DM204
SCHEDULING, TIMETABLING AND ROUTING

Lecture 2
Scheduling and Complexity

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Outline

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Complexity Hierarchy

Reduction

A search problem Π is (polynomially) reducible to a search problem Π' ($\Pi \longrightarrow \Pi'$) if there exists an algorithm $\mathcal{A}$ that solves Π by using a hypothetical subroutine $\mathcal{S}$ for Π' and except for $\mathcal{S}$ everything runs in polynomial time. [Garey and Johnson, 1979]

NP-hard

A search problem Π' is NP-hard if

1. it is in NP
2. there exists some NP-complete problem Π that reduces to Π'

In scheduling, complexity hierarchies describe relationships between different problems.

Ex: $1||\sum C_j \longrightarrow 1||\sum w_j C_j$

Interest in characterizing the borderline: polynomial vs NP-hard problems

Problems Involving Numbers

Partition

- **Input:** finite set A and a size $s(a) \in \mathbf{Z}^+$ for each $a \in A$
- **Question:** is there a subset $A' \subseteq A$ such that

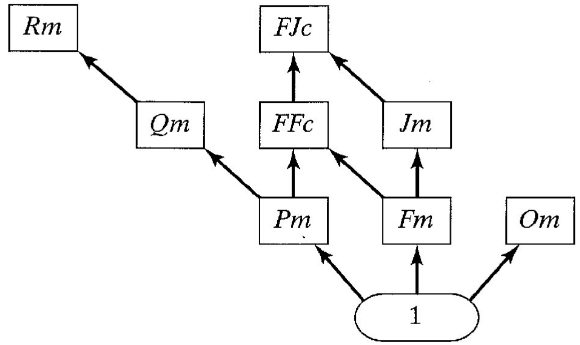
$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)?$$

3-Partition

- **Input:** set A of $3m$ elements, a bound $B \in \mathbf{Z}^+$, and a size $s(a) \in \mathbf{Z}^+$ for each $a \in A$ such that $B/4 < s(a) < B/2$ and such that $\sum_{a \in A} s(a) = mB$
- **Question:** can A be partitioned into m disjoint sets $A_1, \ldots, A_m$ such that for $1 \leq i \leq m$, $\sum_{a \in A_i} s(a) = B$ (note that each A_i must therefore contain exactly three elements from A)?

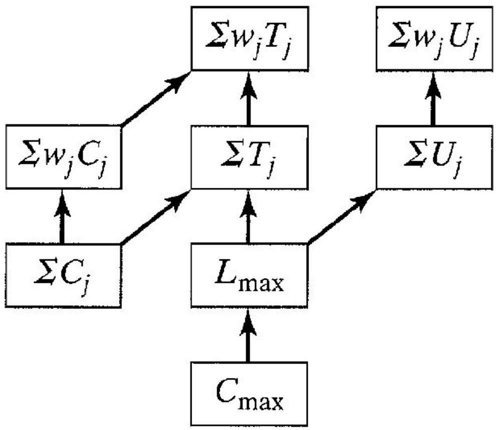
Complexity Hierarchy

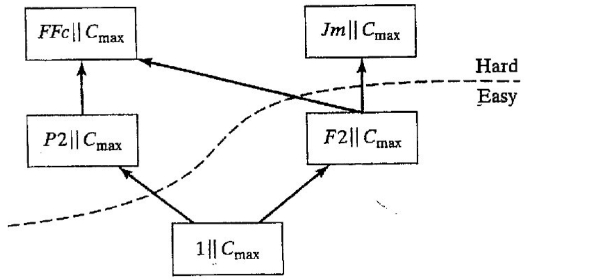
Elementary reductions for machine environment

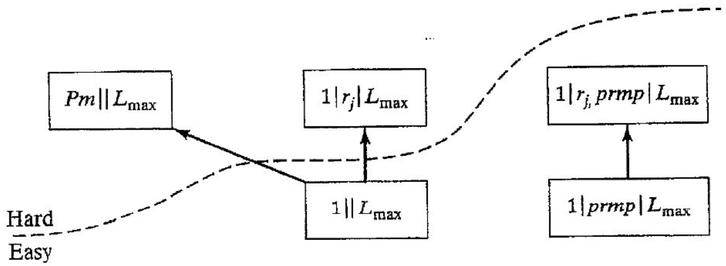


Complexity Hierarchy

Elementary reductions for regular objective functions







Polynomial time solvable problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid r_j, p_j = 1, prec \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid L_{\max}$	$O2 \mid\mid C_{\max}$
$1 \mid r_j, prmp \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid \sum C_j$	
$1 \mid tree \mid \sum w_j C_j$		$Om \mid r_j, prmp \mid L_{\max}$
	$Pm \mid p_j = 1, tree \mid C_{\max}$	
$1 \mid prec \mid L_{\max}$	$Pm \mid prmp, tree \mid C_{\max}$	$F2 \mid block \mid C_{\max}$
$1 \mid r_j, prmp, prec \mid L_{\max}$	$Pm \mid p_j = 1, outtree \mid \sum C_j$	$F2 \mid nwt \mid C_{\max}$
	$Pm \mid p_j = 1, intree \mid L_{\max}$	
$1 \mid\mid \sum U_j$	$Pm \mid prmp, intree \mid L_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum C_j$
$1 \mid r_j, prmp \mid \sum U_j$		$Fm \mid p_{ij} = p_j \mid L_{\max}$
$1 \mid r_j, p_j = 1 \mid \sum w_j U_j$	$Q2 \mid prmp, prec \mid C_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum U_j$
	$Q2 \mid r_j, prmp, prec \mid L_{\max}$	
$1 \mid r_j, p_j = 1 \mid \sum w_j T_j$		$J2 \mid\mid C_{\max}$
	$Qm \mid r_j, p_j = 1 \mid C_{\max}$	
	$Qm \mid p_j = 1, M_j \mid C_{\max}$	
	$Qm \mid r_j, p_j = 1 \mid \sum C_j$	
	$Qm \mid prmp \mid \sum C_j$	
	$Qm \mid p_j = 1 \mid \sum w_j C_j$	
	$Qm \mid p_j = 1 \mid L_{\max}$	
	$Qm \mid prmp \mid \sum U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j T_j$	
	$Rm \mid\mid \sum C_j$	
	$Rm \mid r_j, prmp \mid L_{\max}$	

NP-hard problems in the ordinary sense

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid\mid \sum w_j U_j$ (*)	$P2 \mid\mid C_{\max}$ (*)	$O2 \mid prmp \mid \sum C_j$
$1 \mid r_j, prmp \mid \sum w_j U_j$ (*)	$P2 \mid r_j, prmp \mid \sum C_j$	
	$P2 \mid\mid \sum w_j C_j$ (*)	$O3 \mid\mid C_{\max}$
$1 \mid\mid \sum T_j$ (*)	$P2 \mid r_j, prmp \mid \sum U_j$	$O3 \mid prmp \mid \sum w_j U_j$
	$Pm \mid prmp \mid \sum w_j C_j$	
	$Qm \mid\mid \sum w_j C_j$ (*)	
	$Rm \mid r_j \mid C_{\max}$ (*)	
	$Rm \mid\mid \sum w_j U_j$ (*)	
	$Rm \mid prmp \mid \sum w_j U_j$	

Strongly NP-hard problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid s_{jk} \mid C_{\max}$	$P2 \mid chains \mid C_{\max}$	$F2 \mid r_j \mid C_{\max}$
	$P2 \mid chains \mid \sum C_j$	$F2 \mid r_j, prmp \mid C_{\max}$
$1 \mid r_j \mid \sum C_j$	$P2 \mid prmp, chains \mid \sum C_j$	$F2 \mid\mid \sum C_j$
$1 \mid prec \mid \sum C_j$	$P2 \mid p_j = 1, tree \mid \sum w_j C_j$	$F2 \mid prmp \mid \sum C_j$
$1 \mid r_j, prmp, tree \mid \sum C_j$		$F2 \mid\mid L_{\max}$
$1 \mid r_j, prmp \mid \sum w_j C_j$	$R2 \mid prmp, chains \mid C_{\max}$	$F2 \mid prmp \mid L_{\max}$
$1 \mid r_j, p_j = 1, tree \mid \sum w_j C_j$		
$1 \mid p_j = 1, prec \mid \sum w_j C_j$		$F3 \mid\mid C_{\max}$
		$F3 \mid prmp \mid C_{\max}$
$1 \mid r_j \mid L_{\max}$		$F3 \mid nwt \mid C_{\max}$
$1 \mid r_j \mid \sum U_j$		$O2 \mid r_j \mid C_{\max}$
$1 \mid p_j = 1, chains \mid \sum U_j$		$O2 \mid\mid \sum C_j$
		$O2 \mid prmp \mid \sum w_j C_j$
$1 \mid r_j \mid \sum T_j$		$O2 \mid\mid L_{\max}$
$1 \mid p_j = 1, chains \mid \sum T_j$		
$1 \mid\mid \sum w_j T_j$		$O3 \mid prmp \mid \sum C_j$
		$J2 \mid rcrc \mid C_{\max}$
		$J3 \mid p_{ij} = 1, rcrc \mid C_{\max}$

Web Archive

Complexity results for scheduling problems
by Peter Brucker and Sigrid Knust
http://www.mathematik.uni-osnabrueck.de/research/OR/class/