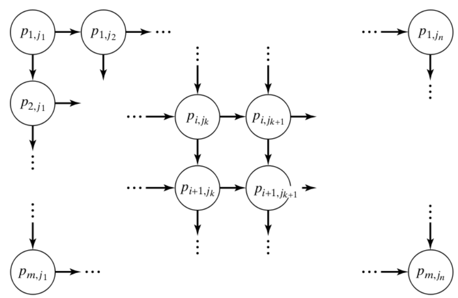
Given a sequence: operation-on-node network, jobs on columns, and machines on rows








# Directed Graph Representation




Recursion for $C_{max}$

$$C_{i,\pi(1)} = \sum_{l=1}^{i} p_{l,\pi(1)}$$

$$C_{1,\pi(j)} = \sum_{l=1}^{j} p_{l,\pi(l)}$$

$$C_{i,\pi(j)} = \max\{C_{i-1,\pi(j)}, C_{i,\pi(j-1)}\} + p_{i,\pi(j)}$$

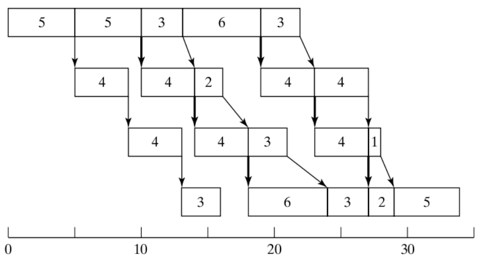
Computation cost?

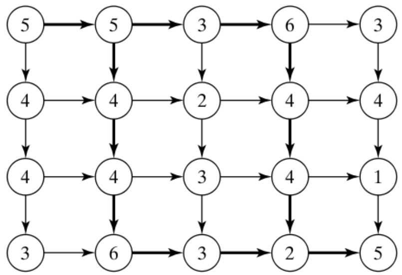
## Example

| jobs | $j_1$ | $j_2$ | $j_3$ | $j_4$ | $j_5$ |
|---|---|---|---|---|---|
| $p_{1,j_k}$ | 5 | 5 | 3 | 6 | 3 |
| $p_{2,j_k}$ | 4 | 4 | 2 | 4 | 4 |
| $p_{3,j_k}$ | 4 | 4 | 3 | 4 | 1 |
| $p_{4,j_k}$ | 3 | 6 | 3 | 2 | 5 |

$C_{max} = 34$

corresponds to longest path





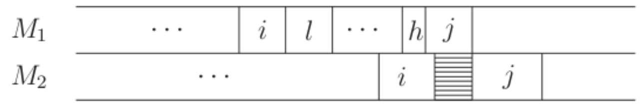


## $Fm \,|\,|\, C_{max}$

**Theorem**

*There always exist an optimum sequence without change in the first two and last two machines.*

**Proof:** By contradiction.




**Corollary**

*$F2 \,|\,|\, C_{max}$ and $F3 \,|\,|\, C_{max}$ are permutation flow shop*

**Note:** $F3 \,|\,|\, C_{max}$ is strongly NP-hard





## $F2 \,|\,|\, C_{max}$

**Intuition:** give something short to process to 1 such that 2 becomes operative and give something long to process to 2 such that its buffer has time to fill.

Construct a sequence $T : T(1), \ldots, T(n)$ to process in the same order on both machines by concatenating two sequences:
a left sequence $L : L(1), \ldots, L(t)$, and a right sequence $R : R(t+1), \ldots, R(n)$, that is, $T = L \circ R$

[Selmer Johnson, 1954, Naval Research Logistic Quarterly]

Let $J$ be the set of jobs to process
Let $T, L, R = \emptyset$
Step 1 Find $(i^*, j^*)$ such that $p_{i^*,j^*} = \min\{p_{ij} \mid i \in 1, 2, j \in J\}$
Step 2 If $i^* = 1$ then $L = L \circ \{i^*\}$
else if $i^* = 2$ then $R = R \circ \{i^*\}$
Step 3 $J := J \setminus \{j^*\}$
Step 4 If $J \neq \emptyset$ go to Step 1 else $T = L \circ R$





**Theorem**

*The sequence $T : T(1), , \ldots, T(n)$ is optimal.*

**Proof**

- Assume at one iteration of the algorithm that job $k$ has the min processing time on machine 1. Show that in this case job $k$ has to go first on machine 1 than any other job selected later.
- By contradiction, show that if in a schedule $S$ a job $j$ precedes $k$ on machine 1 and has larger processing time on 1, then $S$ is a worse schedule than $S'$.
  There are three cases to consider.
- Iterate the proof for all jobs in $L$.
- Prove symmetrically for all jobs in $R$.

# Construction Heuristics (1)

$Fm \,|\, prmu \,|\, C_{max}$

Slope heuristic

- schedule in decreasing order of $A_j = -\sum_{i=1}^{m}(m-(2i-1))p_{ij}$

Campbell, Dudek and Smith's heuristic (1970)

extension of Johnson's rule to when permutation is not dominant

- recursively create 2 machines 1 and $m-1$

$$p'_{ij} = \sum_{k=1}^{i} p_{kj} \qquad p''_{ij} = \sum_{k=m-i+1}^{m} p_{kj}$$

  and use Johnson's rule
- repeat for all $m-1$ possible pairings
- return the best for the overall $m$ machine problem





# Construction Heuristics (2)

$Fm \,|\, prmu \,|\, C_{max}$

Nawasz, Enscore, Ham's heuristic (1983)

Step 1: order in decreasing $\sum_{i=1}^{m} p_{ij}$

Step 2: schedule the first 2 jobs at best

Step 3: insert all others in best position

Implementation in $O(n^2 m)$

[Framinan, Gupta, Leisten (2004)] examined 177 different arrangements of jobs in Step 1 and concluded that the NEH arrangement is the best one for $C_{max}$.





# Iterated Greedy

$Fm \,|\, prmu \,|\, C_{max}$

Iterated Greedy [Ruiz, Stützle, 2007]

Destruction: remove $d$ jobs at random

Construction: reinsert them with NEH heuristic in the order of removal

Local Search: insertion neighborhood
(first improvement, whole evaluation $O(n^2 m)$)

Acceptance Criterion: random walk, best, SA-like

Performance on up to $n = 500 \times m = 20$ :

- NEH average gap 3.35% in less than 1 sec.
- IG average gap 0.44% in about 360 sec.



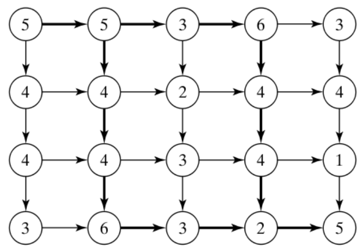


# Efficient local search for $Fm \,|\, prmu \,|\, C_{max}$

Tabu search (TS) with insert neighborhood.

TS uses best strategy. ➨ need to search efficiently!

Neighborhood pruning [Novicki, Smutnicki, 1994, Grabowski, Wodecki, 2004]




A sequence $t = (t_1, t_2, \ldots, t_{m-1})$ defines a path in $\pi$:

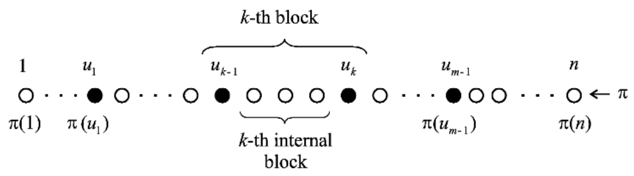
$$C(\pi, t) = \sum_{j=1}^{t_1} p_{\pi(j)1} + \sum_{j=t_1}^{t_2} p_{\pi(j)2} + \cdots + \sum_{j=t_{m-1}}^{n} p_{\pi(j)m}.$$

$C_{max}$ expression through critical path:

$$C_{\max}(\pi) = \max_{1 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_{m-1} \leqslant n} \left( \sum_{j=1}^{t_1} p_{\pi(j)1} + \sum_{j=t_1}^{t_2} p_{\pi(j)2} + \cdots + \sum_{j=t_{m-1}}^{n} p_{\pi(j)m} \right)$$

critical path: $\vec{u} = (u_1, u_2, \dots, u_m) : C_{max}(\pi) = C(\pi, u)$

Block $B_k$ and Internal Block $B_k^{Int}$




**Theorem (Werner, 1992)**

*Let $\pi, \pi' \in \Pi$, if $\pi'$ has been obtained from $\pi$ by a job insert so that $C_{max}(\pi') < C_{max}(\pi)$ then in $\pi'$:*

a) *at least one job $j \in B_k$ precedes job $\pi(u_{k-1})$, $k = 1, \dots, m$, or*

b) *at least one job $j \in B_k$ succeeds job $\pi(u_k)$, $k = 1, \dots, m$*

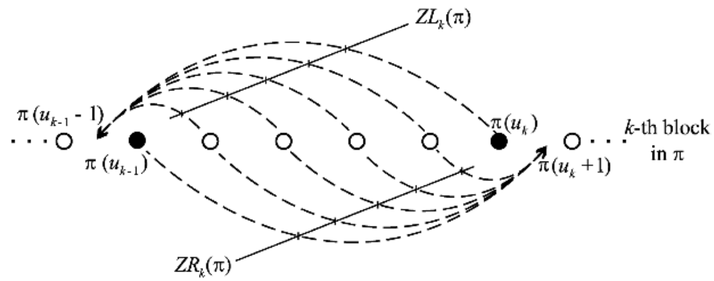




**Corollary (Elimination Criterion)**

*If $\pi'$ is obtained by $\pi$ by an "internal block insertion" then $C_{max}(\pi') \geq C_{max}(\pi)$.*

Hence we can restrict the search to where the good moves can be:








Further speedup: Use of lower bounds in delta evaluations:
Let $\delta^r_{x,u_k}$ indicate insertion of $x$ after $u_k$ (move of type $ZR_k(\pi)$)

$$\Delta(\delta^r_{x,u_k}) = \begin{cases} p_{\pi(x),k+1} - p_{\pi(u_k),k+1} & x \neq u_{k-1} \\ p_{\pi(x),k+1} - p_{\pi(u_k),k+1} + p_{\pi(u_{k-1}+1),k-1} - p_{\pi(x),k-1} & x = u_{k-1} \end{cases}$$

That is, add and remove from the adjacent blocks
It can be shown that:

$$C_{max}(\delta^r_{x,u_k}(\pi)) \geq C_{max}(\pi) + \Delta(\delta^r_{x,u_k})$$

**Theorem (Nowicki and Smutnicki, 1996, EJOR)**

*The neighborhood thus defined is connected.*



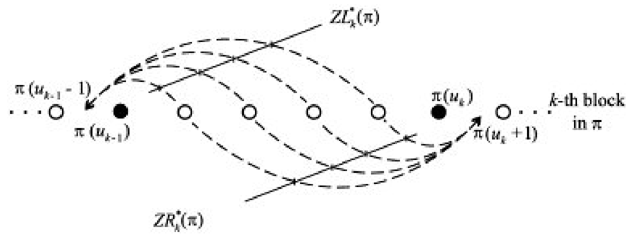


**Metaheuristic details:**

Prohibition criterion:
an insertion $\delta_{x,u_k}$ is tabu if it restores the relative order of $\pi(x)$ and $\pi(x+1)$.

Tabu length: $TL = 6 + \left[\frac{n}{10m}\right]$

Perturbation




- perform all *inserts* among all the blocks that have $\Delta < 0$
- activated after `MaxIdleIter` idle iterations

Tabu Search: the final algorithm:

| | |
|---|---|
| Initialization | : $\pi = \pi_0$, $C^* = C_{max}(\pi)$, set iteration counter to zero. |
| Searching | : Create $UR_k$ and $UL_k$ (set of non tabu moves) |
| Selection | : Find the best move according to lower bound $\Delta$.<br>Apply move. Compute true $C_{max}(\delta(\pi))$.<br>If improving compare with $C^*$ and in case update.<br>Else increase number of idle iterations. |
| Perturbation | : Apply perturbation if `MaxIdleIter` done. |
| Stop criterion | : Exit if `MaxIter` iterations are done. |