

## DM545/DM871 – Linear and integer programming

### Sheet 4, Autumn 2024

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Exercises with the symbol  $+$  are to be done at home before the class. Exercises with the symbol  $*$  will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

#### Exercise 1<sup>+</sup>

Solve the systems  $\mathbf{y}^T E_1 E_2 E_3 E_4 = [1 \ 2 \ 3]$  and  $E_1 E_2 E_3 E_4 \mathbf{d} = [1 \ 2 \ 3]^T$  with

$$E_1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0.5 & 0 \\ 0 & 4 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad E_4 = \begin{bmatrix} -0.5 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

#### Exercise 2\* Sensitivity Analysis and Revised Simplex

A furniture-manufacturing company can produce four types of products using three resources.

- A bookcase requires three hours of work, one unit of metal, and four units of wood and it brings in a net profit of 19 Euro.
- A desk requires two hours of work, one unit of metal and three units of wood, and it brings in a net profit of 13 Euro.
- A chair requires one hour of work, one unit of metal and three units of wood and it brings in a net profit of 12 Euro.
- A bedframe requires two hours of work, one unit of metal, and four units of wood and it brings in a net profit of 17 Euro.
- Only 225 hours of labor, 117 units of metal and 420 units of wood are available per day.

In order to decide how much to make of each product so as to maximize the total profit, the managers solve an LP problem.

- 1) Write the mathematical programming formulation of the problem.
- 2) With the help of a computational environment such as Python for carrying out linear algebra operations, write the optimal tableau, which has  $x_1, x_3$  and  $x_4$  in basis.

Do this task:

- first, using the original simplex method
  - second, using the revised simplex. In this case, start by writing  $A_B, A_N$ , then calculate  $A_B^{-1}A_N$ , and the finally derive the optimal tableau and verify that the solution is indeed optimal.
- 3) What is the increase in price (reduced cost) that would make product  $x_2$  worth to be produced?
  - 4) What is the marginal value (shadow price) of an extra hour of work or amount of metal and wood?
  - 5) Are all resources totally utilized, i.e. are all constraints “binding”, or is there slack capacity in some of them? Answer this question in the light of the complementary slackness theorem.
  - 6) From the economical interpretation of the dual why product  $x_2$  is not worth producing? What is its imputed cost?

Perform a sensitivity analysis for the following variants:

- 7) The net profit brought in by each desk increases from 13 Euro to 15 Euro.
- 8) The availability of metal increases from 117 to 125 units per day
- 9) The company may also produce coffee tables, each of which requires three hours of work, one unit of metal, two units of wood and bring in a net profit of 14 Euro.
- 10) The number of chairs produced must be at most five times the numbers of desks

### **Exercise 3 Factory Planning and Machine Maintenance**

Tasks 1-3 of Factory Planning and Maintenance Case. See Unit 7.