

DM545/DM871 – Linear and integer programming

Sheet 4, Autumn 2025

Exercises with the symbol $^+$ are to be done at home before the class. Exercises with the symbol * will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

Exercise 1$^+$

Solve the systems $\mathbf{y}^T E_1 E_2 E_3 E_4 = [1\ 2\ 3]$ and $E_1 E_2 E_3 E_4 \mathbf{d} = [1\ 2\ 3]^T$ with

$$E_1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0.5 & 0 \\ 0 & 4 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_4 = \begin{bmatrix} -0.5 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Exercise 2* Sensitivity Analysis and Revised Simplex

A furniture-manufacturing company can produce four types of products using three resources.

- A bookcase requires three hours of work, one unit of metal, and four units of wood and it brings in a net profit of 19 Euro.
- A desk requires two hours of work, one unit of metal and three units of wood, and it brings in a net profit of 13 Euro.
- A chair requires one hour of work, one unit of metal and three units of wood and it brings in a net profit of 12 Euro.
- A bedframe requires two hours of work, one unit of metal, and four units of wood and it brings in a net profit of 17 Euro.
- Only 225 hours of labor, 117 units of metal and 420 units of wood are available per day.

In order to decide how much to make of each product so as to maximize the total profit, the managers solve an LP problem.

1) Write the mathematical programming formulation of the problem.
2) With the help of a computational environment such as Python for carrying out linear algebra operations, write the optimal tableau, which has x_1, x_3 and x_4 in basis.

   Do this task:

   - first, using the original simplex method
   - second, using the revised simplex. In this case, start by writing A_B, A_N, then calculate $A_B^{-1} A_N$, and the finally derive the optimal tableau and verify that the solution is indeed optimal.
3) What is the increase in price (reduced cost) that would make product x_2 worth to be produced?
4) What is the marginal value (shadow price) of an extra hour of work or amount of metal and wood?
5) Are all resources totally utilized, i.e. are all constraints "binding", or is there slack capacity in some of them? Answer this question in the light of the complementary slackness theorem.
6) From the economical interpretation of the dual why product x_2 is not worth producing? What is its imputed cost?

Perform a sensitivity analysis for the following variants:

7) The net profit brought in by each desk increases from 13 Euro to 15 Euro.

8) The availability of metal increases from 117 to 125 units per day

9) The company may also produce coffee tables, each of which requires three hours of work, one unit of metal, two units of wood and bring in a net profit of 14 Euro.

10) The number of chairs produced must be at most five times the numbers of desks

Exercise 3* Traffic Light Control

Automobile traffic from three highways, H1, H2, and H3, must stop and wait for a green light before exiting to a toll road. The tolls are 40 kr, 50 kr, and 60 kr for cars exiting from H1, H2, and H3, respectively. The flow rates from H1, H2, and H3 are 550, 650, and 450 cars per hour. The traffic light cycle may not exceed 2.2 minutes, and the green light on any highway must be at least 22 seconds. The yellow light is on for 10 seconds. The toll gate can handle a maximum of 500 cars per hour. Assuming that no cars move on yellow, and that clearly when a highway has the green signal the other two have the red, determine the optimal green time interval for the three highways that will maximize toll gate revenue per traffic cycle.

Exercise 4* Level Terrain for a New Highway

The Highway Department is planning a new 10-km highway on uneven terrain as shown by the profile in Figure 1. The width of the construction terrain is approximately 50 meters and the differences in height measured in meters can be deduced from the y-axis in the figure. To simplify the situation, the terrain profile can be replaced by a step function as shown in the figure. Using heavy machinery, earth removed from high terrain is pulled with effort to fill low areas. There are also two burrow pits, I and II, located at the ends of the 10-km stretch from which additional earth can be pulled, if needed. Pit I is located before km 0 and has a capacity of 15 000 cubic meters, and pit II is located after km 10 and has a capacity of 11 000 cubic meters. The costs of removing earth from pits I and II are, respectively, 15 kr and 19 kr per cubic meter.
The transportation cost per cubic meter per kilometer is 1.5 kr, and the cost of using heavy machinery to load pulling trucks is 2 kr per cubic meter. This means that, for example, three cubic meters extracted from pit I and transported for 1 km will cost a total of $15 \cdot 3 + 1.5 \cdot 3 \cdot 1 + 2 \cdot 3 = 96$ kr and three cubic meters pulled 1 km away from a hill to a fill area will cost $1.5 \cdot 3 \cdot 1 + 2 \cdot 3 = 55.5$ kr.
Write a linear programming model for finding the minimum cost plan for leveling the 10-km stretch.

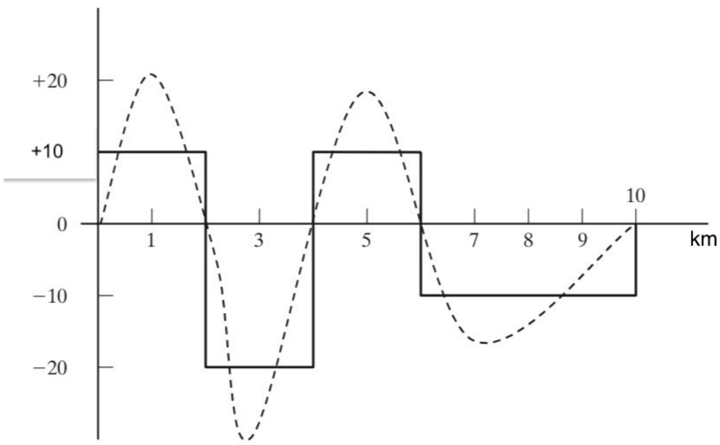

Figure 1:

Exercise 5 Factory Planning and Machine Maintenance

Tasks 1-3 of Factory Planning and Maintainance Case.