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SCHEDULING, TIMETABLING AND ROUTING

Lecture 32  
Local Search Methods for VRPTW

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## Outline

Variants of VRP

1. Variants of VRP

## Course Overview

Variants of VRP

- ✓ Problem Introduction
  - ✓ Scheduling classification
  - ✓ Scheduling complexity
  - ✓ RCPSP
- ✓ General Methods
  - ✓ Integer Programming
  - ✓ Constraint Programming
  - ✓ Heuristics
  - ✓ Dynamic Programming
  - ✓ Branch and Bound
- ✓ Scheduling Models
  - ✓ Single Machine
  - ✓ Parallel Machine and Flow Shop
  - ✓ Job Shop
  - ✓ Resource-Constrained Project Scheduling
- Timetabling
  - ✓ Reservations and Education
  - ✓ Course Timetabling
  - ✓ Workforce Timetabling
  - ✓ Crew Scheduling
- Vehicle Routing
  - ✓ Capacitated Models
  - ✓ Time Windows models
    - Rich Models

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## Outline

Variants of VRP

1. Variants of VRP

## Definition

**Rich Models** are non idealized models that represent the application at hand in an adequate way by including all important optimization criteria, constraints and preferences [Hasle et al., 2006]

## Solution

- Exact methods are often impractical:
  - instances are too large
  - decision support systems require short response times
- Metaheuristics based on local search components are mostly used

## VRP with Pickup and Delivery

## Further Input from CVRP:

- each customer  $i$  is associated with quantities  $d_i$  and  $p_i$  to be delivered and picked up, resp.
- for each customer  $i$ ,  $O_i$  denotes the vertex that is the origin of the delivery demand and  $D_i$  denotes the vertex that is the destination of the pickup demand

## Task:

Find a collection of  $K$  simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the current load of the vehicle along the circuit must be non-negative and may never exceed  $Q$
- for each customer  $i$ , the customer  $O_i$  when different from the depot, must be served in the same circuit and before customer  $i$
- for each customer  $i$ , the customer  $D_i$  when different from the depot, must be served in the same circuit and after customer  $i$

## Further Input from CVRP:

- a partition of customers:
  - $L = \{1, \dots, n\}$  Linehaul customers (deliveries)
  - $B = \{n + 1, \dots, n + m\}$  Backhaul customers (collections)
- precedence constraint:
  - in a route, customers from  $L$  must be served before customers from  $B$

**Task:** Find a collection of  $K$  simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity  $Q$ .
- in any circuit all the linehaul customers precede the backhaul customers, if any.

## Multiple Depots VRP

## Further Input from CVRP:

- multiple depots to which customers can be assigned
- a fleet of vehicles at each depot

## Task:

Find a collection of  $K$  simple circuits for each depot with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the current load of the vehicle along the circuit must be non-negative and may never exceed  $Q$
- vehicles start and return to the depots they belong

Vertex set  $V = \{1, 2, \dots, n\}$  and  $V_0 = \{n + 1, \dots, n + m\}$

Route  $i$  defined by  $R_i = \{l, 1, \dots, l\}$

## Further Input from CVRP:

- planning period of  $M$  days

## Task:

Find a collection of  $K$  simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- each customer vertex is visited by exactly one circuit; and
- the current load of the vehicle along the circuit must be non-negative and may never exceed  $Q$
- A vehicle may not return to the depot in the same day it departs.
- Over the  $M$ -day period, each customer must be visited  $l$  times, where  $1 \leq l \leq M$ .

# Split Delivery VRP

Constraint Relaxation: it is allowed to serve the same customer by different vehicles. (necessary if  $d_i > Q$ )

## Task:

Find a collection of  $K$  simple circuits with minimum costs, such that:

- each circuit visit the depot vertex
- the current load of the vehicle along the circuit must be non-negative and may never exceed  $Q$

Note: a SDVRP can be transformed into a VRP by splitting each customer order into a number of smaller indivisible orders [Burrows 1988].

Three phase approach:

1. Generate feasible alternatives for each customer.  
Example,  $M = 3$  days  $\{d1, d2, d3\}$  then the possible combinations are:  
 $0 \rightarrow 000$ ;  $1 \rightarrow 001$ ;  $2 \rightarrow 010$ ;  $3 \rightarrow 011$ ;  $4 \rightarrow 100$ ;  $5 \rightarrow 101$ ;  $6 \rightarrow 110$ ;  
 $7 \rightarrow 111$ .
- | Customer | Diary mand | De- | Number of Visits | Number of Combina-tions | Possible Combina-tions |
|----------|------------|-----|------------------|-------------------------|------------------------|
| 1        | 30         |     | 1                | 3                       | 1,2,4                  |
| 2        | 20         |     | 2                | 3                       | 3,4,6                  |
| 3        | 20         |     | 2                | 3                       | 3,4,6                  |
| 4        | 30         |     | 2                | 3                       | 1,2,4                  |
| 5        | 10         |     | 3                | 1                       | 7                      |
2. Select one of the alternatives for each customer, so that the daily constraints are satisfied. Thus, select the customers to be visited in each day.
  3. Solve the vehicle routing problem for each day.

# Inventory VRP

## Input:

- a facility, a set of customers and a planning horizon  $T$
- $r_i$  product consumption rate of customer  $i$  (volume per day)
- $C_i$  maximum local inventory of the product for customer  $i$
- a fleet of  $M$  homogeneous vehicles with capacity  $Q$

## Task:

Find a collection of  $K$  daily circuits to run over the planing horizon with minimum costs and such that:

- each circuit visit the depot vertex
- no customer goes in stock-out during the planning horizon
- the current load of the vehicle along the circuit must be non-negative and may never exceed  $Q$

## VRP with Satellite Facilities (VRPSF)

Possible use of [satellite facilities](#) to replenish vehicles during a route.

## Open VRP (OVRP)

The vehicles do not need to return at the depot, hence routes are not circuits but paths

## Dial-a-ride VRP (DARP)

- It generalizes the VRPTW and VRP with Pick-up and Delivery by incorporating time windows and maximum ride time constraints
- It has a human perspective
- Vehicle capacity is normally constraining in the DARP whereas it is often redundant in PDVRP applications (collection and delivery of letters and small parcels)