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DM515 – Introduction to Integer and Linear Programming

Study Plan for the Oral Exam

This is the list of questions that can be posed at the oral exam.

The exam will last 40 minutes. The question number 1 will be the initial question. Other questions will be extracted randomly from the remaining questions in the list. It is allowed to bring hand written notes but not course material (slides, textbook, articles). The notes can be consulted if needed but in this case a correct answer will receive less credit in the final evaluation. At each question there are 2-3 minutes to prepare the answer. Refinement questions on the topics presented are to be expected.

1. Describe the simplex method used to solve the following problem:

$$\begin{array}{rc} \min & c^T x \\ \text{s.t.} & Ax \leq b \\ x \geq 0 & \end{array}$$

   In particular, explain and identify with formal notation, the canonical form, the basis, the non basis, the multipliers, the reduced costs, how to determine a feasible initial solution, the pivot operation, the conditions of optimality. A note on the revised simplex method would be a plus. The use of Dantzig's tableau notation is preferred to Chvàtal's dictionary notation.

2. Revise the different types of degenerancies in the simplex. Describe their meaning and how they are treated.

3. Explain and derive, using one of the four methods presented at the lectures, the dual for the following problem:

$$\begin{array}{rc} \min & 6x_1 + 8x_2 \\ \text{s.t.} & 5x_1 + 10x_2 \leq 60 \\ & 4x_1 + 4x_2 \leq 40 \\ & x_1, x_2 \geq 0 \\ & x \in \mathbb{R} \end{array}$$

4. State without proving the Weak Duality Theorem, the Strong Duality Theorem and the Complementary Slackness Theorem.

5. Formulate by integer programming one the following problems: set covering, set packing, maximum matching, 0-1 Knapsack, traveling salesman problem.

6. Solving integer programs: define what the convex hull of an integer program is and sketch the cutting plane algorithm.

7. Solving integer programs: sketch the branch and bound approach mentioning how to obtain upper bounds, dual bounds, the possibilities for relaxations and pruning.

8. Which properties of the constraint matrix of an integer program ensure that it provides a compact description of the convex hull, or in equivalent terms, that the solution of the linear relaxation has integer solutions? Give examples of network problems whose integer programming formulation exhibit such property.

9. Network flow problems. Introduce terminology and formulate in integer programming terms the min cost flow problem. Show why the following problems can be seen as particular cases of the min cost flow problem: shortest path, max flow problem, assignment problem.

10. A car rental company at the beginning of each month wants to have a certain number of cars in each of the towns in which it operates. For the towns $A, B, C, \ldots, G$ the number of cars desired is $30, 40, 55, 60, 80, 40, 55$, respectively. At the end of the current month there are instead in the stations in these towns $65, 90, 95, 15, 60, 10, 25$ cars, respectively. To move one car from one station to the other causes a cost that we may assume proportional to the distance between the two stations. The table indicates the distances (in hundreds of kilometers) between every city pair of stations.

from .. to..	D	E	F	G
A	5	6	10	9
B	9	11	9	15
C	12	10	14	15

Formulate the problem of deciding the cars to move while minimizing the costs in mathematical programming terms. Which algorithm could you use to solve the problem beside the simplex?