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DM545/DM871 – Linear and integer programming

Sheet 6, Autumn 2025

Exercises with the symbol + are to be done at home before the class. Exercises with the symbol * will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

Exercise 1+

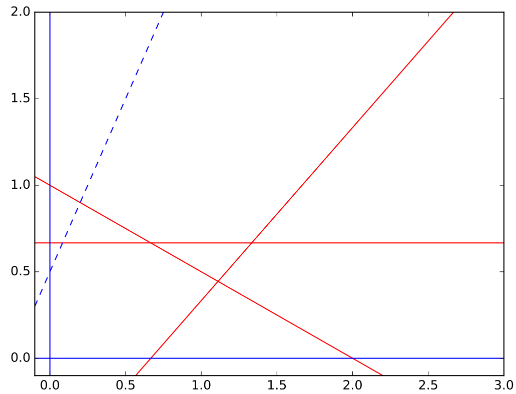
Try solving the following IP problem with the Gomory's fractional cutting plane algorithm, indicating the cut inequalities in the space of the original variables

$$\begin{aligned} \max\ & x_1 + 2x_2 \\ & x_1 - 2x_2 \geq -2 \\ & x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

Exercise 2* — Gomory's Cutting Plane

Consider the following integer linear programming problem

$$\begin{aligned} \max\ & z = 4x_0 - 2x_1 \\ \text{s.t.}\ & x_0 + 2x_1 \leq 2 \\ & 3x_1 \leq 2 \\ & 3x_0 - 3x_1 \leq 2 \\ & x_0, x_1 \geq 0 \text{ and integer} \end{aligned}$$



In the solution of the linear relaxation of the problem the variables x_0, x_1 and the slack variable associated to the second constraint are in basis.

The data in Python format:

```
#%%
from fractions import Fraction as f
import numpy as np
np.set_printoptions(precision=3,suppress=True)

c=np.array([4, -2])
A = np.array([[ 1, 2],
              [ 0, 3],
              [ 3, -3]])

b=np.array([2, 2, 2])
```

Subtask 2.1

Calculate the optimal tableau using the revised simplex method.

Subtask 2.2

Find a Chvatal Gomory's cutting plane

Subtask 2.3

Show that with the cut found the optimal solution of the linear relaxation becomes infeasible.

Exercise 3* — Branch and Bound

This exercise is taken from the exam of 2012.
The Danish Research Council has to decide which research projects to finance. The total budget for the projects is 20 million Dkk. The table below shows the evaluation from 0 (worst) to 2 (best) that the projects received by the external reviewers and the amount of money required.

	1	2	3	4	5
Evaluation score	1	1.8	1.4	0.6	1.4
Investment (in million of DKK)	6	12	10	4	8

Projects 2 and 3 have the same coordinator and the Council decided to grant only one of the two.
The Council wants to select the combination of projects that will maximize the total relevance of the projects, that is, the sum of the evaluation score while remaining within the budget.

Subtask 3.4

Formulate the problem of deciding on which project the Council has to invest as an integer linear programming problem P.

Subtask 3.5

We want the IP instance solved using the branch-and-bound algorithm. What is the optimal solution x^* to the LP relaxation P'? (Hint: use the tool: http://www.zweigmedia.com/simplex/simplex.php to solve the LP problems.]

Subtask 3.6

The rounding heuristic applied to the solution x^* gives a feasible solution x'. Which one? With the knowledge collected until this stage which of the three following statements is correct:

1. x' is certainly optimal
2. x' is certainly not optimal
3. x' might be optimal

(Remember to justify your answer.)

Subtask 3.7

The two subproblems generated by the branch-and-bound algorithm after finding x^* correspond to choosing or not choosing a particular project. Which one?

Subtask 3.8

Suppose the branch-and-bound algorithm considers first the subproblem corresponding to not choosing this project. Let's call this subproblem and its corresponding node in the search tree SP1. What is the optimal solution to its LP relaxation?

Subtask 3.9

Next, the branch-and-bound algorithm considers the subproblem corresponding to choosing the project, i.e., subproblem SP2. Find the optimal solution to its LP relaxation. Which are the active nodes (i.e., open subproblems) at this point?

Subtask 3.10

How does the branch and bound end?

Exercise 4 — Branch and bound

Consider the following ILP:

$$\begin{array}{rrrrrrrrrrrrr} \max & z = & 5x_1 & + & 5x_2 & + & 8x_3 & - & 2x_4 & - & 4x_5 & & \\ & \text{s.t.} & -3x_1 & + & 6x_2 & - & 7x_3 & + & 9x_4 & + & 9x_5 & \geq & 10 \\ & & x_1 & + & 2x_2 & & & - & x_4 & - & 3x_5 & \leq & 0 \\ & & \multicolumn{11}{l}{x_1, x_2, x_3, x_4, x_5 \in \{0, 1\}} \end{array}$$

Solve the problem by branch and bound:

- Use objective-value bounding for pruning subproblems.
- At each node use linear programming to find dual bounds.
- Use the *most fractional variable* rule for branching.
- Follow a depth first search strategy and expand first the *greater-or-equal* branch.

Answer guidelines:

- Make sure that you indicate which is the final solution and its objective function value.
- Use this tool to solve the linear relaxations at each node:
  http://www.zweigmedia.com/simplex/simplex.php
- In the next pages you are given a template for the search tree. Write the search tree first on the paper version of the exam that you received and then digitalize your answer in one of the following ways:
  - scan the tree that you have handwritten (make sure you write all the information needed — see example in the next pages)
  - annotate the tree template provided in the next pages, make a screenshot and include it in your document.
  - use the text format explained in the next page.

Other reporting examples
Drawing: The following is an example of drawing for describing the search tree. The example is not taken from the problem object of this task.

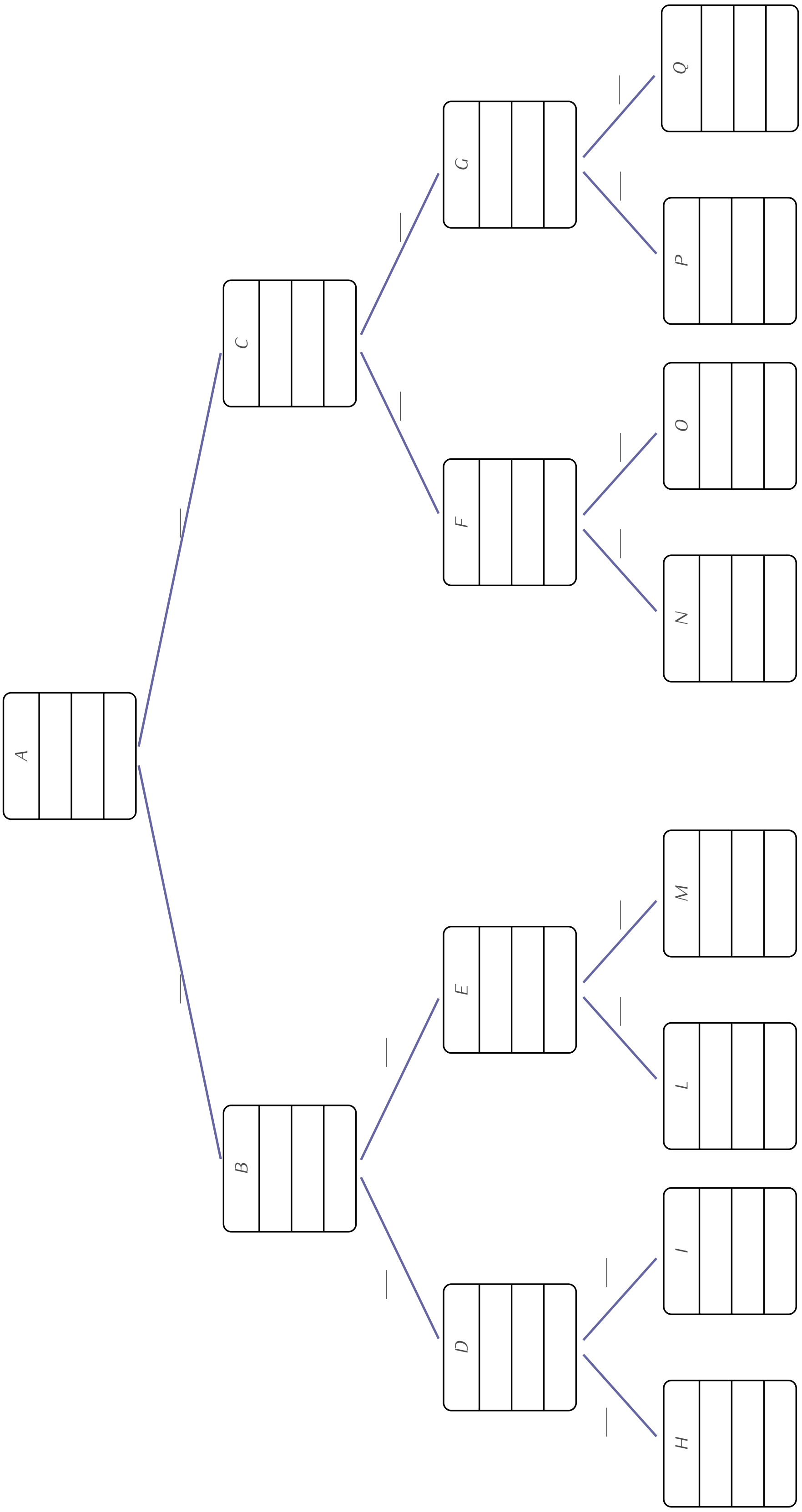
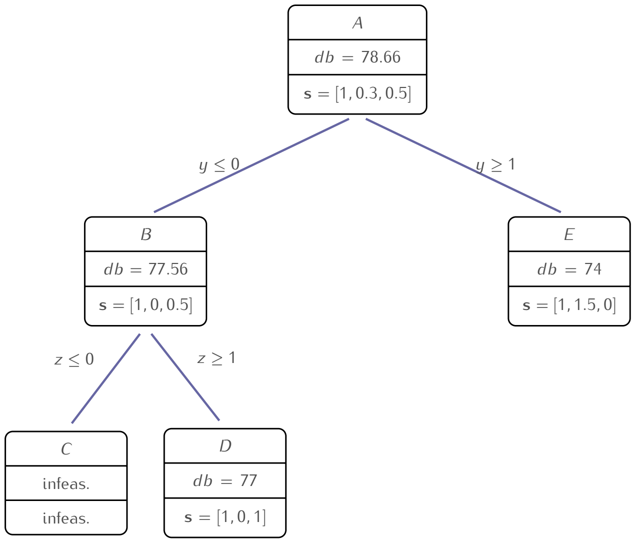


Figure 1: A template for a search tree that you can annotate



Text format: The search tree above can be described in text format as shown below.

```
---
- name: A
  parent: 'null'
  constraint_added: ''
  dual_bound: 78.66
  solution: [1, 0.3, 0.5]
  children:
  - name: B
    parent: A
    constraint_added: y<=0
    dual_bound: 77.56
    solution: [1,0,0.5]
    children:
    - name: C
      parent: B
      constraint_added: z<=0
      dual_bound: infeasible
      solution: infeasible
      children: pruned
    - name: D
      parent: B
      constraint_added: z>=1
      dual_bound: 77
      solution: [1,0,1]
      children: pruned
  - name: E
    parent: A
    constraint_added: y>=1
    dual_bound: 74
    solution: [1, 1.5, 0]
    children: pruned
```