DM204, 2010
SCHEDULING, TIMETABLING AND ROUTING

Lecture 31
Construction Heuristics and Local Search Methods for VRP/VRPTW

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Course Overview

- ✔ Problem Introduction
  - ✔ Scheduling classification
  - ✔ Scheduling complexity
  - ✔ RCPSP
- ✔ General Methods
  - ✔ Integer Programming
  - ✔ Constraint Programming
  - ✔ Heuristics
  - ✔ Dynamic Programming
  - ✔ Branch and Bound
- ✔ Scheduling Models
  - ✔ Single Machine
  - ✔ Parallel Machine and Flow Shop
  - ✔ Job Shop
  - ✔ Resource-Constrained Project Scheduling
- Timetabling
  - ✔ Reservations and Education
  - ✔ Course Timetabling
  - ✔ Workforce Timetabling
  - ✔ Crew Scheduling
- Vehicle Routing
  - Capacitated Models
  - Time Windows models
  - Rich Models

Outline

Outline

Construction Heuristics for CVRP

- TSP based heuristics
- Saving heuristics (Clarke and Wright)
- Insertion heuristics
- Cluster-first route-second
  - Sweep algorithm
  - Generalized assignment
  - Location based heuristic
  - Petal algorithm
- Route-first cluster-second

Cluster-first route-second seems to perform better than route-first
(Note: distinction construction heuristic / iterative improvement is often blurred)

Construction heuristics for TSP

They can be used for route-first cluster-second or for growing multiple tours subject to capacity constraints.

- Heuristics that Grow Fragments
  - Nearest neighborhood heuristics
  - Double-Ended Nearest Neighbor heuristic
  - Multiple Fragment heuristic (aka, greedy heuristic)
- Heuristics that Grow Tours
  - Nearest Addition
  - Farthest Addition
  - Random Addition
  - Nearest Insertion
  - Farthest Insertion
  - Random Insertion
  - Clarke-Wright saving heuristic
- Heuristics based on Trees
  - Minimum spanning tree heuristic
  - Christofides' heuristics

(But recall! Concorde: http://www.tsp.gatech.edu/)

[Bentley, 1992]

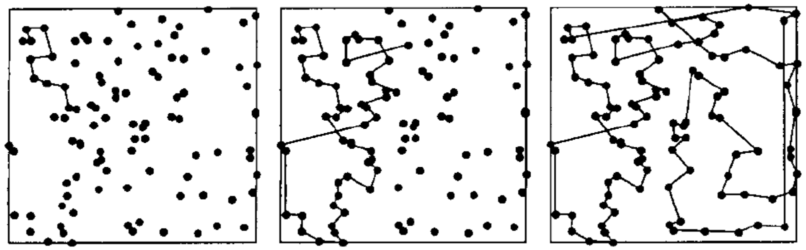


Figure 1. The Nearest Neighbor heuristic.

NN (Flood, 1956)

1. Randomly select a starting node
2. Add to the last node the closest node until no more node is available
3. Connect the last node with the first node

Running time $O(N^2)$

[Bentley, 1992]

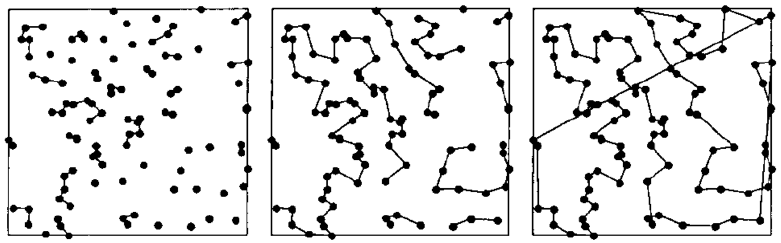


Figure 5. The Multiple Fragment heuristic.

Add the cheapest edge provided it does not create a cycle.

[Bentley, 1992]

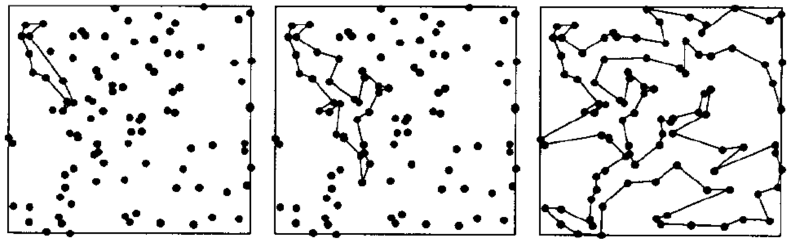


Figure 8. The Nearest Addition heuristic.

NA

1. Select a node and its closest node and build a tour of two nodes
2. Insert in the tour the closest node v until no more node is available

Running time $O(N^3)$

[Bentley, 1992]

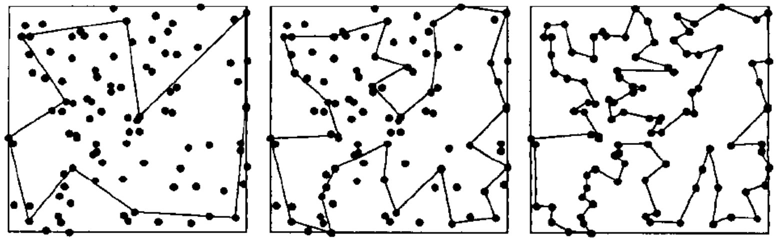


Figure 11. The Farthest Addition heuristic.

FA

1. Select a node and its farthest and build a tour of two nodes
2. Insert in the tour the farthest node v until no more node is available

FA is more effective than NA because the first few farthest points sketch a broad outline of the tour that is refined after.

Running time $O(N^3)$

[Bentley, 1992]

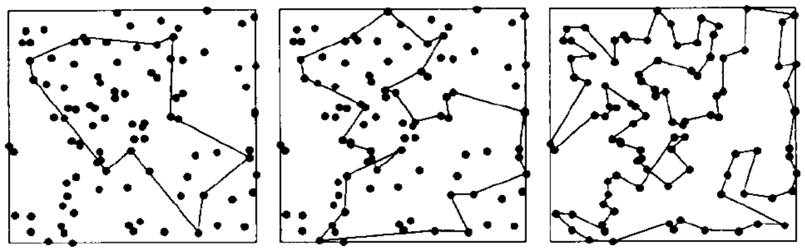


Figure 14. The Random Addition heuristic.

[Bentley, 1992]

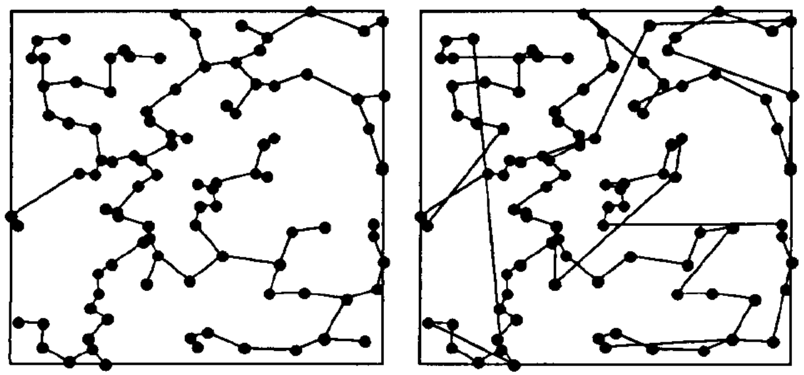


Figure 18. The Minimum Spanning Tree heuristic.

1. Find a minimum spanning tree $O(N^2)$
2. Append the nodes in the tour in a depth-first, inorder traversal

Running time $O(N^2)$ $A = MST(I)/OPT(I) \leq 2$

[Bentley, 1992]

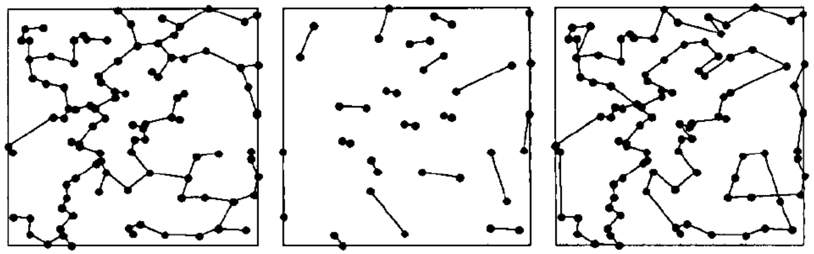
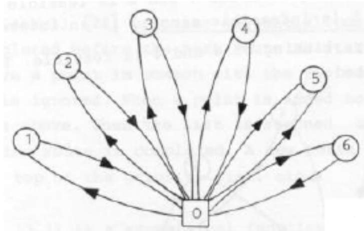


Figure 19. Christofides' heuristic.

1. Find the minimum spanning tree T. $O(N^2)$
2. Find nodes in T with odd degree and find the cheapest perfect matching M in the complete graph consisting of these nodes only. Let G be the multigraph of all nodes and edges in T and M. $O(N^3)$
3. Find an Eulerian walk (each node appears at least once and each edge exactly once) on G and an embedded tour. $O(N)$

Running time $O(N^3)$ $A = CH(I)/OPT(I) \leq 3/2$

Construction Heuristics Specific for VRP



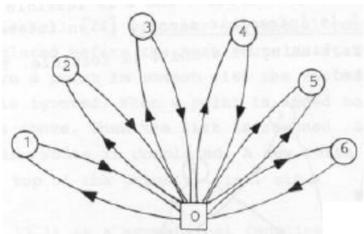
Clarke-Wright Saving Heuristic (1964)

1. Start with an initial allocation of one vehicle to each customer (0 is the depot for VRP or any chosen city for TSP)

Sequential:

2. consider in turn route $(0, i, \ldots, j, 0)$ determine s_{ki} and s_{jl}
3. merge with $(k, 0)$ or $(0, l)$

Construction Heuristics Specific for VRP

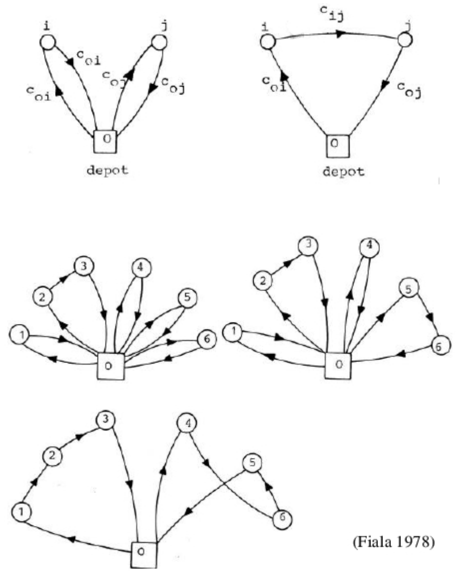


Clarke-Wright Saving Heuristic (1964)

1. Start with an initial allocation of one vehicle to each customer (0 is the depot for VRP or any chosen city for TSP)

Parallel:

2. Calculate saving $s_{ij} = c_{0i} + c_{0j} - c_{ij}$ and order the saving in non-increasing order
3. scan s_{ij}
   merge routes if i) i and j are not in the same tour ii) neither i and j are interior to an existing route iii) vehicle and time capacity are not exceeded



Matching Based Saving Heuristic

1. Start with an initial allocation of one vehicle to each customer (0 is the depot for VRP or any chosen city for TSP)
2. Compute $s_{pq} = t(S_p) + t(S_q) - t(S_p \cup S_q)$ where $t(\cdot)$ is the TSP solution
3. Solve a max weighted matching on the S_k with weights s_{pq} on edges. A connection between a route p and q exists only if the merging is feasible.

Insertion Heuristic

$$\alpha(i,k,j) = c_{ik} + c_{kj} - \lambda c_{ij}$$

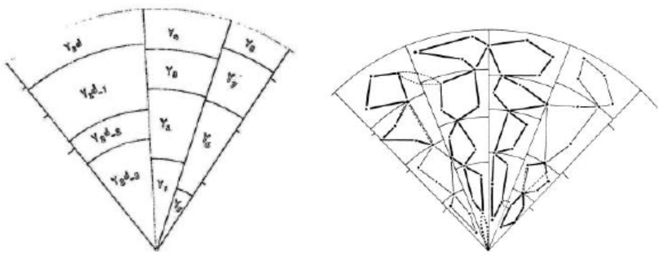
$$\beta(i,k,j) = \mu c_{0k} - \alpha(i,k,j)$$

1. construct emerging route $(0, k, 0)$
2. compute for all k unrouted the feasible insertion cost:

$$\alpha^*(i_k, k, j_k) = \min_p \{\alpha(i_p, k, i_{p+1})\}$$

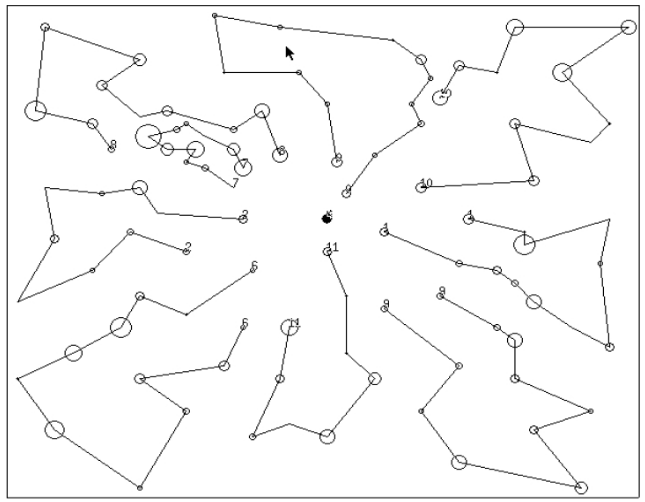
if no feasible insertion go to 1 otherwise choose k^* such that

$$\beta^*(i_k^*, k^*, j_k^*) = \max_k \{\beta(i_k, k, j_k)\}$$



Cluster-first route-second: Sweep algorithm [Wren & Holliday (1971)]

1. Choose i^* and set $\theta(i^*) = 0$ for the rotating ray
2. Compute and rank the polar coordinates (θ, ρ) of each point
3. Assign customers to vehicles until capacity not exceeded. If needed start a new route. Repeat until all customers scheduled.



Cluster-first route-second: Generalized-assignment-based algorithm [Fisher & Jaikumur (1981)]

1. Choose a j_k at random for each route k
2. For each point compute

$$d_{ik} = \min\{c_{0,i} + c_{i,j_k} + c_{j_k,0}, c_{0j_k} + c_{j_k,i} + c_{i,0}\} - (c_{0,j_k} + c_{j_k,0})$$

3. Solve GAP with d_{ik}, Q and q_i

Cluster-first route-second: Location based heuristic [Bramel & Simchi-Levi (1995)]

1. Determine seeds by solving a capacitated location problem (k-median)

2. Assign customers to closest seed

(better performance than insertion and saving heuristics)

Cluster-first route-second: Petal Algorithm

1. Construct a subset of feasible routes

2. Solve a set partitioning problem

Route-first cluster-second [Beasley, 1983]

1. Construct a TSP tour over all customers

2. Choose an arbitrary orientation of the TSP;
   partition the tour according to capacity constraint;
   repeat for several orientations and select the best
   Alternatively, solve a shortest path in an acyclic digraph with costs on arcs: $d_{ij} = c_{0i} + c_{0j} + l_{ij}$ where l_{ij} is the cost of traveling from i to j in the TSP tour.

(not very competitive)

Exercise

Which heuristics can be used to minimize K
and which ones need to have K fixed a priori?

Construction Heuristics for VRPTW

Extensions of those for CVRP [Solomon (1987)]

- Saving heuristics (Clarke and Wright)
- Time-oriented nearest neighbors
- Insertion heuristics
- Time-oriented sweep heuristic

Time-Oriented Nearest-Neighbor

- Add the unrouted node "closest" to the depot or the last node added without violating feasibility
- Metric for "closest":

$$c_{ij} = \delta_1 d_{ij} + \delta_2 T_{ij} + \delta_3 v_{ij}$$

d_{ij} geographical distance

T_{ij} time distance

v_{ij} urgency to serve j

Insertion Heuristics

Step 1: Compute for each unrouted costumer u the *best feasible position* in the route:

$$c_1(i(u), u, j(u)) = \min_{p=1,\dots,m}\{c_1(i_{p-1}, u, i_p)\}$$

(c_1 is a composition of increased time and increase route length due to the insertion of u)
(use push forward rule to check feasibility efficiently)

Step 2: Compute for each unrouted customer u which can be feasibly inserted:

$$c_2(i(u^*), u^*, j(u^*)) = \max_u\{\lambda d_{0u} - c_1(i(u), u, j(u))\}$$

(max the benefit of servicing a node on a partial route rather than on a direct route)

Step 3: Insert the customer u^* from Step 2

- Let's assume waiting is allowed and s_i indicates service times
- $[e_i, l_i]$ time window, w_i waiting time
- $b_i = \max\{e_i, b_j + s_j + t_{ji}\}$ begin of service
- insertion of u: $(i_0, i_1, \ldots, i_p, \mathbf{u}, i_{p+1}, \ldots, i_m)$
- $PF_{i_{p+1}} = b^{new}_{i_{p+1}} - b_{i_{p+1}} \geq 0$ push forward
- $PF_{i_{r+1}} = \max\{0, PF_{i_r} - w_{i_{r+1}}\}, \quad p \leq r \leq m-1$

Theorem

The insertion is feasible if and only if:

$$b_u \leq l_u \quad \text{and} \quad PF_{i_r} + b_{i_r} \leq l_{i_r} \quad \forall p < r \leq m$$

Check vertices k, $u \leq k \leq m$ sequentially.

- if $b_k + PF_k > l_k$ then stop: the insertion is infeasible
- if $PF_k = 0$ then stop: the insertion is feasible

Outline

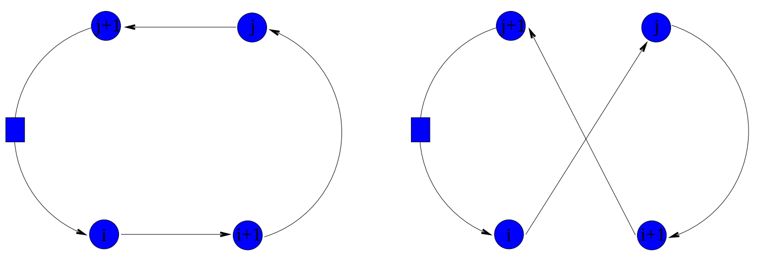
Local Search for CVRP and VRPTW

- Neighborhood structures:
  - Intra-route: 2-opt, 3-opt, Lin-Kernighan (not very well suited), Or-opt (2H-opt)
  - Inter-routes: λ-interchange, relocate, exchange, cross, 2-opt*, b-cyclic k-transfer (ejection chains), GENI
- Solution representation and data structures
  - They depend on the neighborhood.
  - It can be advantageous to change them from one stage to another of the heuristic

Intra-route Neighborhoods

2-opt

$$\{i, i+1\}\{j, j+1\} \longrightarrow \{i, j\}\{i+1, j+1\}$$

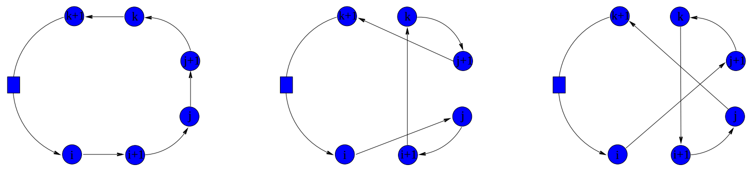


$O(n^2)$ possible exchanges
One path is reversed

Intra-route Neighborhoods

3-opt

$$\{i, i+1\}\{j, j+1\}\{k, k+1\} \longrightarrow \dots$$

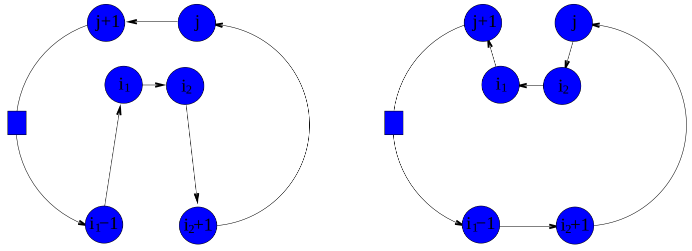


$O(n^3)$ possible exchanges
Paths can be reversed

Intra-route Neighborhoods

Or-opt [Or (1976)]

$$\{i_1-1, i_1\}\{i_2, i_2+1\}\{j, j+1\} \longrightarrow \{i_1-1, i_2+1\}\{j, i_1\}\{i_2, j+1\}$$



sequences of one, two, three consecutive vertices relocated
$O(n^2)$ possible exchanges — No paths reversed

Inter-route Neighborhoods

[Savelsbergh, ORSA (1992)]

Figure 6. The exchange neighborhood.

Inter-route Neighborhoods

[Savelsbergh, ORSA (1992)]

Figure 5. The relocate neighborhood.

Inter-route Neighborhoods

[Savelsbergh, ORSA (1992)]

Figure 7. The cross neighborhood.

GENI: generalized insertion [Gendreau, Hertz, Laporte, Oper. Res. (1992)]

- select the insertion restricted to the neighborhood of the vertex to be added (not necessarily between consecutive vertices)
- perform the best 3- or 4-opt restricted to reconnecting arc links that are close to one another.

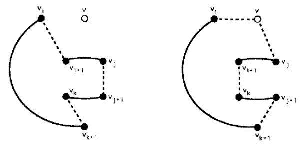


Figure 1. Type I insertion of vertex v between v_i and v_j.

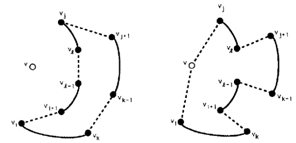


Figure 2. Type II insertion of vertex v between v_i and v_j.

Efficient Implementation

Intra-route

Time windows: Feasibility check

In TSP verifying k-optimality requires $O(n^k)$ time
In TSPTW feasibility has to be tested then $O(n^{k+1})$ time

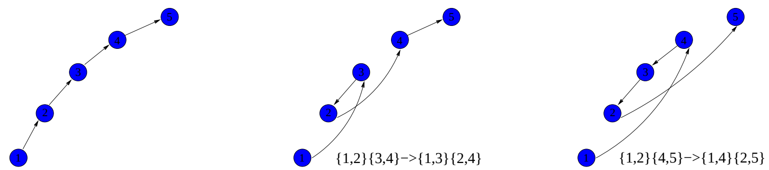
(Savelsbergh 1985) shows how to verify constraints in constant time
Search strategy + Global variables

$\Downarrow$

$O(n^k)$ for k-optimality in TSPTW

Search Strategy

- Lexicographic search, for 2-exchange:
  - $i = 1, 2, \dots, n-2$ (outer loop)
  - $j = i+2, i+3, \dots, n$ (inner loop)



Previous path is expanded by the edge $\{j-1, j\}$

Global variables (auxiliary data structure)

- Maintain auxiliary data such that it is possible to:
  - handle single move in constant time
  - update their values in constant time

Ex.: in case of time windows:

- total travel time of a path
- earliest departure time of a path
- latest arrival time of a path

Efficient Local Search

[Irnich (2008)] uniform model

Outline

Metaheuristics

Many and fancy examples, but first thing to try:

- Variable Neighborhood Search + Iterated greedy

Basic Variable Neighborhood Descent (BVND)

Procedure VND
input : $\mathcal{N}_k$, $k = 1, 2, \ldots, k_{max}$, and an initial solution s
output: a local optimum s for $\mathcal{N}_k$, $k = 1, 2, \ldots, k_{max}$

```
k ← 1
repeat
    s' ← FindBestNeighbor(s,N_k)
    if g(s') < g(s) then
        s ← s'
        k ← 1
    else
        k ← k + 1
until k = k_max ;
```


Variable Neighborhood Descent (VND)

Procedure VND
input : $\mathcal{N}_k$, $k = 1, 2, \ldots, k_{max}$, and an initial solution s
output: a local optimum s for $\mathcal{N}_k$, $k = 1, 2, \ldots, k_{max}$

```
k ← 1
repeat
    s' ← IterativeImprovement(s,N_k)
    if g(s') < g(s) then
        s ← s'
        k ← 1
    else
        k ← k + 1
until k = k_max ;
```


- Final solution is locally optimal w.r.t. all neighborhoods
- First improvement may be applied instead of best improvement
- Typically, order neighborhoods from smallest to largest
- If iterative improvement algorithms II_k, $k = 1, \ldots, k_{max}$ are available as black-box procedures:
  - order black-boxes
  - apply them in the given order
  - possibly iterate starting from the first one
  - order chosen by: *solution quality* and *speed*

General recommendation: use a combination of 2-opt* + or-opt
[Potvin, Rousseau, (1995)]

However,

- Designing a local search algorithm is an engineering process in which learnings from other courses in CS might become important.
- It is important to make such algorithms as much efficient as possible.
- Many choices are to be taken (search strategy, order, auxiliary data structures, etc.) and they may interact with instance features. Often a trade-off between examination cost and solution quality must be decided.
- The assessment is conducted through:
  - analytical analysis (computational complexity)
  - experimental analysis

Table 5.6. *The effect of 3-opt on the Clarke and Wright algorithm.*

Problem	Sequential: No 3-opt[1]	Sequential: + 3-opt FI[2]	Sequential: + 3-opt BI[3]	Sequential: K^4	Parallel: No 3-opt[5]	Parallel: + 3-opt FI[6]	Parallel: + 3-opt BI[7]	Parallel: K^8
E051-05e	625.56	624.20	624.20	5	584.64	578.56	578.56	6
E076-10e	1005.25	991.94	991.94	10	900.26	888.04	888.04	10
E101-08e	982.48	980.93	980.93	8	886.83	878.70	878.70	8
E101-10c	939.99	930.78	928.64	10	833.51	824.42	824.42	10
E121-07c	1291.33	1232.90	1237.26	7	1071.07	1049.43	1048.53	7
E151-12c	1299.39	1270.34	1270.34	12	1133.43	1128.24	1128.24	12
E200-17c	1708.00	1667.65	1669.74	16	1395.74	1386.84	1386.84	17
D051-06c	670.01	663.59	663.59	6	618.40	616.66	616.66	6
D076-11c	989.42	988.74	988.74	12	975.46	974.79	974.79	12
D101-09c	1054.70	1046.69	1046.69	10	973.94	968.73	968.73	9
D101-11c	952.53	943.79	943.79	11	875.75	868.50	868.50	11
D121-11c	1646.60	1638.39	1637.07	11	1596.72	1587.93	1587.93	11
D151-14c	1383.87	1374.15	1374.15	15	1287.64	1284.63	1284.63	15
D200-18c	1671.29	1652.58	1652.58	20	1538.66	1523.24	1521.94	19

[1] Sequential savings.
[2] Sequential savings + 3-opt and first improvement.
[3] Sequential savings + 3-opt and best improvement.
[4] Sequential savings: number of vehicles in solution.
[5] Parallel savings.
[6] Parallel savings + 3-opt and first improvement.
[7] Parallel savings + 3-opt and best improvement.
[8] Parallel savings: number of vehicles in solution.

What is best?

Iterated Greedy

Key idea: use the VRP cosntruction heuristics

- alternation of Construction and Deconstruction phases
- an acceptance criterion decides whether the search continues from the new or from the old solution.

Iterated Greedy (IG):
determine initial candidate solution s
while termination criterion is not satisfied **do**
  $r := s$
  greedily destruct part of s
  greedily reconstruct the missing part of s
  apply subsidiary iterative improvement procedure (eg, VNS)
  based on acceptance criterion,
    keep s or revert to $s := r$

In the literature, the overall heuristic idea received different names:

- Removal and reinsertion
- Ruin and repair
- Iterated greedy
- Fix and re-optimize

Removal procedures
Remove some related customers
(their re-insertion is likely to change something, if independent would be reinserted in same place)

Relatedness measure r_{ij}

- belong to same route
- geographical
- temporal and load based
- cluster removal
- history based

Dispersion sub-problem:
choose q customers to remove with minimal r_{ij}

$$\begin{aligned} \min \quad & \textstyle\sum_{ij} r_{ij} x_i x_j \\ & \textstyle\sum_j x_j = q \\ & x_j \in \{0,1\} \end{aligned}$$

Heuristic stochastic procedure:

- select i at random and find j that minimizes r_{ij}
- Kruskal like, plus some randomization
- history based
- random

Reinsertion procedures

- Greedy (cheapest insertion)
- Max regret:

  Δf_i^q due to insert i into its best position in its q^{th} best route

  $i = \arg\max(\Delta f_i^2 - \Delta f_i^1)$
- Constraint programming (max 20 costumers)

Advantages of remove-reinsert procedure with many side constraints:

- the search space in local search may become disconnected
- it is easier to implement feasibility checks
- no need of computing delta functions in the objective function

Further ideas

- Adaptive removal: start by removing 1 pair and increase after a certain number of iterations
- use of roulette wheel to decide which removal and reinsertion heuristic to use (π past contribution)

  $$p_i = \frac{\pi_i}{\sum \pi_i} \qquad \text{for each heuristic } i$$
- SA as accepting criterion after each reconstruction

Outline

Performance of exact methods

Current limits of exact methods [Ropke, Pisinger (2007)]:

CVRP: up to 135 customers by branch and cut and price

VRPTW: 50 customers (but 1000 customers can be solved if the instance has some structure)

CP can handle easily side constraints but hardly solve VRPs with more than 30 customers.

Large Neighborhood Search

Other approach with CP: [Shaw, 1998]

- Use an over all local search scheme
- Moves change a large portion of the solution
- CP system is used in the exploration of such moves.
- CP used to check the validity of moves and determine the values of constrained variables
- As a part of checking, constraint propagation takes place. Later, iterative improvement can take advantage of the reduced domains to speed up search by performing fast legality checks.

Solution representation:

- Handled by local search:
  Next pointers: A variable n_i for every customer i representing the next visit performed by the same vehicle

$$n_i \in N \cup S \cup E$$

  where $S = \bigcup S_k$ and $E = \bigcup E_k$ are additional visits for each vehicle k marking the start and the end of the route for vehicle k

- Handled by the CP system: time and capacity variables.

Insertion

by CP:

- constraint propagation rules: time windows, load and bound considerations
- search heuristic most constrained variable + least constrained valued (for each *v* find cheapest insertion and choose *v* with largest such cost)
- Complete search: ok for 15 visits (25 for VRPTW) but with heavy tails
- Limited discrepancy search

[Shaw, 1998]

```
Reinsert(RoutingPlan plan, VisitSet visits, integer discrep)
    if |visits| = 0 then
        if Cost(plan) < Cost(bestplan) then
            bestplan := plan
        end if
    else
        Visit v := ChooseFarthestVisit(visits)
        integer i := 0
        for p in rankedPositions(v) and i ≤ discrep do
            Store(plan) // Preserve plan on stack
            InsertVisit(plan, v, p)
            Reinsert(plan, visits - v, discrep - i)
            Restore(plan) // Restore plan from stack
            i := i + 1
        end for
    end if
end Reinsert
```