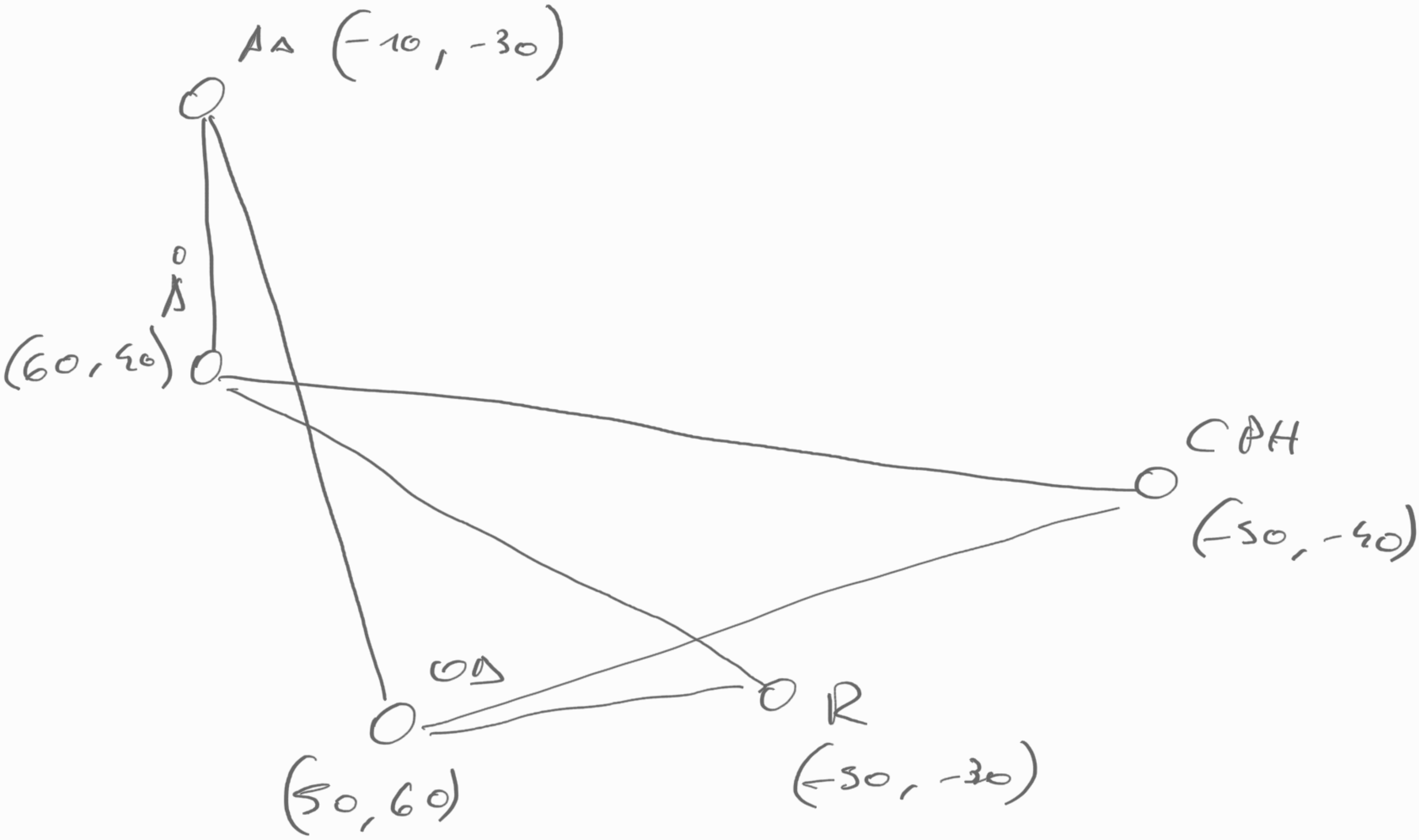
MULTICOMMODITY NETWORK FLOW



$$\min \sum_{1 \le k \le K} c^k x^k$$

Bundle constr. $\sum x_{ij}^k \le u_{ij} \quad \forall (ij) \in A$

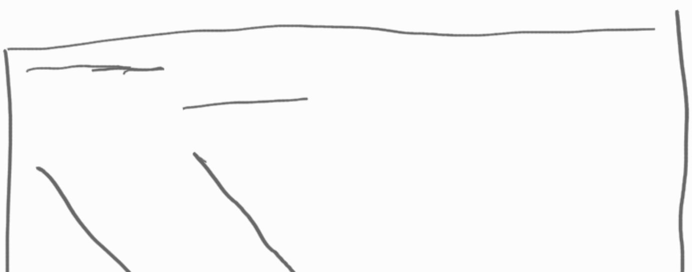
mass bal. $N x^k = b^k \quad \forall k = 1 \dots K$

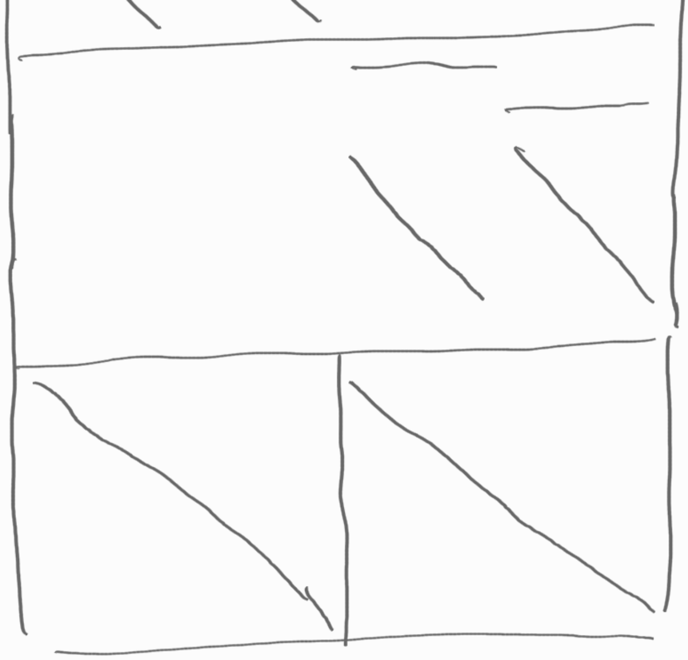
$$0 \le x_{ij}^k \le u_{ij}^k \quad \forall (ij) \in A,\ k = 1 \dots K$$

$$x_{ij}^k \text{ integral}$$

Divisible goods $\Rightarrow$ LP

Indivisible goods $\Rightarrow$ IP





Simplification:

- single source and target $\forall k$, s^k, t^k, d^k
- no flow bounds on individual comm.

• Reformulation

let P^k be set of all directed paths from s^k to t^k in G

let $\lambda(p)$ be the flow associated to $p \in P^k$

let $\delta_{ij}(p)$ be the arc-path indicator

Hence we can decompose

$$x^k_{ij} = \sum_{p \in P^k} \delta_{ij}(p) \lambda(p)$$

let $c^k(p) = \sum_{ij \in A} c^k_{ij} \delta_{ij}(p) = \sum_{ij \in p} c^k_{ij}$ the cost of the path

$$= \sum \sum c^k(p) \lambda(p)$$

$$\min \quad \sum_k \sum_{p\in P^k} c(p)\lambda(p)$$

$$w_{ij} \quad m \qquad \sum_k \sum_{p\in P^k} \delta_{ij}(p)\lambda(p) \le u_{ij} \qquad \forall\, ij \in A$$

$$\mu^k \quad K \qquad \sum_{p\in P^k} \lambda(p) = d^k \qquad \forall\, k = 1..K$$

$$\lambda(p) \ge 0 \qquad \forall\, k,\ \forall\, p \in P^k$$

$O(m+K)$ against $O(m+mK)$ of the other form.

$$\bar{c}_p = c^k(p) + \sum_{ij\in p} w_{ij} - \mu^k =$$

$$= \sum_{ij\in p} \left(c^k_{ij} + w_{ij} \right) - \mu^k$$

if $\bar{c}_p < 0$ add column p

Solve $\bar{c}_p^* = \min \sum_{ij\in p} \left(c^k_{ij} + w_{ij} \right)$

if $\bar{c}_p^* < 0$ add the column.