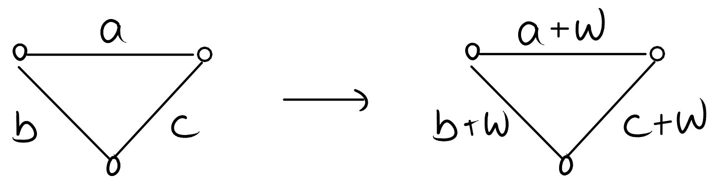
Sheet 4

1. a) Add $W = \max_{e \in E} \{w_e\}$ to all edge weights.

The resulting weights are metric:



$$a + W \leq 2W \leq (b+W) + (c+W)$$

b) For any tour T

$\underline{T}$ optimal in $G \iff T$ optimal in G'

For any tour T in G, let $w(T)$ be the total weight of T in G and let $w'(T)$ be the total weight of T with the modified weights.

Then $w'(T) = w(T) + \underbrace{nW}$

only this part can vary

This part is the same for any tour

Hence, w' is minimized when w is minimized

c) Contradiction with inapproximability?

The reduction to the metric case is not approx. factor preserving.

Ex:

$W = 4$ $\longrightarrow$ $W = 4004$

$W = 1003$ $\quad$ $W = 5003$

ratio ≈ 250 $\quad$ ratio $\approx 5/4$

2. Alg. for Euler tour in connected graph

v ← any vertex in the graph
Follow non-traversed edges, starting in v, until reaching a vertex with no non-traversed edges
While ∃ non-traversed edges
    v ← vertex with both traversed and non-traversed edges
    Follow non-traversed edges, starting in v, until reaching a vertex with no non-traversed edges

Correctness:

When reaching a vertex with no non-traversed edges, the vertex has an even # traversed edges. This can only be v, so we have produced a tour.

Since the graph is connected, there must be a non-traversed edge leaving the tour, if there are still non-traversed edges.

Ex: