DM812
METAHEURISTICS

Lecture 5

Scatter Search and Path Relinking

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Outline

Resume

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1. Resume

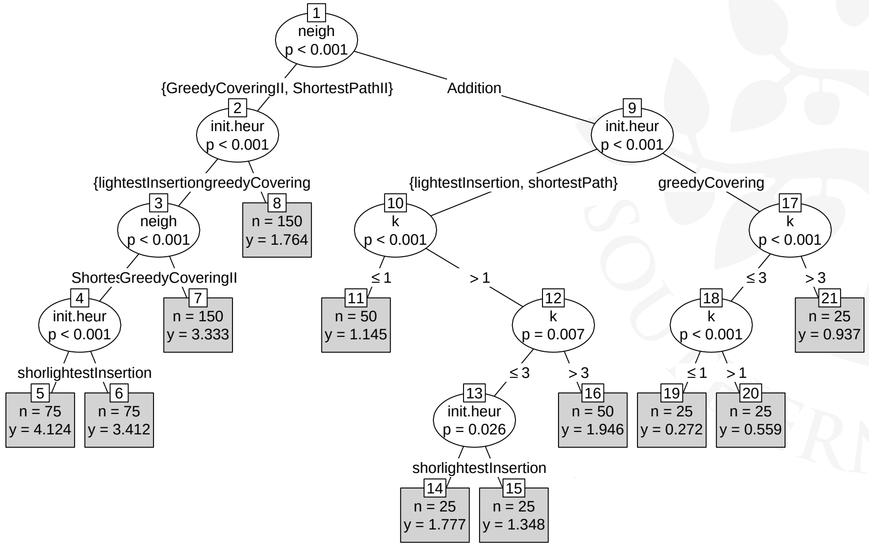
2. Scatter Search and Path Relinking

Resume

Methods for Tuning

- ANOVA
- Regression trees [Bartz-Beielstein and Markon, 2004]
- Racing algorithms [Birattari et al., 2002]
- Response surface models, DACE
  [Bartz-Beielstein, 2006; Ridge and Kudenko, 2007a,b]
- Search approaches
  [Minton 1993, 1996, Cavazos & O'Boyle 2005, Adenso-Diaz & Laguna 2006, Audet & Orban 2006, Hutter et al., 2007]

```
> library(party)
> plot(ctree(gap~init.heur*neigh*k,data=AOPR),type="simple")
```



NOTE

> *A fine tuning of parameters will never balance a bad choice of the neighborhood structure or of the objective function. On the other hand, an effective modelling should lead to robust techniques that are not too sensitive to different parameter settings.*
>
> Hertz, Taillard, de Werra

Outline

Classification of Metaheuristics

- Trajectory methods *vs* discontinuous methods
- Population-based *vs* single-point search
- Memory usage *vs* memory-less methods
- One *vs* various neighborhood structures
- Dynamic *vs* static objective function
- Nature-inspired *vs* non-nature inspiration
- Instance based *vs* probabilistic modeling based

Scatter Search and Path Relinking

Originally proposed by Glover 1977 in the context of integer programming

Key idea: maintain a small population of reference solutions and combine them to create new solutions.

Orient search systematically towards reference points that are good solutions obtained by previous search.

Examples of combination: linear combination of solutions followed by rounding for integer values. Or path relinking in a neighborhood space.

Basic Procedure

Scatter Search:
generate set P of solutions with a diversification generation method
perform subsidiary local search on each $x \in P$ and add the new sol. in P
update reference set $RefSet \subset P$
while *termination criterion* is not satisfied: **do**
  generate subset $NewSubset$ from $RefSet$
  apply solution combination to $S \subseteq NewSubset$ to obtain S'
  perform subsidiary local search on each $x \in S'$ and add new sol. to S'
  update reference set $RefSet$ from $RefSet \cup S'$

Components

- Diversification generation: a large number of solutions is generated by the method while about 1/10 of them are chosen for the reference set.
- update $RefSet$: selects the b solutions that are best in quality or maximally diverse or a combination thereof.
  Example: $b = b_1 + b_2$
  b_1 best solutions in $RefSet$
  b_2 solutions such $\text{argmax}_{s \in P \setminus RefSet}\{d_{\mathcal{N}}(s, s') \mid s' \in RefSet\}$
- Generate subset: generates all pair combinations or in more complex implementations $|NewSubset| > 2$

Components
Management of RefSet

$NewSubset$ creation:

- static update: improve all combinations of solution in $RefSet$ before update RefSet.
  Examination order of the combinations is not important
- dynamic update: $RefSet$ is immediately updated after each combination has been improved, and new combination is generated.
  Examination order of the combinations may cause differences

$RefSet$ rebuilding:

When no solutions can be added anymore then:

Step 1 keep b_1 solutions in $RefSet$
Step 2 use diversification generation method to make P
Step 3 select sequentially b_2 solutions from $P \setminus RefSet$ with maximal diversity from $s \in RefSet$

Components
Management of RefSet

$RefSet$ tiers

- $RefSet_1$: $s^1 \dots s^{b_1}$ kept ordered by f and updated by increasing quality
- $RefSet_2$: $s^{b_1+1} \dots s^{b_2}$ kept ordered by d and updated by increasing diversity
- $RefSet_3$: $s^{b_2+1} \dots s^{b_3}$ (good generators) kept ordered by g (objective function value of the best solution ever created from a combination of $s \in RefSet_1$) and updated by those leaving $RefSet_1$

Diversity control

- Hashing function to avoid repetitions
  Example for permutation representations: $H(\pi) = \sum_{i=1}^{m} i\pi_i^2$
- Add to $RefSet_1$ iff "enough" distant

Components
Subset Generation

Subset generation method (Scatter Search)

- systematic and deterministic combinations of 2 or more solutions
- subsets:
  - subset type 1: all 2-element subset
  - subset type 2: 3-element subsets derived by augmenting each 2-element subset to include the best solutions not in this subset
  - subset type 3: 4-element subsets derived by augmenting each 3-element subset to include the best solutions not in this subset
  - subset type 4: the subsets consisting of the best i elements, for $i = 5 \dots b$
- Solutions are encoded as points of an Euclidean space and new solutions are created by building linear combinations of reference solutions using both positive and negative coefficients.

Components
Subset Generation

Subset generation method (Path Relinking)

- systematic and deterministic combinations of 2 or more solutions
- subsets:
  - subsets: all pair combinations of solutions in $RefSet$
  - combinations are reinterpreted as paths between solutions in a neighborhood space. Starting from an *initiating solution* moves are performed that introduces components of a *guiding solution*.

Path Relinking
Refinements

Path relinking can be interpreted more loosely as a paths from solutions (initiating and guiding) to other solutions

- At step i choose the solution s_{i+1} from the neighborhood of s_i that minimizes the number of moves remaining to reach the guiding solution.
- Alternatively, choose the best move (according to f) from a restricted set of moves
- Each visited solution constitutes a "point of access", hence all neighborhood is explored in search of good solutions
- An aspiration criterion can prefer good quality solution to minimization of distance from the guiding solution
- Apply subsidiary local search along the way:
  - every `NumImp` iterations
  - collect a few best solutions and then return to them

- Strategic oscillation in feasibility problems: allow the process of path relinking to cross the boundary and visit solutions both feasible and infeasible. (at least one guiding solution must be feasible)
- Constructive neighborhoods implement the path relinking phase by destruction of the initiating solution and reconstruction towards the guiding solution