

INSTITUT FOR MATEMATIK OG DATALOGI  
SYDDANSK UNIVERSITET

# Obligatory Problems 2012

**M M 508**

The Problems should be handed in either to me or to the secretary not later than **December 27., 2012 at 12 noon.**

The problems consist of the one below and some of the earlier exam problems.

## OPGAVE

Let  $(X, d)$  be a metric space.

(i) Let  $x, y, x_1, y_1 \in X$ . Prove that

$$d(x, y) - d(x_1, y_1) \leq d(x, x_1) + d(y, y_1) \quad (1)$$

and show next that

$$|d(x, y) - d(x_1, y_1)| \leq d(x, x_1) + d(y, y_1). \quad (2)$$

(ii) Use equation (2) to prove that  $d$  is a uniformly continuous function from  $X \times X$  to  $[0, \infty[$  when  $X \times X$  is equipped with one of the equivalent metrics defined on page 47 of the notes.

### Earlier exam questions:

- January 2001: Problem 2.
- January 2006: Problem 1: 1, 2, 3, 5 (determine only  $\text{int}(S)$ ).
- January 2007: Problem 4.
- January 2010: Problem 2: 1, 2. Problem 3: 2.
- June 2010: Problem 2: 1, 2. Problem 3: 2. Problem 5.
- January 2011: Problem 2: 1, 3. Problem 5.
- June 2011: Problem 2: 1, 3.