

Solving CSP – Overview

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[Based on Slides by Helmut Simonis, University College Cork]

Outline

1. Solving CSP – Overview
 - Constraint Propagation
 - Search

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General Purpose Algorithms

Search algorithms

organize and explore the search tree

- ▶ Search tree with branching factor at the top level nd and at the next level $(n-1)d$. The tree has $n! \cdot d^n$ leaves even if only d^n possible complete assignments.
- ▶ Insight: CSP is commutative in the order of application of any given set of action (the order of the assignment does not influence final answer)
- ▶ Hence we can consider search algs that generate successors by considering possible assignments for only a single variable at each node in the search tree.
The tree has d^n leaves.

Backtracking search

depth first search that chooses one variable at a time and backtracks when a variable has no legal values left to assign.

Backtrack Search

function BACKTRACKING-SEARCH(*csp*) **returns** a solution, or failure
 return RECURSIVE-BACKTRACKING({ }, *csp*)

function RECURSIVE-BACKTRACKING(*assignment*, *csp*) **returns** a solution, or failure
 if *assignment* is complete **then return** *assignment*
 var \leftarrow SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*], *assignment*, *csp*)
 for each *value* **in** ORDER-DOMAIN-VALUES(*var*, *assignment*, *csp*) **do**
 if *value* is consistent with *assignment* according to CONSTRAINTS[*csp*] **then**
 add {*var* = *value*} to *assignment*
 result \leftarrow RECURSIVE-BACKTRACKING(*assignment*, *csp*)
 if *result* \neq failure **then return** *result*
 remove {*var* = *value*} from *assignment*
 return failure

Backtrack Search

- ▶ No need to copy solutions all the times but rather extensions and undo extensions
- ▶ Since CSP is standard then the alg is also standard and can use general purpose algorithms for initial state, successor function and goal test.
- ▶ Backtracking is uninformed and complete. Other search algorithms may use information in form of heuristics

General Purpose Backtracking

Implementation refinements

- 1) [Search] Which variable should we assign next, and in what order should its values be tried?
- 2) [Propagation] What are the implications of the current variable assignments for the other unassigned variables?
- 3) [Search] When a path fails – that is, a state is reached in which a variable has no legal values can the search avoid repeating this failure in subsequent paths?

Search

1) Which variable should we assign next, and in what order should its values be tried?

▶ **Select-Initial-Unassigned-Variable**

degree heuristic (reduces the branching factor) also used as tie breaker

▶ **Select-Unassigned-Variable**

Most constrained variable (DSATUR); fail-first heuristic;

Minimum remaining values (MRV) heuristic (speeds up pruning)

▶ **Order-Domain-Values**

least-constraining-value heuristic (leaves maximum flexibility for subsequent variable assignments)

NB: If we search for all the solutions or a solution does not exist, then the ordering does not matter.

Search

Branching (aka, Labelling)

1. Pick a variable x with at least two values
2. Pick value v from $D(x)$
3. Branch with

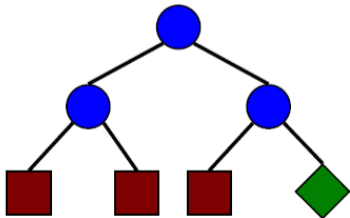
$$x = v$$

$$x \leq v$$

$$x \neq v$$

$$x > v$$

The constraints for branching become part of the model in the subproblems generated



The inner nodes (blue circles) are choices, the red square leaf nodes are failures, and the green diamond leaf node is a solution.

Outline

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Constraint Propagation

2) What are the implications of the current variable assignments for the other unassigned variables?

Definition (Domain consistency)

For a CSP, a constraint C on the variables X_1, \dots, X_k is called **domain consistent** if for each variable X_i and each value $v_i \in D(X_i)$ ($i = 1, \dots, k$), there exists a value $v_j \in D(X_j)$ for all $j \neq i$ such that $(v_1, \dots, v_i, \dots, v_k) \in C$.

Loose definition

Domain **filtering** is the removal of values from variable domains that are not consistent with an individual constraint.

Constraint **propagation** is the repeated application of all domain filtering of individual constraints until no domain reduction is possible anymore.

Constraint Propagation

Three consistency levels

Trade off between speed and propagation

- ▶ Forward checking
- ▶ Bounds consistency
- ▶ Domain consistency

Constraint Propagation

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

Problem shown as matrix

Each cell corresponds to a variable

Instantiated: Shows integer value
(large)

Uninstantiated: Shows values in
domain

Currently active constraint highlighted

Values removed at a step shown in blue

Values assigned at a step shown in red

alldifferent (distinct)

- ▶ Argument: list of variables
- ▶ Meaning: variables are pairwise different
- ▶ Reasoning: Forward Checking (FC)
 - ▶ When variable is assigned to value, remove the value from all other variables
 - ▶ If a variable has only one possible value, then it is assigned
 - ▶ If a variable has no possible values, then the constraint fails
 - ▶ Constraint is checked whenever one of its variables is assigned
 - ▶ Equivalent to decomposition into binary disequality constraints

Forward checking

H. Simonis' demo

Initial State (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
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1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
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4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3	1 2 3 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 2	5	1 2 3 4 5 6 7 8 9	
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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	7 4 5 6 7 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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1	3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 8 9	1 1	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 9	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	
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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	
	3 5 6 8 9	2 4 5 6 9	2 3 4 5 9	1 4 5 6 7 9	7 8 9	2 3 4 5 6 7 9	2 3 4 5 6 7 9	2 3 4 5 6 7 9	2 3 4 5 6 7 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9	
1 3 7 9	4	6 4 5 7 9	1 3 4 7 9	1 3 4 7 9	8 4 5 6 7 9	2 4 5 6 7 9	5 4 5 6 7 9	1 3 4 7 9	
1 3 5 6 7 8 9	9 4 5 7 8 9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 3 5 6 7 8 9	3 4 5 7 8 9	7 4 5 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2 4 5 7 8 9	7 4 5 9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	4 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4 4 5 6 7 8 9	

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	1 5 6 7 9	4 5 6 4 5 9	4 5 4 5 9	8	4 5 6 7 9	4 5 6 7 9	3 2	
1 3 7 9	1 4	6	4 3 7 9	8 2 5	1 3 4	1 3 4	1 3 4	1 3 4
1 3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3 7 6	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	2 7	1 2 3 4 5 9	2 3 4 5 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	1 5 6 7 9	4 5 6 4 5 9	4 5 4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	
1 3 7 9	1 4	6	4 3 7 9	1 3 4 7 9	8 2 5	1 3 4 7 9		
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3 7 6	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 7	1 2 3 4 5 9	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	2
1 3 7 9	1 4	6	4 3 7 9	4 3 8	2 3 9	8 2 5	1 3 4 7 9	3
1 3 5 6 7 9	9	1 2 3 4 5 9	2 3 4 5 7	2 3 4 6	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7 9
1 3 5 6 7 8 9	3	7	6	4 6 9	2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 7 9	1 2 3 6 5 6 9 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 4 5 8	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	2 3 4 7 8 9	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 5 6 9	1 4 5 6 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 9	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 4	1 2 3 4 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8 9	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 6 9	1 2 3 5 6 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2	
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5 9	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7	
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8	
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 6 4 8 9	1 3 6 4 8 9	1 3 6 4 8 9	
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4	

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 3 5 6 9	1 2 3 5 6 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	1 5 6 7 9	4 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	4 5 7	3 2
1 3 7 9	4	6	4 3 7 9	4 3 9	8 2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	4 3 8 9	5	1 3 6 4 9	1 3 8 9	1 3 4 6 4 6 8 9 8 9	1 3 4 6 4 6 8 9 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 7	1 2 3 5 6 7	1 2 3 5 6 7
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3 2	
1 3 7 9	4	6	4 3 7 9	4 3 9	8 2	5	1 3 4 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1 4	5 7 8	3 7 8	2 3 5 6 7 8 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 7	1 2 6 9	1 2 3 5 6 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 5 6 9	1 4 5 7	3 2	
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7 9	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7 9
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8 9	2 3 5 9	2 3 5 9	1 4	2 3 4 5 9	5 3 7 8	2 6 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 9	2 3 4 6 9	1 2 3 5 6 9	6	1 2 4 6 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 7	1 2 6 9	1 2 3 5 6 9
	3 2 5 6 4 5 6 8 9	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 6 8 9	2 4 6 9	2 3 4 5 6 8 9	
1 5 6 7 9	1 4 5 6 7 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7 9	3 2	
1 3 7 9	4	6	4 3 7 9	4 3 9	8 2 5	1 3 4 7 9		
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3 7 6	1 3 4 9	2 3 4 8 9	4	1 2 5 9	9	1 2 4 5 8	
2 7	1 3 4 9	4 3 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 4 6 8 9	
3 2 5 6 7 8 9	2 3 5 8	2 3 5 9	2 3 5 8 9	1 4	5 3 7 8	2 6 7 9	2 3 5 6 7 8 9	
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 8 9	2 3 5 9	1 2 3 5 9	6	1 2 7 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 7	1 2 6 9	1 3 5 6 9
	3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3 4 5 9	2 8 9
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8 2 9	5 9	1 3 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8 9	1 3 5 6 7
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 9	1 3 5 6 9	
	3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 5 6 9	1 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2	
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 7 9	
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7	
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4	1 5 8	
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9	1 3 6 8 9	
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 9	3 5 6 7 8 9	
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4	

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9 9	1 3 5 7	1 2 6 7 9 9	1 3 5 6 9 9	
	3 5 6 9 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9 9	3 4 5 8 9	2 4 6 9 8 9	3 5 6 8 9
1 5 6 7 9	1 5 6	1 5 9	4 5 9 9	8	5 6 9 7	1 4 5 9 7	3	2	
1 3 7 9	1 4	6	4 3 7 9	4 3 9 9	8	2	5	1 3 7 9	
1 3 5 6 7 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6	2 3 5 6 7 7	1 2 3 4 5 7	1 3 6 7	8	1 3 5 6 7 7
1 5 8	3	7	6	2 4	1 2 5 5	9	1 2 4 4	1 5 8	
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 9 9	1 3 6 8 9	
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	3 5 6 7 8 9	
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9 7 9	1 2 3 5 9	6	1 2 7 9	4	

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	^{2 3} 6 9	^{2 3} 5 6 9 7	¹ 5	¹ 7	¹ 6 5 6 9 7
³ 5 6 9	² 5 6	^{2 3} 5 9	1	7	^{2 3} 5 6 9 8	^{4 5} 8	⁴ 6 9	^{5 6} 8 9
¹ 5 6 7 9	¹ 5 6	¹ 5 9	^{4 5} 9	8	^{5 6} 9 7	^{4 5} 8	3	2
1	4	6	⁴ 7 9	³ 4 9	8	2	5	^{1 3} 7 9
5	9	2	^{2 3} 4 5 7	^{2 3} 4 6	^{1 2 3} 5 6 7	^{1 3} 4 5 7	8	^{1 3} 5 6 7
8	3	7	² 4	^{1 2} 5	9	^{1 2} 4	¹ 5 8	
2	7	^{1 3} 4 9	³ 4 8 9	5	^{1 3} 6 4 9 8	^{1 3} 4 6 9 8	^{1 3} 6 8 9	
³ 5 6 7 8 9	² 5 6 8	^{2 3} 5 9 7 8 9	^{2 3} 5 9 7 8 9	1	4	^{5 3} 7 8	² 6 7 9	³ 5 6 7 8 9
¹ 5 7 8 9	^{1 2} 5 8	^{1 2 3} 5 9 7 8 9	^{2 3} 5 9 7 8 9	^{2 3} 9 7 9	^{1 2 3} 5 9	6	^{1 2} 7 9	4

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	^{2 3} 6 9	^{2 3} 5 6 9 7	¹ 5	¹ 7	¹ 6 9 7	¹ 5 6 9
	³ 6 9	² 5 6	³ 5	1	7	^{2 3} 5 6 9 8	^{4 5} 4	⁶ 6 9	^{5 6} 5 6 8 9
	¹ 6 9	¹ 5 6	¹ 5	^{4 5} 4 5	8	^{5 6} 5 6 9 7	^{4 5} 4 5	3	2
1	4	6	³ 7 9	³ 9	8	2	5	³ 7 9	
5	9	2	⁴ 7	³ 4	^{3 1} 6 7	^{3 1} 6 7	³ 8	^{1 3} 7 6	
8	3	7	⁶ 4	² 4	^{1 2} 5	9	^{1 2} 4	¹ 5	
2	7	^{1 3} 4	³ 4 8 9	³ 5	^{1 3} 6 9	^{1 3} 4 8	¹ 6 9	^{1 3} 6 8 9	
	³ 6 9	² 5 6 8	³ 5 9	^{2 3} 5 7 8 9	1	4	⁵ 7 8	² 6 7 9	³ 5 6 7 8 9
	³ 5 9	^{1 2} 5 8	¹ 5	³ 5 7 8 9	^{2 3} 5 9	^{1 2 3} 5 7 9	6	^{1 2} 7 9	4

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	^{2 3} 6 9	^{2 3} 5 6 9 7	¹ 5 7	¹ 6 7 9 7	¹ 5 6 9 9
	³ 6 9	² 5 6	³ 5 9	1 7	^{2 3} 5 6 9 9	^{4 5} 4 8	⁴ 6 9	^{5 6} 5 6 8 9
	¹ 6 9	¹ 5 6	^{4 5} 4 5 9 9	8	^{5 6} 5 6 9 7	^{4 5} 4 5 9 7	3 2	
1 4 6			³ 7 9	³ 9 9	8	2 5	³ 7 9	
5 9 2			³ 4 7	^{3 1} 4 7	^{3 1} 3 7	^{1 3} 4 7	^{1 3} 8 7	⁶ 6 7
8 3 7 6 2 5						9 4 1		
2 7	^{1 3} 4 9	⁴ 4 8 9 9	³ 4 8 9 9	5	^{1 3} 6 9	^{1 3} 4 8 9 8	^{1 3} 4 6 9 8 9 9	^{1 3} 6 8 9 9
	³ 6 9	² 5 6	³ 5 9	^{2 3} 5 8 9 9	1 4	^{5 3} 7 8	² 7 9	³ 6 7 8 9
	³ 6 9	^{1 2} 5 8	¹ 5 9	³ 7 8 9 9	^{2 3} 5 9	^{2 3} 5 9	^{1 2} 6 7	4

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Propagation Steps (Forward Checking)

4	<small>1 2 5 6</small>	8	<small>2 3 5 9</small>	<small>3 6 9</small>	<small>2 3 6 9</small>	<small>1 5 6 7</small>	<small>1 6 7</small>	<small>5 6 7 9</small>
<small>3 6 9</small>	<small>2 5 6</small>	<small>3 5 9</small>	1	7	<small>2 3 6 8</small>	<small>4 5 9</small>	<small>6 9</small>	<small>5 6 8 9</small>
<small>7 6 9</small>	<small>1 5 6</small>	<small>1 5 9</small>	<small>4 5 9</small>	8	<small>6 9</small>	<small>4 5 7</small>	3	2
1	4	6	<small>7 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>4 7</small>	<small>3 4</small>	<small>3 7</small>	<small>1 7</small>	8	<small>3 6</small>
8	3	7	6	2	5	9	4	1
2	7	<small>1 3 4 9</small>	<small>3 4 8 9</small>	5	<small>1 3 6 9</small>	<small>1 3 4 8</small>	<small>1 6 9</small>	<small>3 6 8 9</small>
<small>3 6 7</small>	<small>2 5 8</small>	<small>3 5 9</small>	<small>2 3 7 8 9</small>	1	4	<small>5 7 8</small>	<small>3 7 9</small>	<small>2 6 7 8 9</small>
<small>3 7 9</small>	<small>1 2 5 8</small>	<small>1 5 9</small>	<small>3 7 8 9</small>	<small>2 3 5 9</small>	<small>3 7 9</small>	<small>1 2 3 9</small>	6	<small>1 2 7 9</small>
							4	

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	3	^{2 3} 6 9	¹ 5 7	¹ 6 7	^{5 6} 9
	³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 9	^{4 5} 8	⁶ 5 8
	¹ 6 9	¹ 5 6	^{4 5} 9	8		¹ 6 9	^{4 5} 7	3 2
1	4	6	³ 7 9	³ 9	8	2	5	³ 7
5	9	2	⁴ 7	³ 4	^{3 1} 7	³ 7	8	³ 6 7
8	3	7	6	2	5	9	4	1
2	7	^{1 3} 4 9	³ 4 8 9	5	^{1 3} 6 9	^{1 3} 8	¹ 6 9	³ 6 8 9
³ 6 9	^{5 6} 8	⁵ 9	^{2 3} 5 7 8 9	1	4	^{5 3} 7 8	² 7 9	³ 5 6 7 8 9
^{3 1} 9	⁵ 8	^{1 3} 5 9	^{2 3} 7 8 9	⁵ 9	^{3 1 2 3} 7 9	6	^{1 2} 7 9	4

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	³ 6 9	^{2 3} 6 9	¹ 5 7	¹ 6 7	^{5 6} 7 9
³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 8	^{4 5} 7 9	⁶ 7 9	^{5 6} 8 9
¹ 6 7	^{5 6} 7 9	¹ 5 9	^{4 5} 7 9	8	^{6 4 5} 7 9	³ 7 9	2	
1	4	6	³ 7 9	³ 7 9	8	2	5	³ 7 9
5	9	2	^{4 3} 7 9	⁴ 7 9	^{3 1 3} 7 9	³ 7 9	8	^{3 6} 7 9
8	3	7	6	2	5	9	4	1
2	7	^{1 3} 4 9	³ 8 9	5	^{3 1 3 1} 6 8	³ 7 9	⁶ 7 9	^{3 6} 8 9
³ 6 9	^{5 6} 7 8	⁵ 7 9	^{2 3} 7 8 9	1	4	^{5 3} 7 8	^{2 6} 7 9	^{3 5 6} 7 8 9
^{3 1} 9 8	^{1 3} 5 9	^{2 3} 7 8 9	³ 7 9	^{2 3} 7 9	6	^{1 2} 7 9	4	

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Propagation Steps (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	3	^{2 3} 6 9	¹ 5 7	¹ 6 7	^{5 6} 9
	³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 4 5 9 8		⁶ 5 6 8 9
	¹ 6 7	¹ 5 6	¹ 5 9	4 5	8	¹ 6 4 5 9 7	3	2
1	4	6	³ 7 9	³ 9	8	2	5	³ 7
5	9	2	³ 4 7	³ 4	^{1 3} 7	³ 7	8	³ 6 7
8	3	7	6	2	5	9	4	1
2	7	^{1 3} 4	³ 9 8 9	5	³ 6 9	^{1 3 1} 8	¹ 9	³ 8 9
	³ 6 9	^{5 6} 8	⁵ 9 7 8 9	^{3 2 3} 9 7 8 9	1	4	^{5 3} 7 8	² 7 9 7 8 9
	^{3 1} 9	⁵ 8	^{1 3} 9 7 8 9	^{3 2 3} 9 7 9	3	^{2 3} 9	6	^{1 2} 7 9
								4

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After Setup (Forward Checking)

4	^{1 2} 5 6	8	^{2 3} 5 9	³ 6 9	^{2 3} 6 9	¹ 5 7	¹ 6 7	^{5 6} 9 9
³ 6 9	² 5 6	³ 5 9	1	7	^{2 3} 6 9	^{4 5} 8 8	⁶ 9 9	^{5 6} 8 9
⁶ 7 9	¹ 5 6	¹ 5 9	^{4 5} 9 9	8	^{6 4 5} 9 7	¹ 5 7	3	2
1	4	6	³ 7 9	³ 9 9	8	2	5	³ 7 9
5	9	2	^{4 3} 7 7	⁴ 7 7	^{3 1 3} 7 7	³ 7 7	8	³ 6 7
8	3	7	6	2	5	9	4	1
2	7	^{1 3} 4 9	³ 8 9	5	^{3 1 3 1} 6 9 8 9	³ 9 9	² 9 9	³ 8 9
³ 6 9	^{5 6} 8 8	³ 9 9	^{2 3} 7 8 9	1	4	⁵ 7 8	^{3 2} 7 9	³ 5 7 8 9
^{3 1} 9 8	¹ 5 9	³ 5 9	^{2 3} 7 8 9	³ 9 9	^{2 3} 7 9	6	^{1 2} 7 9	4

Can we do better?

- The alldifferent constraint is missing propagation
 - How can we do more propagation?
 - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables

A Simpler Example

```
:-lib(ic) .
```

```
top:-
```

```
  X :: 1..2,
```

```
  Y :: 1..2,
```

```
  Z :: 1..3,
```

```
  alldifferent ([X, Y, Z]),
```

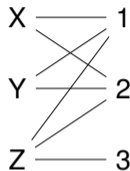
```
  writeln ([X, Y, Z]) .
```


Using Forward Checking

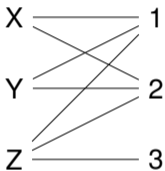
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

Visualization of alldifferent as Graph

- Show problem as graph with two types of nodes
 - Variables on the left
 - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph



A Simpler Example



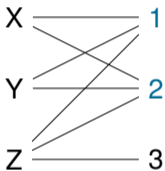
Value Graph for

X :: 1..2,

Y :: 1..2,

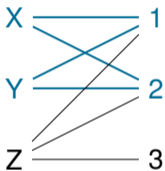
Z :: 1..3

A Simpler Example



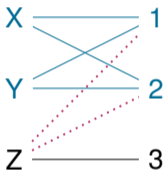
Check interval [1,2]

A Simpler Example



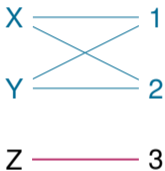
- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

A Simpler Example



No other variable can use that interval

A Simpler Example



Only one value left in domain of Z,
this can be assigned

Idea (Hall Intervals)

- Take each interval of possible values, say size N
- Find all K variables whose domain is completely contained in interval
- If $K > N$ then the constraint is infeasible
- If $K = N$ then no other variable can use that interval
- Remove values from such variables if their bounds change
- If $K < N$ do nothing
- Re-check whenever domain bounds change

Implementation

- Problem: Too many intervals ($O(n^2)$) to consider
- Solution:
 - Check only those intervals which update bounds
 - Enumerate intervals incrementally
 - Starting from lowest(highest) value
 - Using sorted list of variables
- Complexity: $O(n \log(n))$ in standard implementations
- Important: Only looks at min/max bounds of variables

Bounds Consistency

Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

Can we do better?

- Bounds consistency only considers min/max bounds
- Ignores “holes” in domain
- Sometimes we can improve propagation looking at those holes

Another Simple Example

```
:-lib(ic) .
```

```
top:-
```

```
  X :: [1,3],
```

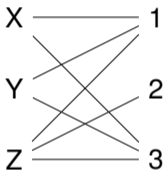
```
  Y :: [1,3],
```

```
  Z :: 1..3,
```

```
  alldifferent ([X,Y,Z]),
```

```
  writeln ([X,Y,Z]) .
```

Another Simple Example



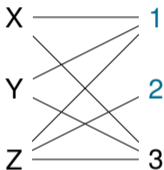
Value Graph for

X :: [1, 3],

Y :: [1, 3],

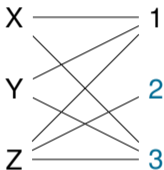
Z :: 1..3

Another Simple Example



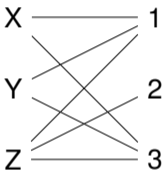
- Check interval $[1,2]$
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



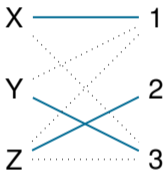
- Check interval $[2,3]$
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



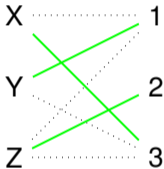
But, more propagation is possible,
there are only two solutions

Another Simple Example



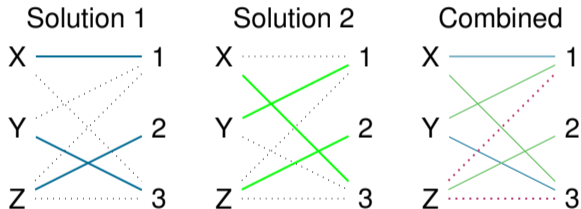
Solution 1: assignment in blue

Another Simple Example



Solution 2: assignment in green

Another Simple Example



Combining solutions shows that $Z=1$ and $Z=3$ are not possible.

Another Simple Example

Can we deduce this without enumerating solutions?

Solutions and maximal matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

Implementation

- Possible to compute all links which belong to some matching
 - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity: $O(n^{1.5}d)$

Domain Consistency

Definition

A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
 - Overkill for simple problems
 - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
 - We have to live with more restricted propagation

Initial State (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4	

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Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3	7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 9 1	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 2 1	2 1 3
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 5 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 7 6	8 7 6	2 1 3	5 4 3	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9 8 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 7 6	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3 2 1	7 6 5	6 5 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 8 7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 1 3	7 6 5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 3	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3	4 3 2	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 5 4	1 2 3 4 5 6 7 8 9	4 3 2

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Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1	3	1 2 3	1 2 3	1	7	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	1	7	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	8	4 5 6	4 5 6	3	2
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	8	2	5	4 5 6	7 8 9
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	9	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	3	7	6	1 2 3	1 2 3	9	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
2	7	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	5	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	1	4	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9
1	3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6	4 5 6
7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	7 8 9	6	7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3	2 3 4 5 6 8 9	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	3 1 2 3 5 6 4 5 6 7 8 9 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1	3 1 2 3 5 6 4 5 6 7 8 9 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 1 2 3 5 6 4 5 6 7 8 9 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 1 2 3 5 6 4 5 6 7 8 9 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 8 9	1 4 5 6 7 8 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 8 9	3 1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3 4 5 6 7 8 9	2 4 5 6 7 8 9
1 5 6 7 8 9	3 1 2 4 5 6 8	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9	2 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 7 8 9	3 4 5 6 7 8 9	7 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 4 5 6 7 8 9	7 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 8 9	2 3 4 5 6 8 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	1 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1	3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1	3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 8	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
	3 5 6 8 9	2 4 5 6 9	2 3 4 5 9	1 4 5 6 7 9	7 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
	1 5 6 7 9	1 4 5 6 9	1 4 5 6 7 9	8 4 5 6 7 9	1 4 5 6 7 9	1 4 5 6 7 9	3 4 5 6 7 9	2 4 5 6 7 9
	1 3 4	6	1 3 4	1 3 4	8 4	2 4	5 4	1 3 4
	1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
	1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9
	2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
	1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
	1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9			
	3 5 6 8 9	2 4 5 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9			
1	1 5 6 7 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2			
1	3 4	6	4 7 9	3 4	1 7 9	3 4	8	2	5	1 4 7 9	3
1	3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9		
1	3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9		
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9			
1	3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 7 8 9	4 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9		
1	3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4		

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5	1 2 3 5 6	1 2 3 5 6	1 2 3 5 6	1 2 3 5 6	1 2 3 5 6
	3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1	1 5 6 7 9	1 4 5 6 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3 2	
1 3 7 9	1 4	6	4 3 7 9	1 3 7 9	8 2 5	1 3 4 7 9		
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2 7	1 2 3 4 5 9	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 5 6	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	4 6 9 7 8 9	2 3 4 5 6 9 7 8 9	1 2 3 4 5 6 9 7 8 9	9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4 4	1 2 4 5 8	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9	1 2 3 5 6 9
5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9	2 3 4 5 6 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4 9	2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 5 6 7 8 9	3 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4 9	2 5	1 2 9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8 9	1 3 4 8 9	1 3 4 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 3 5 6 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 9	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4	2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 3 4 5 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 3 5 6 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4	2 5	1 2 9	9	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 3 4 6 8 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 5 6 7 8 9	2 3 5 6 7 8 9
1 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 6 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	4 9	2 5	1 2 9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	4 3 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 3 7 8	2 6 7 9	2 3 5 6 7 8 9
1 5 6 7 8 9	3 1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 9	6	1 2 4 6 7 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 6 9	1 2 3 5 6 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6 9	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	4	6	4 3 7 9	4 3 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 5 7	2 3 4 6 7	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6 4	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 4 9	1 3 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	5 7 8	3 2 6 7 9	2 3 5 6 7 8 9
1 5 7 8 9	3 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	1 2 3 5 6 9	1 3 5 9	1 2 6 9	1 3 5 6 9
	3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1 7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3 4 5 9	2 8 9
	1 3 7 9	4	4 3 7 9	4 3 9	8	2	5	1 3 7 9
	1 3 5 6 7	9	1 2 3 4 5 7	2 3 4 6 7	2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
	1 5 8	3	7	6	4 2	1 2 5	9	1 5 8
2	7	1 3 4 9	4 3 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
	3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1 4	5 3 7 8	2 6 7 9	3 5 6 7 8 9
	1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 5 9	1 2 3 5 9	6	1 2 7 9
								4

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Propagation Steps (Domain Consistency)

4	^{1 2} _{5 6}	8	^{2 3} _{5 9}	^{2 3} _{6 9}	^{1 2 3} _{5 6 7 9}	^{1 3} _{5 7}	^{1 2} _{6 7 9}	^{1 3} _{5 6 7 9}
³ _{5 6 9}	² _{5 6}	^{2 3} _{5 9}	1	7	^{2 3} _{5 6 9}	³ _{4 5 8}	² _{4 6 9}	³ _{5 6 8 9}
7	¹ _{5 6}	¹ _{5 9}	4	8	¹ _{5 6 9}	^{4 5} ₁	3	2
^{1 3} ₉	4	6	³ _{7 9}	³ ₉	8	2	5	^{1 3} _{7 9}
^{1 3} _{5 6}	9	^{1 2 3} _{4 5 7}	^{2 3} _{4 5 7}	^{2 3} _{4 6 7}	^{1 2 3} _{5 6 7}	^{1 3} _{4 5 7}	8	^{1 3} _{5 6 7}
¹ _{5 8}	3	7	6	² ₄	^{1 2} ₅	9	^{1 2} ₄	¹ _{5 8}
2	7	^{1 3} _{4 9}	⁴ _{8 9}	5	^{1 3} _{6 9}	^{1 3} _{4 8}	^{1 3} _{4 6 9}	^{1 3} _{6 8 9}
³ _{5 6 8 9}	² _{5 6 8}	^{2 3} _{5 9 7 8 9}	^{2 3} _{5 7 8 9}	1	4	⁵ _{7 8}	³ _{7 9}	² _{6 7 8 9}
^{1 3} _{5 8 9}	^{1 2} _{5 8}	^{1 2 3} _{5 9 7 8 9}	^{2 3} _{5 9 7 9}	^{2 3} _{9 7 9}	^{1 2 3} _{5 9}	6	^{1 2} _{7 9}	4

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Propagation Steps (Domain Consistency)

4	^{1 2} _{5 6}	8	^{2 3} _{5 9}	^{2 3} _{6 9}	^{2 3} _{5 6}	^{1 3} _{5 7}	^{1 2} _{7 9}	^{1 3} _{5 6}
³ _{5 6}	² _{5 6}	^{2 3} _{5 9}	1	7	^{2 3} _{5 6}	³ _{4 5}	² _{4 6}	³ _{5 6}
7	¹ _{5 6}	¹ _{5 9}	4	8	^{5 6} ₉	¹ ₅	3	2
^{1 3} ₉	4	6	^{7 9} ₉	³ ₉	8	2	5	^{1 3} _{7 9}
^{1 3} _{5 6}	9	^{1 2 3} _{4 5}	^{2 3} _{7 9}	^{2 3} _{4 6}	^{1 2 3} _{5 6}	^{1 3} ₇	8	^{1 3} _{5 6}
¹ _{5 8}	3	7	6	² ₄	^{1 2} ₅	9	^{1 2} ₄	¹ _{5 8}
2	7	^{1 3} _{4 9}	³ _{8 9}	5	^{1 3} _{6 9}	^{1 3} _{4 8}	¹ _{4 6}	^{1 3} _{9 8}
³ _{5 6}	² _{5 6}	^{2 3} _{5 9}	^{2 3} _{7 8 9}	1	4	^{5 3} _{7 8}	² _{7 9}	³ _{5 6}
^{1 3} _{5 8}	^{1 2} _{5 8}	^{1 2 3} _{5 9}	^{2 3} _{7 8 9}	^{2 3} _{9 7}	^{1 2 3} _{5 9}	6	^{1 2} _{7 9}	4

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Propagation Steps (Domain Consistency)

4	^{1 2} 5 6	8	^{2 3} 5 9	^{2 3} 6 9	^{2 3} 5 6 9 7	¹ 5 7	¹ 6 7 9 7	¹ 5 6 9 7
³ 5 6 9 9	² 5 6	^{2 3} 5 9	1	7	^{2 3} 5 6 9 9	^{4 5} 8 8	⁴ 6 9	^{5 6} 8 9
7	¹ 5 6	¹ 5 9	4	8	^{5 6} 9 9	¹ 5 7	3	2
1	4	6	⁷ 9 9	³ 9 9	8	2	5	^{1 3} 7 9
5	9	2	^{2 3} 7 7	^{2 3} 4 6	^{1 2 3} 5 6 7 7	^{1 3} 4 5 7 7	8	^{1 3} 5 6 7 7
8	3	7	6	² 4 4	^{1 2} 5 5	9	^{1 2} 4 4	¹ 5 8
2	7	4	³ 8 9	5	^{1 3} 6 4 9 8	^{1 3} 4 8	¹ 6 9 7 9	^{1 3} 6 8 9 9
³ 5 6 8 9	² 5 6 8 8	^{2 3} 5 9 7 8 9	^{2 3} 5 9 7 8 9	1	4	⁵ 7 8	² 7 9	³ 5 6 7 8 9
¹ 5 8 9 9	^{3 1 2} 5 8	^{1 2 3} 5 9 7 8 9	^{2 3} 5 9 7 8 9	^{2 3} 9 7 9 9	^{1 2 3} 5 9 9 7 9	6	^{1 2} 7 9	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	¹ ₇	¹ ₉	¹ ₇ ⁵ ₉ ⁶ ₉
³ ₉	5	³ ₉	1	7	2	4	6	8	
7	6	1	4	8	9	5	3	2	
1	4	6	³ ₇ ³ ₉	³ ₉	8	2	5	³ ₇ ³ ₉	
5	9	2	³ ₇	4	1	¹ ₄ ³ ₇	8	6	
8	3	7	6	2	5	9	4	1	
2	7	4	³ ₈ ³ ₉	5	6	8	1	¹ ₈ ³ ₉	⁶ ₉
6	8	³ ₉	2	1	4	⁵ ₇ ³ ₈	² ₇ ⁶ ₉	5	
³ ₉	1	5	8	² ₉ ³ ₉	7	6	2	4	

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9</small>	<small>7 9</small>
<small>3</small>	5	<small>3</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9</small>	<small>3</small>	<small>3</small>	8	2	5
5	9	2	<small>7</small>	<small>3</small>	4	1	<small>7</small>	8
8	3	7	6	2	5	9	4	1
2	7	4	<small>3</small>	5	6	8	1	<small>3</small>
6	8	<small>3</small>	2	1	4	<small>7</small>	<small>7 9</small>	5
<small>3</small>	1	5	8	<small>3</small>	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9</small>	<small>7 9</small>
<small>3</small>	5	<small>3</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9</small>	<small>3</small>	<small>3</small>	8	2	5
5	9	2	<small>7</small>	<small>3</small>	4	1	<small>7</small>	8
8	3	7	6	2	5	9	4	1
2	7	4	<small>3</small>	5	6	8	1	<small>3</small>
6	8	<small>3</small>	2	1	4	<small>7</small>	<small>7 9</small>	5
<small>3</small>	1	5	8	<small>3</small>	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	<small>7 9 7 9</small>	
<small>3 9</small>	5	<small>3 9</small>	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	<small>7 9 3 9</small>	<small>3 9</small>	8	2	5	<small>3 7</small>
5	9	2	<small>7 3</small>	4	1	<small>7 3</small>	8	6
8	3	7	6	2	5	9	4	1
2	7	4	<small>3 9</small>	5	6	8	1	<small>3 9</small>
6	8	<small>3 9</small>	2	1	4	<small>7 3 7 9</small>		5
<small>3 9</small>	1	5	8	<small>3 9</small>	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1		
	5		1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6			8	2	5	
5	9	2		4	1		8	6
8	3	7	6	2	5	9	4	1
2	7	4		5	6	8	1	
6	8		2	1	4			5
	1	5	8		7	6	2	4

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After Setup (Domain Consistency)

4	2	8	5	6	3	1		
	5	1	7	2	4	6	8	
7	6	1	4	8	9	5	3	2
1	4	6			8	2	5	
5	9	2		4	1		8	6
8	3	7	6	2	5	9	4	1
2	7	4		5	6	8	1	
6	8		2	1	4			5
	1	5	8		7	6	2	4

Comparison

Forward Checking

4	2	8	5	6	3	1	7	9
3	5	1	7	8	4	5	6	2
7	6	1	8	9	4	5	3	2
1	4	6	7	8	2	5	7	3
5	9	2	4	7	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	5	8	9	1	3	6
6	5	1	7	8	4	5	6	2
3	1	5	2	3	2	3	1	4

Bounds Consistency

4	2	8	5	6	3	1	7	9
3	5	1	7	8	4	5	6	2
7	6	1	8	9	4	5	3	2
1	4	6	7	8	2	5	7	3
5	9	2	4	7	1	3	8	6
8	3	7	6	2	5	9	4	1
2	7	4	5	8	9	1	3	6
6	5	1	7	8	4	5	6	2
3	1	5	2	3	2	3	1	4

Domain Consistency

4	2	8	5	6	3	1	7	9
3	5	1	7	2	4	6	8	9
7	6	1	4	8	9	5	3	2
1	4	6	7	8	2	5	7	3
5	9	2	4	1	8	6	7	3
8	3	7	6	2	5	9	4	1
2	7	4	5	6	8	1	3	9
6	8	2	1	4	7	5	3	9
3	1	5	8	7	6	2	4	9

Typical?

- ▶ This does not always happen
- ▶ Sometimes, two methods produce same amount of propagation
- ▶ Possible to predict in certain special cases
- ▶ In general, tradeoff between speed and propagation
- ▶ Not always fastest to remove inconsistent values early
- ▶ But often required to find a solution at all

In MiniZinc

You can specify the level of consistency enforced by constraint annotations (sec. 4.3.1.10 page 444)

```
alldiff(x) :: domain
```

Constraint annotations

Annotations can be placed on constraints advising the solver how the constraint should be implemented. Here are some constraint annotations supported by some solvers:

bounds or boundsZ	Use integer bounds propagation.
boundsR	Use real bounds propagation.
boundsD	A tighter version of boundsZ where support for the bounds must exist.
domain	Use domain propagation.
priority(k)	where k is an integer constant indicating propagator priority.

Outline

1. Solving CSP – Overview
 - Constraint Propagation
 - Search

Search

3) When a path fails – that is, a state is reached in which a variable has no legal values can the search avoid repeating this failure in subsequent paths?

Backtracking-Search

- ▶ chronological backtracking, the most recent decision point is revisited
- ▶ backjumping, backtracks to the most recent variable in the conflict set (set of previously assigned variables connected to X by constraints).

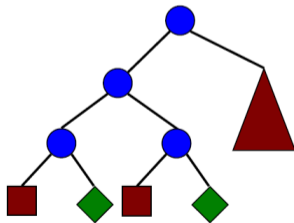
every branch pruned by backjumping is also pruned by forward checking

idea remains: backtrack to reasons of failure.

Optimization Problems

Objective function to minimize $F(X_1, X_2, \dots, X_n)$

- ▶ Naive approach: find all solutions and choose the best
- ▶ Branch and Bound approach
 - ▶ Solve a modified Constraint Satisfaction Problem by setting an (upper) bound z^* in the objective function



Dichotomic search: U upper bound, L lower bound $M = \frac{U+L}{2}$

Types of Variables and Values

- ▶ Discrete variables with finite domain:
complete enumeration is $O(d^n)$
- ▶ Discrete variables with infinite domains:
Impossible by complete enumeration.
Propagation by reasoning on bounds.
Eg, project planning.

$$S_j + p_j \leq S_k$$

NB: if only linear constraints, then integer linear programming

- ▶ Variables with continuous domains (time intervals)
branch and reduce
NB: if only linear constraints or convex functions then mathematical programming
- ▶ structured domains (eg, sets, graphs)

Basic Framework

Solve(CSP)

bool continue;

continue:= TRUE;

while continue AND NOT Done() **do**

 Preprocess;

 Constraint Propagation;

if NOT Done() **then**

if Atomic() **then**

 | continue:= FALSE

else

 | Split();

 | Proceed by Cases

- ▶ Atomic() checks if the CSP can be further split or further search is necessary
- ▶ Proceed by Cases calls **Solve** recursively
- ▶ Split() by enumeration, labelling, bisection, disjunctive constraints, compound constraints