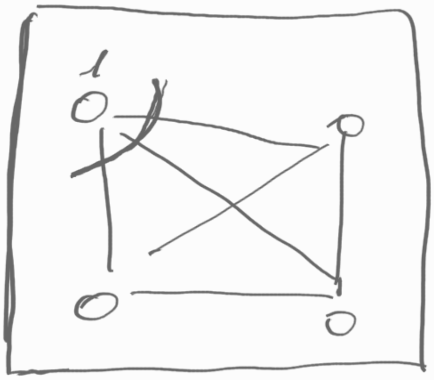
Practice on TSP

Notes associated to Sheet 2

- SYM formulation is not TUM

$$\min \; c^T x$$
$$Ax \leq b$$
$$x \geq 0$$



$= 2 \quad \forall i$

$$\begin{array}{c|cccccc} & 12 & 13 & 14 & 23 & 24 & 34 \\ \hline 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 & 0 \\ \hline 3 & 0 & 1 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 2 \end{bmatrix}$$

$\boxed{\text{SYM}}$

1: $x_{12} + x_{13} + x_{14} = 2$

2: $x_{12} + x_{23} + x_{24} = 2$

3: $x_{13} + x_{23} + x_{34} = 2$

4: ...

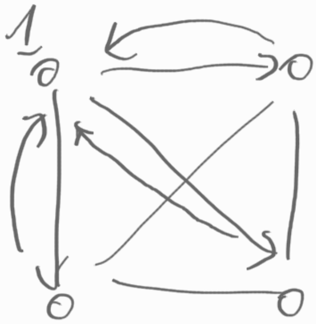
- ASYM formulation is TUM
(but has twice as many vars)

ASYM

MASS BALANCE CONSTRAINTS

ON NODES

$$\min \sum x_{ij} c_{ij}$$



$$\begin{cases} ⓐ \ \sum_j x_{ij} = \sum_j x_{ji} & \forall i \in V \\ ⓑ \ \sum_j x_{ij} = 1 & \forall i \in N \end{cases}$$

m.display()   ⓒ $\cancel{\sum_{ij \in S} x_{ij} \leq |S| - 1 \quad \forall\ 1 \leq |S| \leq |V| - 1}$

$S \subset V$
$S \neq \phi$

$$x_{ij} \in \mathbb{B}$$

$$\begin{cases} 1: & x_{12} + x_{13} + x_{14} - x_{21} - x_{31} - x_{41} = 0 \\ 2: & \\ 3: & \\ 4: & \end{cases}$$

$$\begin{cases} 1: & x_{12} + x_{13} + x_{14} = 1 \\ 2: & \end{cases}$$

ⓐ

$$\begin{array}{cc|ccccccccc|c} & & x_{12} & 13 & 14 & 21 & 23 & 24 & \cdots & & & \\ \hline \multirow{4}{*}{S} & 1 & 1 & 1 & 1 & -1 & & & -1 & & -1 & \\ & 2 & -1 & 0 & 0 & 1 & & & & & & 0 \\ & 3 & 0 & 0 & -1 & 0 & & & & & & \\ & 4 & 0 & 0 & 0 & 0 & & & & & & \end{array}$$

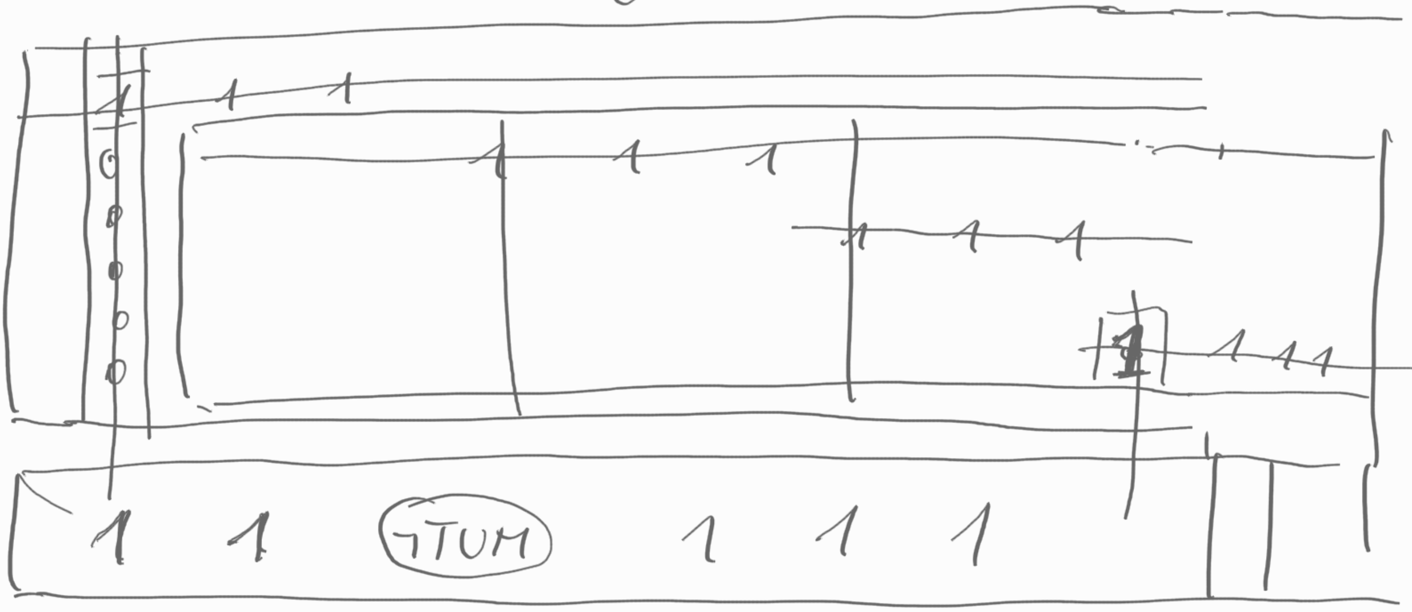
$S = \emptyset$

$\Uparrow$

TUM

$\Downarrow$

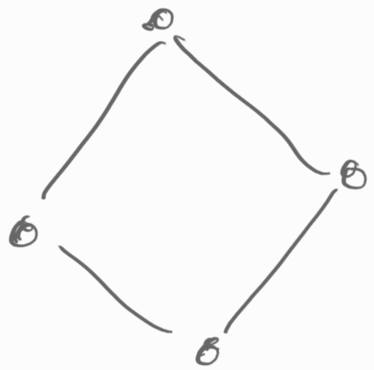
(b)

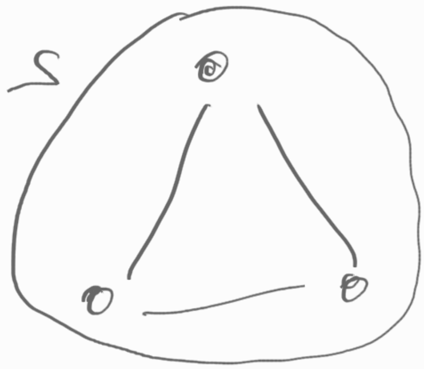


SEPARATION PROBLEM

(find violated subtour elimination const.)

Given solution x^* $\Rightarrow$





Find S:

$$\sum_{e\in E(S)} x_e^* \le |S|-1 \qquad \forall\, S \neq V,\ S\neq\emptyset$$

$$\Rightarrow \sum_{e\in E(S)} x_e^* > |S|-1$$

$V_0 = V\setminus\{0\}$

SEP2

$\underline{z_i \in B}\ \forall\, i \in V$

$$\sum_{i,j\in V} x_{ij}^* \boxed{z_i z_j} - \sum_{i\in V_0} z_i + 1 = \zeta_k$$

$k \in V_0$ arbitrarily chosen

$$\boxed{\max \; \xi_k \;\Big|\; - \sum_{i \in V_0 \setminus \{k\}} z_i \qquad z_k = 1}$$

Binary Quadratic Progr. Prob.

$$z_i = 1 \quad \forall\, i \in V$$

$$\sum_{ij} x^*_{ij} = n$$

$$\boxed{\sum_{ij \in A} x^*_{ij} \, z_i \, z_j}$$

$$\vec{X}^* = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{array}{l} x_{12} \\ x_{13} \\ x_{14} \\ x_{21} \\ \vdots \\ \\ \\ \\ \\ \end{array}$$

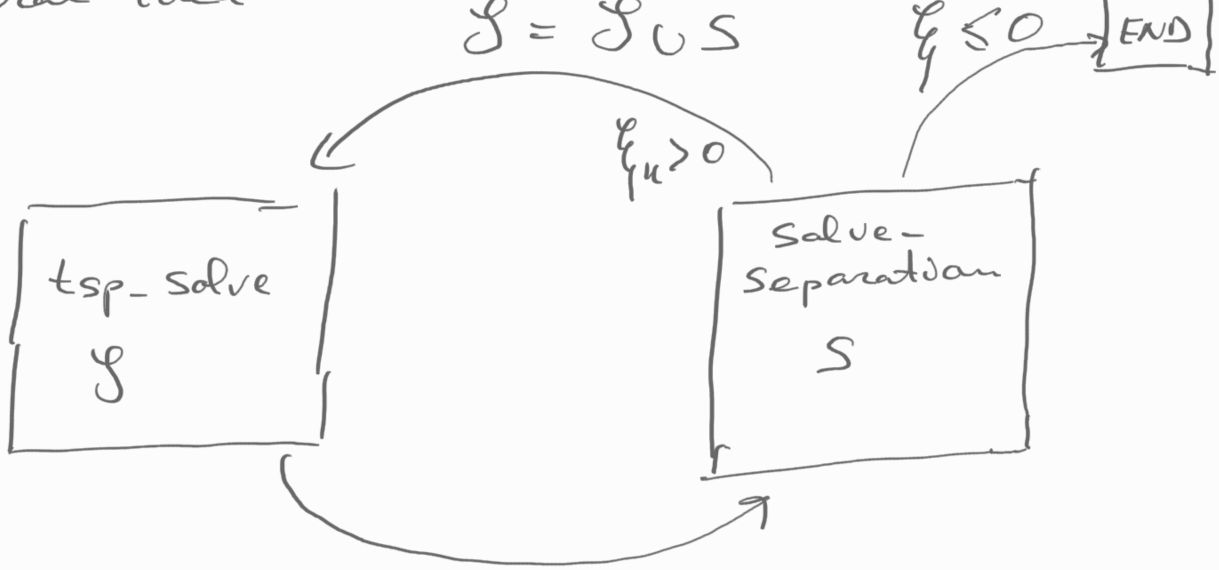
$$0 \cdot z_1 \cdot z_2 + 1 \cdot z_1 \cdot z_3 + 0\, z_1 z_4 + \ldots$$

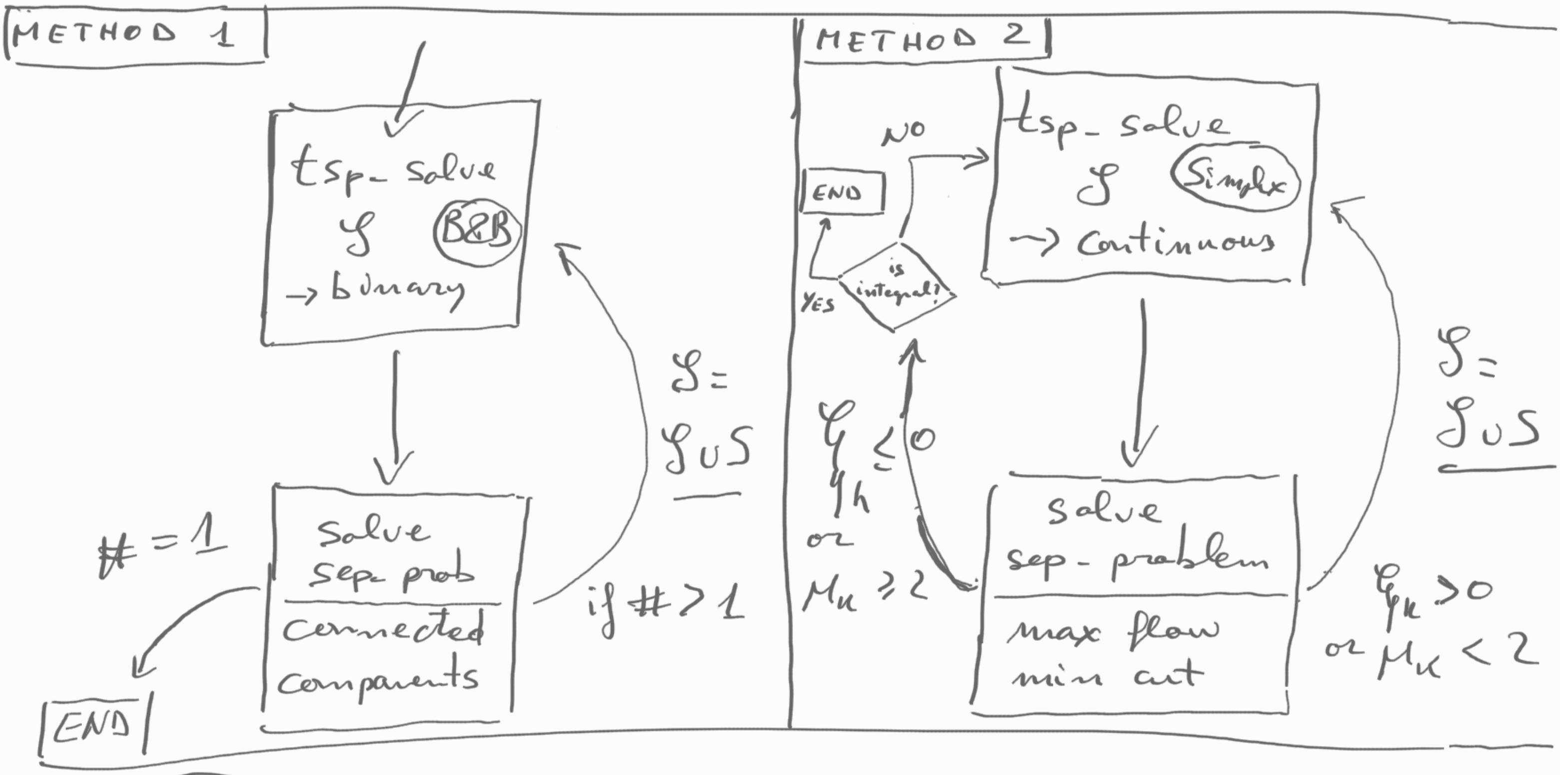
Alternative separation problem:
find all connected components — SEP 1

OK: 1 connected component

NO: 2 connected components

General idea





SEP2

$$\rightarrow \quad y_e = \min(z_i, z_j)$$

$$\min(z_i, z_j) \geq z_i + z_j - 1$$

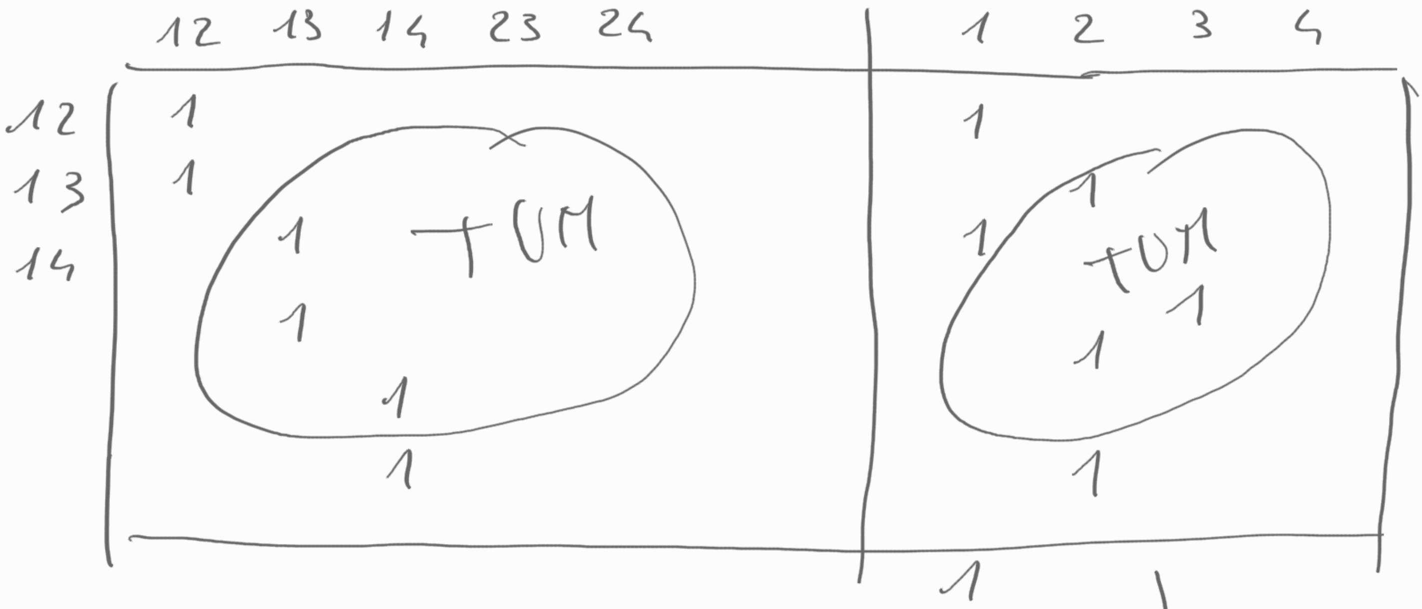
Always true hence remove

$z_i \quad z_j \quad LHS \quad RHS$

0	0	0	-1	✓
1	0	1	0	✓
0	1	1	0	✓
1	1	1	1	✓

Problem left is TUM:

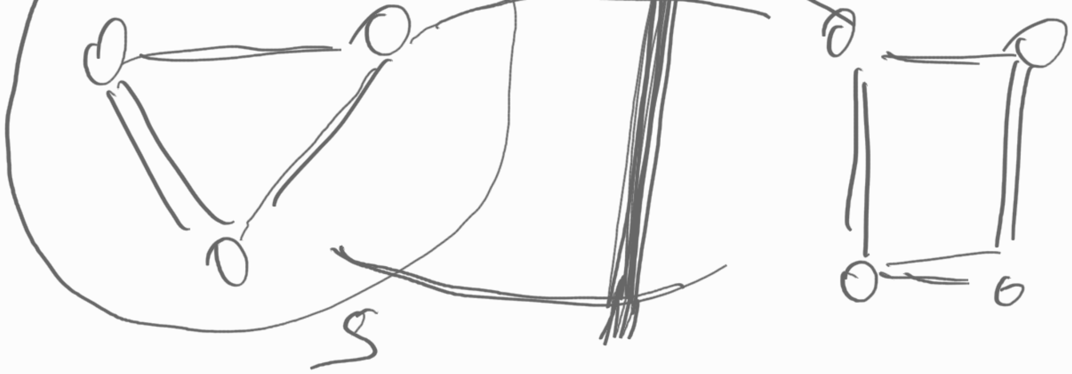
$$y_e \le z_i \quad \forall e \in E$$
$$y_e \le z_j$$
$$z_k = 1$$



transpose is TUM

$$\begin{array}{c|ccccccccccc}
 & 12 & 13 & 14 & 23 & 24 & & & & & & \\
\hline
1 & 1 & 1 & 1 & & & & & & & & \\
2 & & & & 1 & 1 & 1 & & & & & \\
3 & & & & & & & 1 & 1 & 1 & & \\
4 & & & & & & & & & & 1 & 1
\end{array}$$

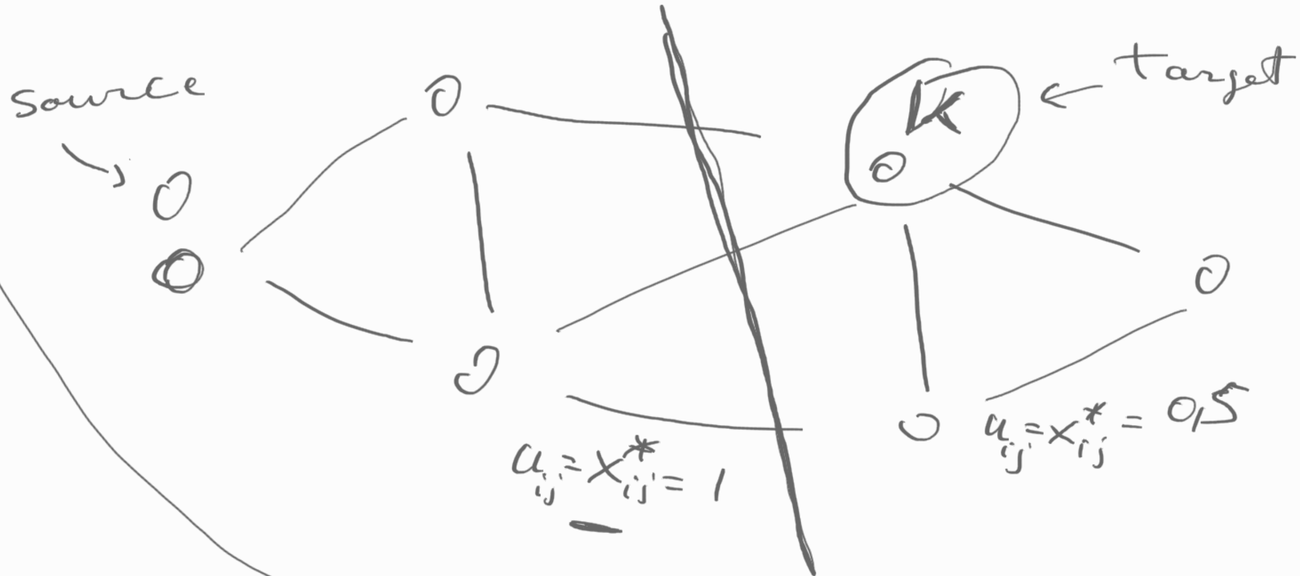
- SEP2 corresponds to a max st-flow:



$$\boxed{\sum_{e\in\delta(S)} x_e \geq 2 \qquad \forall\, \emptyset \subset S \subset V}$$

$$\delta(S) = \{ e = \{u, v\} \mid u \in S,\ v \notin S \}$$

max (st) flow $\cong$ min cut



$$\mu_k = \min_S \boxed{\sum_{e\in\delta(S)} x^*_e}$$ min cut ok

if $\mu_k < 2$ then we have found a cut

Try all different $k \in V_0$ if $\mu_k < 2$ then add constraint(s)

that violates
the requirement
of gooding ≥ 2

if $M_u \geq 2$ for all $u \in V_0$
then no constraint
to add.