DM204, 2010
SCHEDULING, TIMETABLING AND ROUTING

Lecture 20
Job Shop Models

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Outline

1. Job Shop
   - Modelling
   - Exact Methods
   - Shifting Bottleneck Heuristic
   - Local Search Methods

2. Job Shop Generalizations

Course Overview

- ✔ Problem Introduction
  - ✔ Scheduling classification
  - ✔ Scheduling complexity
  - ✔ RCPSP
- ✔ General Methods
  - ✔ Integer Programming
  - ✔ Constraint Programming
  - ✔ Heuristics
  - ✔ Dynamic Programming
  - ✔ Branch and Bound
- ✔ Scheduling
  - ✔ Single Machine
  - ✔ Parallel Machine and Flow Shop Models
  - Job Shop
  - Resource Constrained Project Scheduling Model
- Timetabling
  - Reservations and Education
  - University Timetabling
  - Crew Scheduling
  - Public Transports
- Vechicle Routing
  - Capacited Models
  - Time Windows models
  - Rich Models

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2. Job Shop Generalizations

Job Shop

General Shop Scheduling:

- $J = \{1, \ldots, N\}$ set of jobs; $M = \{1, 2, \ldots, m\}$ set of machines
- $J_j = \{O_{ij} \mid i = 1, \ldots, n_j\}$ set of operations for each job
- p_{ij} processing times of operations O_{ij}
- $\mu_{ij} \subseteq M$ machine eligibilities for each operation
- precedence constraints among the operations
- one job processed per machine at a time,
  one machine processing each job at a time
- C_j completion time of job j

➨ Find feasible schedule that minimize some regular function of C_j

Job shop

- $\mu_{ij} = l, l = 1, \ldots, n_j$ and $\mu_{ij} \neq \mu_{i+1,j}$ (one machine per operation)
- $O_{1j} \rightarrow O_{2j} \rightarrow \ldots \rightarrow O_{n_j,j}$ precedences (without loss of generality)
- without repetition and with unlimited buffers

Task:

- Find a schedule $S = (S_{ij})$, indicating the starting times of O_{ij}, such that:

  it is feasible, that is,

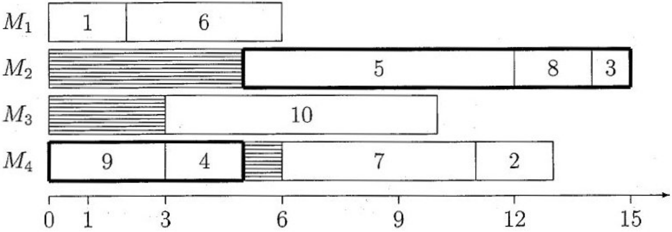
  - $S_{ij} + p_{ij} \leq S_{i+1,j}$ for all $O_{ij} \rightarrow O_{i+1,j}$
  - $S_{ij} + p_{ij} \leq S_{uv}$ or $S_{uv} + p_{uv} \leq S_{ij}$ for all operations with $\mu_{ij} = \mu_{uv}$.

  and has minimum makespan: $\min\{\max_{j \in J}(S_{n_j,j} + p_{n_j,j})\}$.

A schedule can also be represented by an m-tuple $\pi = (\pi^1, \pi^2, \ldots, \pi^m)$ where π^i defines the processing order on machine i.

There is always an optimal schedule that is semi-active.

(semi-active schedule: for each machine, start each operation at the earliest feasible time.)


M_1
M_2
M_3
M_4
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6
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8
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4
7
2
0
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3
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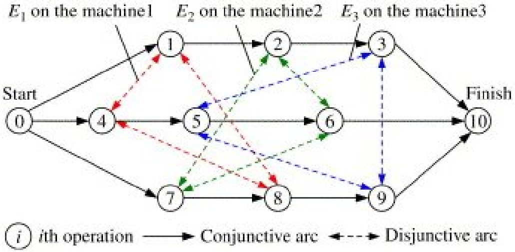
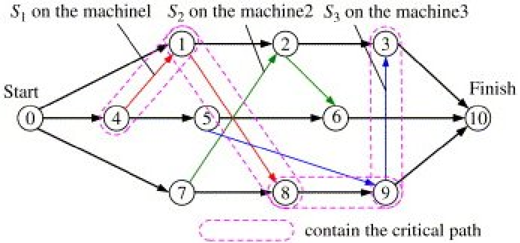
9
12
15

- Often simplified notation: $N = \{1, \ldots, n\}$ denotes the set of operations
- Disjunctive graph representation: $G = (N, A, E)$
  - vertices N: operations with two dummy operations 0 and $n+1$ denoting "start" and "finish".
  - directed arcs A, conjunctions
  - undirected arcs E, disjunctions
  - length of (i, j) in A is p_i



- A complete selection corresponds to choosing one direction for each arc of E.
- A complete selection that makes D acyclic corresponds to a feasible schedule and is called consistent.
- Complete, consistent selection $\Leftrightarrow$ semi-active schedule (feasible earliest start schedule).
- Length of longest path $0–(n+1)$ in D corresponds to the makespan



Longest path computation

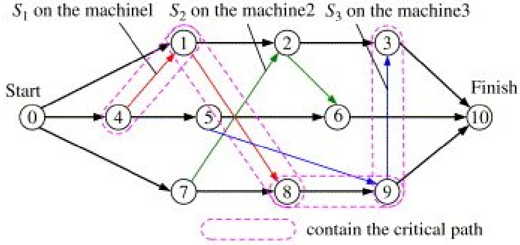
In an acyclic digraph:

- construct topological ordering ($i < j$ for all $i \to j \in A$)
- recursion:

$$r_0 = 0$$
$$r_l = \max_{\{j \mid j \to l \in A\}} \{r_j + p_j\} \qquad \text{for } l = 1, \ldots, n+1$$

- A block is a maximal sequence of adjacent critical operations processed on the same machine.

- In the Fig. below: $B_1 = \{4, 1, 8\}$ and $B_2 = \{9, 3\}$



- Any operation, u, has two immediate predecessors and successors:
  - its job predecessor $JP(u)$ and successor $JS(u)$
  - its machine predecessor $MP(u)$ and successor $MS(u)$

Exact methods

- Disjunctive programming

$$
\begin{array}{lll}
\min & C_{max} & \\
s.t. & x_{ij} + p_{ij} \leq C_{max} & \forall\, O_{ij} \in N \\
 & x_{ij} + p_{ij} \leq x_{lj} & \forall\, (O_{ij}, O_{lj}) \in A \\
 & x_{ij} + p_{ij} \leq x_{ik} \vee x_{ij} + p_{ij} \leq x_{ik} & \forall\, (O_{ij}, O_{ik}) \in E \\
 & x_{ij} \leq 0 & \forall\, i = 1, \ldots, m \;\; j = 1, \ldots, N
\end{array}
$$

- Constraint Programming
- Branch and Bound [Carlier and Pinson, 1983]

Typically unable to schedule optimally more than 10 jobs on 10 machines.
Best result is around 250 operations.

Branch and Bound [Carlier and Pinson, 1983] [B2, p. 179]

Let Ω contain the first operation of each job;
Let $r_{ij} = 0$ for all $O_{ij} \in \Omega$

Machine Selection Compute for the current partial schedule

$$t(\Omega) = \min_{ij \in \Omega}\{r_{ij} + p_{ij}\}$$

and let i^* denote the machine on which the minimum is achieved

Branching Let Ω' denote the set of all operations O_{i^*j} on machine i^* such that

$$r_{i^*j} < t(\Omega) \qquad \text{(i.e. eliminate } r_{i^*j} \geq t(\Omega))$$

For each operation in Ω', consider an (extended)partial schedule with that operation as the next one on machine i^*.
For each such (extended) partial schedule, delete the operations from Ω, include its immediate follower in Ω and return to Machine Selection.

Lower Bounding:

- longest path in partially selected disjunctive digraph
- solve $1|r_{ij}|L_{max}$ on each machine i like if all other machines could process at the same time (see later shifting bottleneck heuristic) + longest path.

Shifting Bottleneck Heuristic

- A complete selection is made by the union of selections S_k for each clique E_k that corresponds to machines.
- **Idea**: use a priority rule for ordering the machines.
  chose each time the bottleneck machine and schedule jobs on that machine.
- Measure bottleneck quality of a machine k by finding optimal schedule to a certain single machine problem.
- Critical machine, if at least one of its arcs is on the critical path.

- $M_0 \subset M$ set of machines already sequenced.
- $k \in M \setminus M_0$
- $P(k, M_0)$ is problem $1 \mid r_j \mid L_{max}$ obtained by:
  - the selections in M_0
  - removing any disjunctive arc in $p \in M \setminus M_0$
- $v(k, M_0)$ is the optimum of $P(k, M_0)$
- bottleneck $m = \arg \max_{k \in M \setminus M_0} \{v(k, M_0)\}$
- $M_0 = \emptyset$

Step 1: Identify bottleneck m among $k \in M \setminus M_0$ and sequence it optimally. Set $M_0 \leftarrow M_0 \cup \{m\}$

Step 2: Reoptimize the sequence of each critical machine $k \in M_0$ in turn: set $M'_o = M_0 - \{k\}$ and solve $P(k, M'_0)$.
Stop if $M_0 = M$ otherwise Step 1.

- Local Reoptimization Procedure

Construction of $P(k, M_0)$

$1 \mid r_j \mid L_{max}$:

- $r_j = L(0, j)$
- $d_j = L(0, n) - L(j, n) + p_j$

$L(i, j)$ length of longest path in G: Computable in $O(n)$

acyclic complete directed graph $\iff$ transitive closure of its unique directed Hamiltonian path.

Hence, only predecessors and successor are to be checked.
The graph is not constructed explicitly, but by maintaining a list of jobs per machines and a list machines per jobs.

$1 \mid r_j \mid L_{max}$ can be solved optimally very efficiently.
Results reported up to 1000 jobs.

$1 \mid r_j \mid L_{max}$

From one of the past lectures

[Maximum lateness with release dates]

- Strongly NP-hard (reduction from 3-partition)
- might have optimal schedule which is not non-delay
- Branch and bound algorithm (valid also for $1 \mid r_j, prec \mid L_{max}$)
  - **Branching**:
    schedule from the beginning (level k, $n!/(k-1)!$ nodes)
    elimination criterion: do not consider job j_k if:

$$r_j > \min_{l \in J} \{\max(t, r_l) + p_l\} \qquad J \text{ jobs to schedule, } t \text{ current time}$$

  - **Lower bounding**: relaxation to preemptive case for which EDD is optimal

Efficient local search for job shop

Solution representation:
m-tuple $\pi = (\pi^1, \pi^2, \ldots, \pi^m) \Longleftrightarrow$ oriented digraph $D_\pi = (N, A, E_\pi)$

Neighborhoods
Change the orientation of certain disjunctive arcs of the current complete selection

Issues:

1. Can it be decided easily if the new digraph $D_{\pi'}$ is acyclic?
2. Can the neighborhood selection S' improve the makespan?
3. Is the neighborhood connected?

Swap Neighborhood [Novicki, Smutnicki]

Reverse one oriented disjunctive arc (i, j) on some critical path.

Theorem

All neighbors are consistent selections.

Note: If the neighborhood is empty then there are no disjunctive arcs, nothing can be improved and the schedule is already optimal.

Theorem

The swap neighborhood is weakly optimal connected.

Insertion Neighborhood [Balas, Vazacopoulos, 1998]

For some nodes u, v in the critical path:

- move u right after v (forward insert)
- move v right before u (backward insert)

Theorem: If a critical path containing u and v also contains $JS(v)$ and

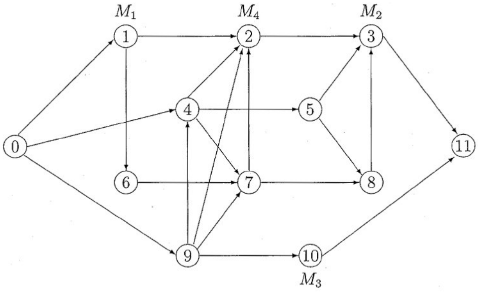
$$L(v, n) \geq L(JS(u), n)$$

then a forward insert of u after v yields an acyclic complete selection.

Theorem: If a critical path containing u and v also contains $JS(v)$ and

$$L(0, u) + p_u \geq L(0, JP(v)) + p_{JP(v)}$$

then a backward insert of v before v yields an acyclic complete selection.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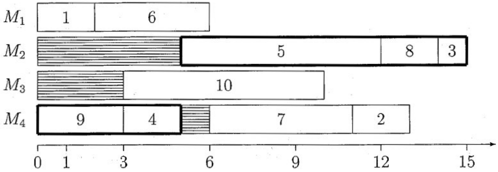
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Theorem: (Elimination criterion) If $C_{max}(S') < C_{max}(S)$ then at least one operation of a machine block B on the critical path has to be processed before the first or after the last operation of B.

- Swap neighborhood can be restricted to first and last operations in the block
- Insert neighborhood can be restricted to moves similar to those saw for the flow shop. [Grabowski, Wodecki]

Tabu Search requires a best improvement strategy hence the neighborhood must be search very fast.

Neighbor evaluation:

- exact recomputation of the makespan $O(n)$
- approximate evaluation (rather involved procedure but much faster and effective in practice)

The implementation of Tabu Search follows the one saw for flow shop.

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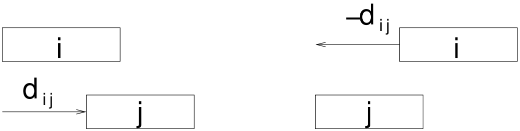
Generalizations: Time Lags



Generalized time constraints

They can be used to model:

- Release time:

$$S_0 + r_i \leq S_i \qquad \Longleftrightarrow \qquad d_{0i} = r_i$$

- Deadlines:

$$S_i + p_i - d_i \leq S_0 \qquad \Longleftrightarrow \qquad d_{i0} = p_i - d_i$$

- Modelling

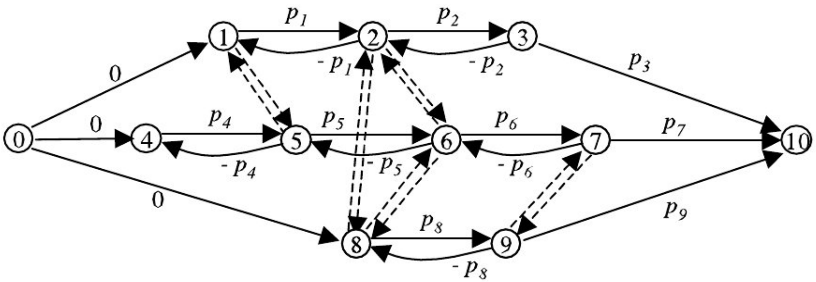
$$
\begin{array}{lll}
\min & C_{max} & \\
s.t. & x_{ij} + d_{ij} \leq C_{max} & \forall\, O_{ij} \in N \\
 & x_{ij} + d_{ij} \leq x_{lj} & \forall\, (O_{ij}, O_{lj}) \in A \\
 & x_{ij} + d_{ij} \leq x_{ik} \vee x_{ij} + d_{ij} \leq x_{ik} & \forall\, (O_{ij}, O_{ik}) \in E \\
 & x_{ij} \geq 0 & \forall\, i = 1, \ldots, m \;\; j = 1, \ldots, N
\end{array}
$$

- In the disjunctive graph, d_{ij} become the lengths of arcs

- Exact relative timing (perishability constraints):
  if operation j must start l_{ij} after operation i:

$$S_i + p_i + l_{ij} \leq S_j \qquad \text{and} \qquad S_j - (p_i + l_{ij}) \leq S_i$$

($l_{ij} = 0$ if no-wait constraint)



- Set up times:

$$S_i + p_i + s_{ij} \leq S_j \qquad \text{or} \qquad S_j + p_j + s_{ji} \leq S_i$$

- Machine unavailabilities:
  - Machine M_k unavailable in $[a_1, b_1], [a_2, b_2], \ldots, [a_v, b_v]$
  - Introduce v artificial operations with $\lambda = 1, \ldots, v$ with $\mu_\lambda = M_k$ and:

    $p_\lambda = b_\lambda - a_\lambda$
    $r_\lambda = a_\lambda$
    $d_\lambda = b_\lambda$

- Minimum lateness objectives:

$$L_{max} = \max_{j=1}^{N}\{C_j - d_j\} \qquad \Longleftrightarrow \qquad d_{n_j,n+1} = p_{n_j} - d_j$$

Arises with limited buffers:
after processing, a job remains on the machine until the next machine is freed

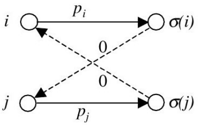
- Needed a generalization of the disjunctive graph model
  $\Longrightarrow$ Alternative graph model $G = (N, E, A)$ [Mascis, Pacciarelli, 2002]

1. two non-blocking operations to be processed on the same machine

$$S_i + p_i \leq S_j \qquad \text{or} \qquad S_j + p_j \leq S_i$$

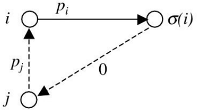
2. Two blocking operations i, j to be processed on the same machine $\mu(i) = \mu(j)$

$$S_{\sigma(j)} \leq S_i \qquad \text{or} \qquad S_{\sigma(i)} \leq S_j$$



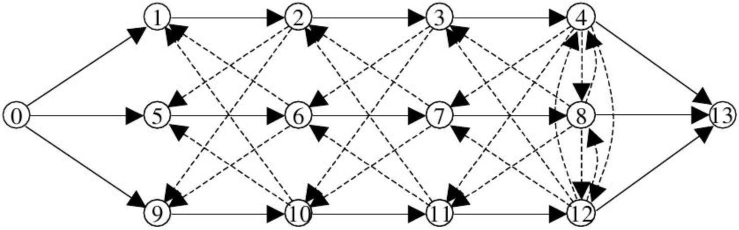
3. i is blocking, j is non-blocking (ideal) and i, j to be processed on the same machine $\mu(i) = \mu(j)$.

$$S_i + p_i \leq S_j \qquad \text{or} \qquad S_{\sigma(j)} \leq S_i$$

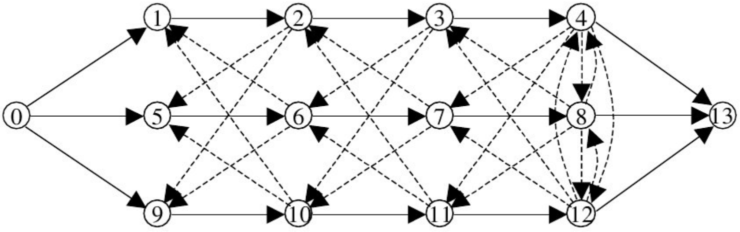


Example

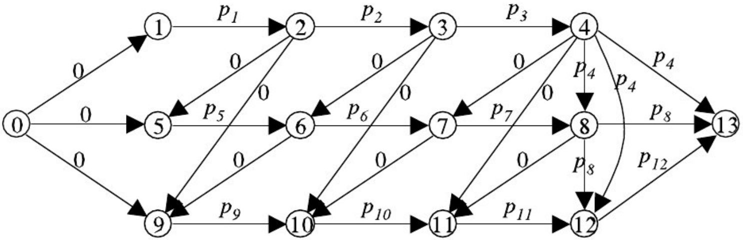
- $O_0, O_1, \ldots, O_{13}$
- $M(O_1) = M(O_5) = M(O_9)$
  $M(O_2) = M(O_6) = M(O_{10})$
  $M(O_3) = M(O_7) = M(O_{11})$



- Length of arcs can be negative
- Multiple occurrences possible: $((i,j),(u,v)) \in A$ and $((i,j),(h,k)) \in A$
- The last operation of a job j is always non-blocking.



- A complete selection S is consistent if it chooses alternatives from each pair such that the resulting graph does not contain positive cycles.



Example:

- $p_a = 4$
- $p_b = 2$
- $p_c = 1$

- b must start at least 9 days after a has started
- c must start at least 8 days after b is finished
- c must finish within 16 days after a has started

$$
\begin{aligned}
S_a + 9 &\leq S_b \\
S_b + 10 &\leq S_c \\
S_c - 15 &\leq S_a
\end{aligned}
$$

This leads to an absurd.
In the alternative graph the cycle is positive.

- The Makespan still corresponds to the longest path in the graph with the arc selection $G(S)$.
- Problem: now the digraph may contain cycles. Longest path with simple cyclic paths is NP-complete. However, here we have to care only of non-positive cycles.
- If there are no cycles of length strictly positive it can still be computed efficiently in $O(|N||E \cup A|)$ by Bellman-Ford (1958) algorithm.
- The algorithm iteratively considers all edges in a certain order and updates an array of longest path lengths for each vertex. It stops if a loop over all edges does not yield any update or after $|N|$ iterations over all edges (in which case we know there is a positive cycle).
- Possible to maintain incremental updates when changing the selection [Demetrescu, Frangioni, Marchetti-Spaccamela, Nanni, 2000].

Heuristic Methods

- The search space is highly constrained + detecting positive cycles is costly
- Hence local search methods not very successful
- Rely on the construction paradigm
- Rollout algorithm [Meloni, Pacciarelli, Pranzo, 2004]

Rollout

- Master process: grows a partial selection S^k:
  decides the next element to fix based on an heuristic function
  (selects the one with minimal value)

- Slave process: evaluates heuristically the alternative choices.
  Completes the selection by keeping fixed what passed by the master
  process and fixing one alternative at a time.

- Slave heuristics
  - *Avoid Maximum Current Completion time*
    find an arc (h, k) that if selected would increase most the length of the longest path in $G(S^k)$ and select its alternative

    $$\max_{(uv)\in A} \{l(0,u) + a_{uv} + l(v,n)\}$$

  - *Select Most Critical Pair*
    find the pair that, in the worst case, would increase least the length of the longest path in $G(S^k)$ and select the best alternative

    $$\max_{((ij),(hk))\in A} \min\{l(0,u) + a_{hk} + l(k,n), l(0,i) + a_{ij} + l(j,n)\}$$

  - *Select Max Sum Pair*
    find the pair with greatest potential effect on the length of the longest path in $G(S^k)$ and select the best alternative

    $$\max_{((ij),(hk))\in A} |l(0,u) + a_{hk} + l(k,n) + l(0,i) + a_{ij} + l(j,n)|$$

Trade off quality *vs* keeping feasibility

Implementation details of the slave heuristics

- Once an arc is added we need to update all $L(0,u)$ and $L(u,n)$. Backward and forward visit $O(|F|+|A|)$
- When adding arc a_{ij}, we detect positive cycles if $L(i,j)+a_{ij}>0$. This happens only if we updated $L(0,i)$ or $L(j,n)$ in the previous point and hence it comes for free.
- Overall complexity $O(|A|(|F|+|A|))$

Speed up of Rollout:

- Stop if partial solution overtakes upper bound
- limit evaluation to say 20% of arcs in A