DM810

Computer Game Programming II: AI

# Lecture 6
# Pathfinding

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# Outline

1. Pathfinding

2. Heuristics

3. World Rerpresentations

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3. World Rerpresentations

# Best first search

State Space Search
We assume:

- A start state
- A successor function
- A goal state or a goal test function

- Choose a metric of best
  Expand states in order from best to worst

- Requires:
  Sorted open list/priority queue
  closed list
  unvisited nodes

# Best first search

Definitions

- Node is expanded/processed when taken off queue
- Node is generated/visited when put on queue
- $g$-cost is the cost from the start to the current node
- $c(a, b)$ is the edge cost between $a$ and $b$

Algorithm Measures

- Complete
  Is it guaranteed to find a solution if one exists?
- Optimal
  Is it guaranteed the find the optimal solution?
- Time
- Space

# Best-First Algorithms

## Best-First Pseudo-Code

```
Put start on OPEN
While(OPEN is not empty)
  Pop best node n from OPEN
  if (n == goal) return path(n, goal)
  for each child of n: # generate children
    put/update value on OPEN/CLOSED
return NO PATH
```

## Best-First child update

```
If child on OPEN, and new cost is less
  Update cost and parent pointer
If child on CLOSED, and new cost is less
  Update cost and parent pointer, move node
        to OPEN
Otherwise
  Add to OPEN list
```

# Search Algorithms

**Dijkstra's algorithm** $\equiv$ **Uniform-Cost Search (UCS)**
$\rightsquigarrow$ Best-first with $g$-cost
Complete? Finite graphs yes, Infinite yes if $\exists$ finite cost path + weights $> \epsilon$
Optimal? yes

*Idea*: reduce fill nodes: Heuristic: estimate of the cost from a given state to the goal

Pure Heuristic Search / **Greedy Best-first Search (GBFS)**
$\rightsquigarrow$ Best-first with $h$-cost
Complete? Only on finite graph
Optimal? No

**A**$^*$
$\rightsquigarrow$ best-first with $f$-cost, $f = g + h$
Optimal? depends on heuristic

# Theorem

If the heuristic is:

- admissible, i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from $n$ ($h(n) \geq 0$, so $h(G) = 0$ for any goal $G$)
- consistent (triangular inequality holds, see later)

then when A* selects a node for expansion (smallest estimated-total-cost), the optimal path to that node has been found.

E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Note:

- consistent $\Rightarrow$ admissible
- if the graph is a tree, then admissible is enough.

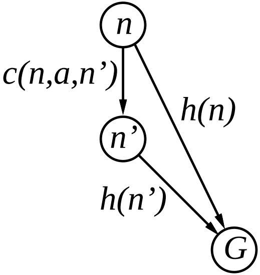
# Consistency

A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If $h$ is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

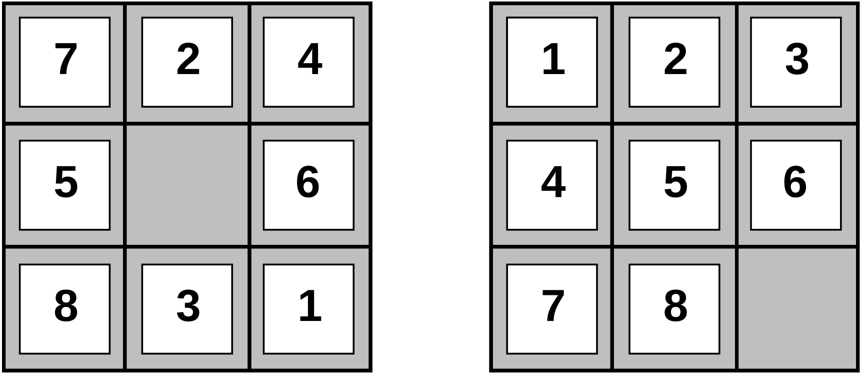



I.e., $f(n)$ is nondecreasing along any path.

# Admissible heuristics

E.g., for the 8-puzzle:

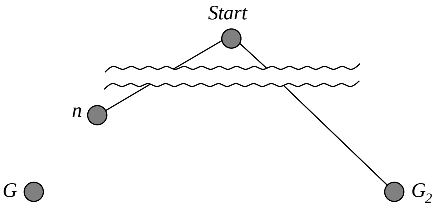
$h_1(n)$ = number of misplaced tiles
1 $h_2(n)$ = total Manhattan distance(i.e., no. of squares from desired location of each tile)




Start State Goal State

# Optimality of A* (standard proof)

Suppose some suboptimal goal $G_2$ has been generated and is in the queue. Let $n$ be an unexpanded node on a shortest path to an optimal goal $G_1$.

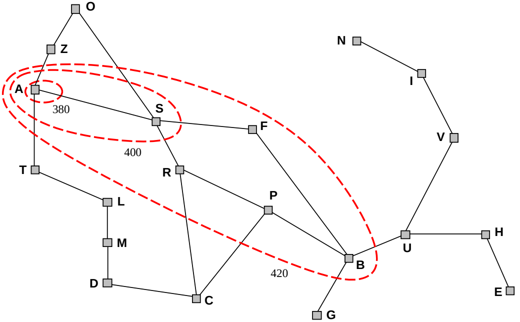



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

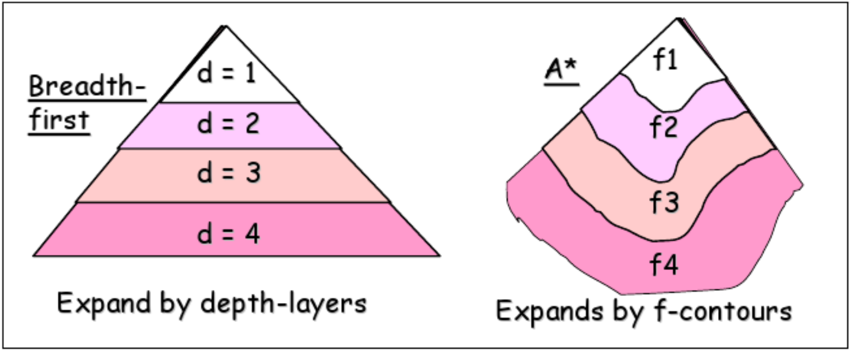
Since $f(G_2) > f(n)$, A* will never select $G_2$ for expansion

# Optimality of A*

Lemma: A* expands nodes in order of increasing $f$ value*
Gradually adds "$f$-contours" of nodes (cf. breadth-first adds layers)
Contour $i$ has all nodes with $f = f_i$, where $f_i < f_{i+1}$



# A* vs. Breadth First Search



# Properties of A$^*$



Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Optimal? Yes—cannot expand $f_{i+1}$ until $f_i$ is finished

A$^*$ expands all nodes with $f(n) < C^*$
A$^*$ expands some nodes with $f(n) = C^*$
A$^*$ expands no nodes with $f(n) > C^*$

Time $O(lm)$, Exponential in [relative error in $h \times$ length of sol.]
$l$ number of nodes whose total estimated-path-cost is less than that of the goal.
Space $O(lm)$ Keeps all nodes in memory

# Data Structures

Same as Dijkstra:

*list used to accumulate the final path*: not crucial, basic linked list

*graph* : not critical: adjacency list, best if arcs are stored in contiguous memory, in order to reduce the chance of cache misses when scanning

*open and closed lists*: critical!

1. push
2. remove
3. extract min
4. find an entry

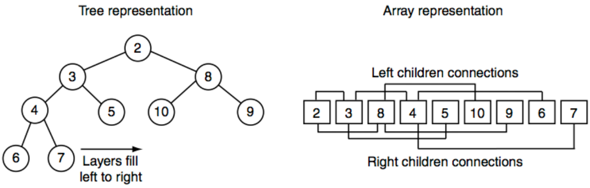
**Priority queues**

keep list sorted by finding right insertion point when adding.
If we use an array rather than a linked list, we can use a binary search

**Priority heaps**

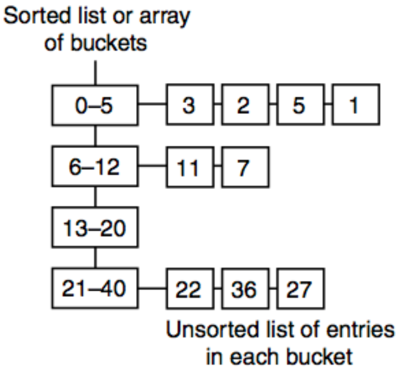
- array-based data structure which represents a tree of elements.
- each node has up to two children, both with higher values.
- balanced and filled from left to right
- node $i$ has children in positions $2i$ and $2i+1$

- extract min in $O(1)$
- adding $O(\log n)$
- find $O(\log n)$
- remove $O(\log n)$



# Bucketed Priority Queues

- partially sorted data structure
- buckets are small lists that contain unsorted items within a specified range of values.
- buckets are sorted but their contents not
- exctract min: go to the first non-empty bucket and search its contents
- find, add and remove depend on numebr of buckets and can be tuned.
- extensions: multibuckets



# Implementation Details

Data structures:

- author: depends on the size of the graph with million of nodes bucket priority list may outperform priority buffer
  But see http://stegua.github.com/blog/2012/09/19/dijkstra/

Heuristics:

- implemented as functions or class.
- receive a goal so no code duplication
- `pathfindAStar(graph, start, end, new Heuristic(end))`
- efficiency is critical for the time of pathfind
  Problem background, Pattern Databases, precomputed memory-based heuristic

Other:

- overall must be very fast, eg, 100ms split in 1ms per frame
- 10MB memory

- Break ties towards states with higher g-cost
- If a successor has f-cost as good as the front of OPEN
  Avoid the sorting operations
- Make sure heuristic matches problem representation
  With 8-connected grids don't use straight-line heuristic
- weighted A$^*$: $f(n) = (1 - w)g(n) + wh(n)$

# Node Array $A^*$

- Improvement of A$^*$ when nodes are numbered with sequential integers.
- Trade memory for speed
- Allocate array of pointers to records for all nodes of the graph. (many nodes will be not used)
- Thus Find in $O(1)$
- A field in the record indicates: unvisited, open, or closed
- Closed list can be removed
- Open list still needed

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# Heuristics

Admissible (underestimating):

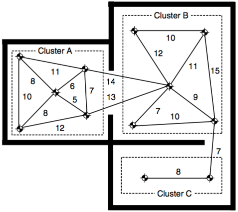
- has the nice properties of optimality
- more influence by cost-so-far
- increases the runtime, gets close to Dijkstra
- in practice beliviability is more important than optimality

Inadmissible (overestimating)

- less influence by cost-so-far
- if overestimate by $\epsilon$ then path at most $\epsilon$ worse

Common heuristics

- Euclidean heuristic (straght line without obstacles, underestimating) good in outdoor, bad in indoor
- Octile distance
- Cluster heuristic: group nodes together in clusters (eg, cliques) representing some highly interconnected region.
  Precompute lookup table with shortest path between all pairs of clusters.
  If nodes in same cluster then Euclidean distance else lookup table

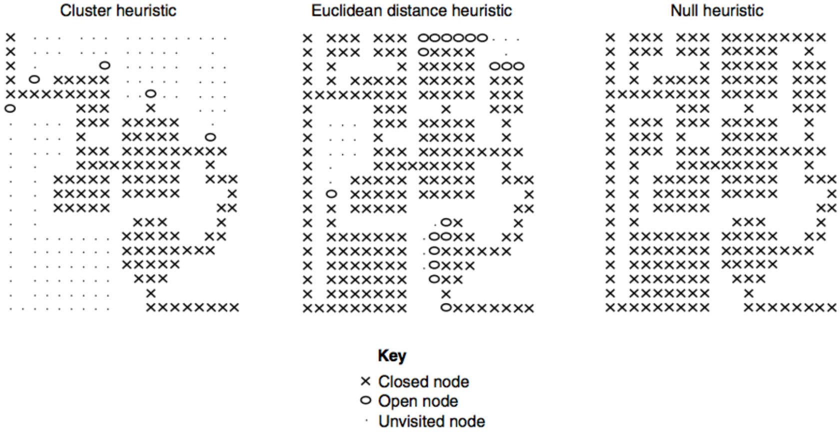



| | A | B | C |
|---|---|---|---|
| A | x | 13 | 29 |
| B | 13 | x | 7 |
| C | 29 | 7 | x |

Lookup table

Problems: all nodes of a cluster will have the same heuristic. Maybe add Euclidean heuristic in the cluster?

## Visualization of the fill



# Dominance

If $h_2(n) \geq h_1(n)$ for all $n$ (both admissible)
then $h_2$ dominates $h_1$ and is better for search

Typical search costs:

| | |
|---|---|
| $d = 14$ | IDS = 3,473,941 nodes |
| | A$^*(h_1)$ = 539 nodes |
| | A$^*(h_2)$ = 113 nodes |
| $d = 24$ | IDS $\approx$ 54,000,000,000 nodes |
| | A$^*(h_1)$ = 39,135 nodes |
| | A$^*(h_2)$ = 1,641 nodes |

Given any admissible heuristics $h_a$, $h_b$,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates $h_a$, $h_b$

# Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

- If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution
- Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

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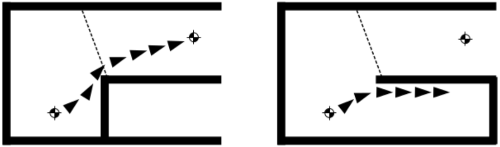
# World Representations

Division scheme: the way the game level is divided up into linked reagions that make the nodes and edges.
Properties of division schemes:

- quantization/localization
  from game world locations to graph nodes and viceversa

- generation
  how a continous space is split in regions
  *manual techniques*: Dirichlet domain
  *algorithmic techniques*: tile graphs, points of visibility, and navigation meshes

- validity
  all points in two connected reagions must be reachable from each other.

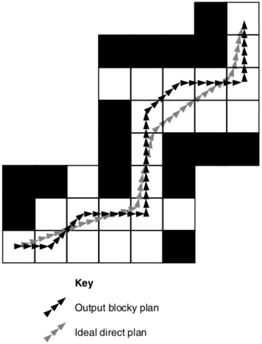


# Tile graphs

Division scheme:
Tile-based levels split world into regular square (or exagonal) regions.
(in 3D, for outdoor games graphs based on height and terrain data.)
Nodes represent tiles, connections with 8 neighboring tiles

Quantization and Localization
Each point is mapped in a tile by:

```
tileX = floor(x / tileSize)
tileZ = floor(z / tileSize)
```



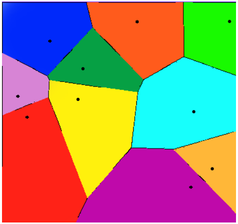

Generation:
automatic at run time, no need to store separately. Allow blocked tiles.

Validity:
with partial blockage might be not guaranteed.

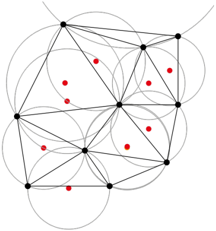
Remarks:
it may end up with large number of tiles
paths may look blocky and irregular

# Dirichelet Tassellation

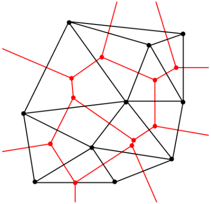
Way of dividing space into a number of regions (aka Vornoi diagram/decomposition)
A set of points (called seeds or sites) is specified beforehand.
For each seed there will be a corresponding region consisting of all points closer to that seed than to any other.



Dual of Delaunay triangulation



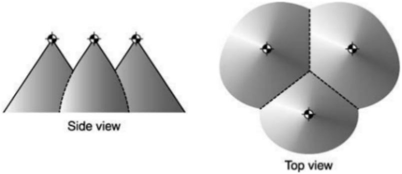
no point inside circumcircles of triangles (their centers in red).



connecting circumcircles ⇝ Vornoi decomposition

**Division scheme**:
Seeds (characteristic points) usually specified by a level designer as part of the level data
connections between bordering domains

Regions can be also left to define to the designer or cone representation and point of view.
weighted Dirichlet domain: each point has an associated weight value that controls the size of its region.




**Quantization and Localization**
find closest seed: use some kind of spatial partitioning data structure (ex $k$d-trees, as quad-tree, octree, binary space partition, or multi-resolution map)

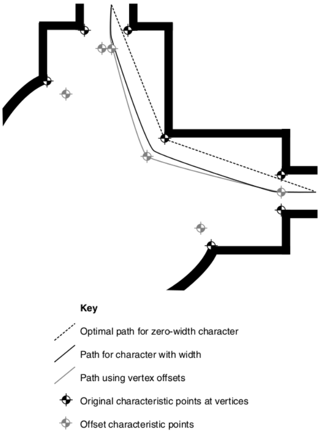
**Validity**
may lead to invalid paths. Leave Obstacle and Wall Avoidance on.

# Points of Visibility

Inflection points: points on the path where the direction changes, may not be feasible for the character due to collision. Need to be moved.

Division scheme:
inflection points: Look at level geometry (maybe costly) or generate specially.
connection is made if the ray doesn't collide with any other geometry

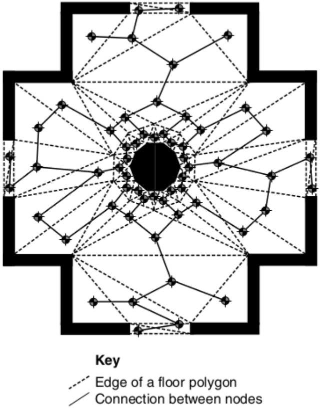
Quantization:
Points of visibility are usually taken to represent the centers of Dirichlet domains



# Navigation Meshes

Navmesh: Designer specifies the way the level is connected and the regions it has by defining the graphical structure made up of polygons connected to other polygons.

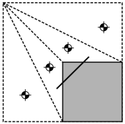
Division scheme:
floor polygons are nodes
connections if polygons share an edge



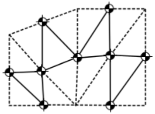

Quantization and Localization:
Coherence refers to the fact that, if we know which location a character was in at the previous frame, it is likely to be in the same node or an immediate neighbor on the next frame. Check first these nodes. (note, polygons must be convex)

Validity:

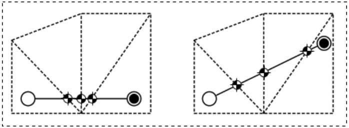
Not always guaranteed



Alternative division scheme: polygon-as-node vs edge-as-node
nodes on the edges between polygons and connections across the face of each polygon.
used in association with portal-based rendering, where nodes are assigned to portals and connections link portals on the same (convex) polygon.



Nodes may move on the edge.

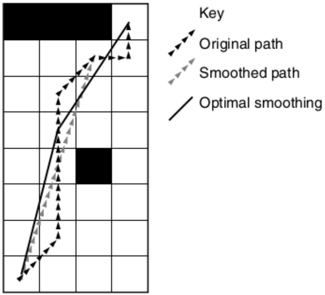


# Other Issues

- Non-translational problems: nodes may indicate not only positions but also orientations
- Cost maybe more than simple distance
- Different cost functions for different characters (tactical pathfinding)
- Erratic paths
  portal representations with points of visibility tend to give smooth paths
  tile-based graphs tend to be erratic.
  steering behaviours can take care of this.

# Path smoothing

```
def smoothPath(inputPath):
  if len(inputPath) == 2: return inputPath
  outputPath = [inputPath[0]]
  # We start at 2, because we assume two adjacent
  # nodes will pass the ray cast
  inputIndex = 2
  while inputIndex < len(inputPath)-1:
    if not rayClear(outputPath[len(outputPath)-1],
      inputPath[inputIndex]):
      outputPath += inputPath[inputIndex-1]
    inputIndex ++
  outputPath += inputPath[len(inputPath)-1]
  return outputPath
```

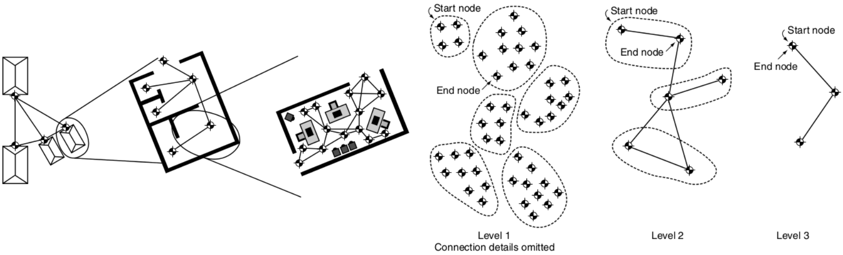



Note: output is a list of nodes that are in line of sight but among which we may have no connection

# Hierarchical Pathfinding



- multi-level plan: plan an overview route first and then refine it as needed.
- we only need to plan the next part of the route when we complete a previous section.
- grouping locations together to form clusters.




- edges between clusters that are connected.
- costs not trivial: heuristics: minimum distance, maximum distance, average minimum distance

Further speed-up:
Consider only nodes that are within the group that is part of the path.