DUAL BOUND DERIVATION FROM COL. GENERATION

z_{IP} from Master P (IP)

↓

z_{LRMP} from LRMP (LP) Lin. Rel.

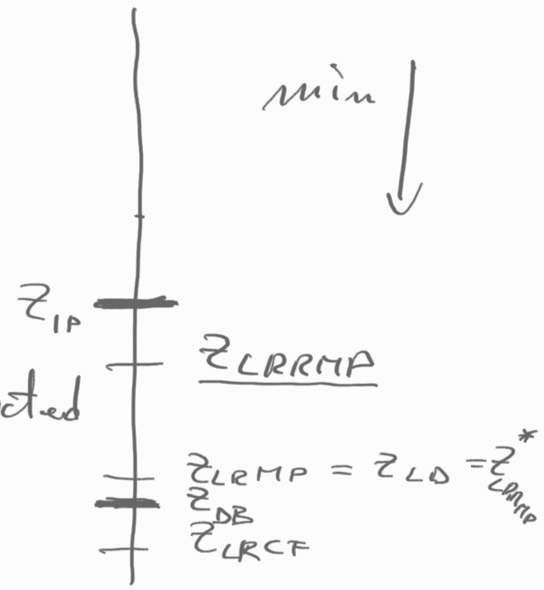
↓

z_{LRRMP} from LRRMP (LP) Lin. Rel. Restricted

z^*_{LRRMP}

subproblem

z_{PP} pricing Problem



$$\boxed{z_{LD} = z^*_{LRRMP} = z_{LRMP}}$$ because of simplex theory

$$\boxed{z_{LRRMP} \geq z_{LRMP}}$$ because LRRMP has not all needed columns

z_{LRRMP} is thus an upper bound.

Let's derive a lower bound.

Assume following LRMP (ie, the one on slide 11 with $K = 1$)

PRIMAL LRMP	DUAL LRMP
$\min c \sum_{t \in T} \lambda_t x^t$	$\max b y_1 + y_0$
$y_1 \quad \sum_{t \in T} \lambda_t A x^t = b$	$A^T x^t y_1 + y_0 \leq c x^t \quad \forall t \in T$
$y_0 \quad \sum_{t \in T} \lambda_t = 1$	$y_1 \in \mathbb{R}$
$\lambda_t \geq 0$	$y_0 \in \mathbb{R}$

Reduced cost for a new col: | Violated constraint

$cx^t - Ax^t y_1 - y_0 < 0$ for a $t \in T$ | $A x^t y_1 + y_0 > cx^t$

$cx^t - Ax^t y - y_0 < 0$

$$z_{PP} = cx^t - Ax^t y_1^* - y_0^*$$

In the reduced problem x^t is not known while (y_1^*, y_0^*) is the optimal solution to the current LP problem in the reduced formulation. We find x^t by solving the subproblem:

$$SB: \quad z_{PP}^* = \min_{x^t} z_{PP}$$

Hence:

$$z_{PP}^* \leq cx^t - Ax^t y_1^* - y_0^* \qquad \forall x^t \in \underbrace{X}_{\text{set of combinatorial structures to choose from}}$$

it means that no other yet unexpressed column has a better reduced cost.

Since we can select columns at most such that $\sum_{t \in T} \lambda^t = 1$ then:

$$z_{DB} = z_{LRRMP} + 1 \cdot z_{PP} \leq z_{LRMP}$$

if we had $\sum_{t \in T} \lambda^t \leq k$ then:

$$\boxed{z_{DB} = z_{LRRMP} + k\, z_{PP} \leq z_{LRMP}}$$

dual bound, here a lower bound!

- An alternative way to derive the dual bound is using the dual problem.

Let as above:

$$z_{PP}^* \leq c x^t - A x^t y_1^* - y_0^*$$

then:

$$c x^t - A x^t y_1^* - y_0^* - z_{PP}^* \geq 0 \qquad \forall x \in X$$

which means that $(y_1^*, y_0^* + z_{PP}^*)$ is a feasible solution for the dual of LRMP (the complete form not the reduced form).

By weak duality theorem (since we have feasible solutions but not yet optimal for LRMP) we can write:

$$\underbrace{b y_1^* + y_0^* + z_{PP}}_{\text{obj func of dual for solution } (y_1^*,\, y_0^* + z_{PP})} \leq Z_{LRMP}$$

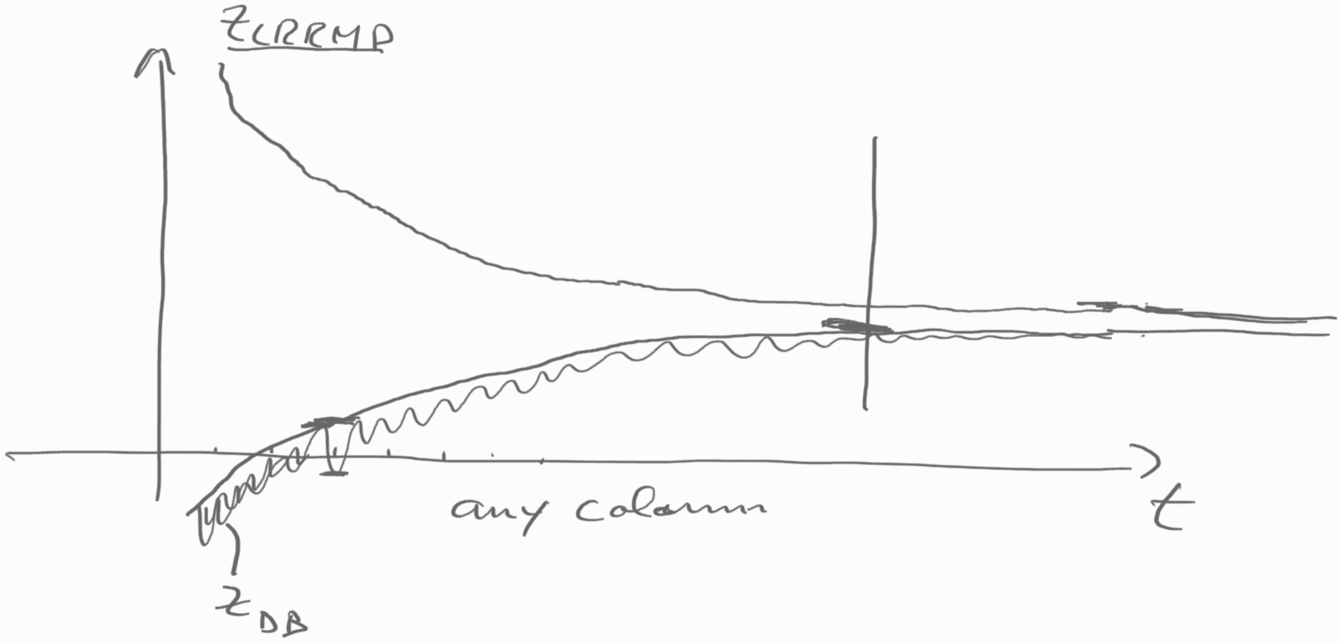
Since (y_1^*, y_0^*) is opt sol to the dual of LRRMP then by strong duality th:

$$z_{LRRMP} = b y_1^* + y_0^*$$

and

$$z_{DB} = z_{LRRMP} + z_{PP} \leq z_{LRMP}$$

During the column generation procedure the values of z_{LRRMP} and z_{DB} go as follows:



When $z_{PP} = 0$ the gap closes and we can stop the generation because of reached optimality.

If we stop earlier we can have however a lower bound that is worth for the B&B of MP.