Institut for Matematik og Datalogi
Syddansk Universitet, Odense 29. november 2012

MM 508 Topologi I
Ugeseddel 4

The home page of this course can be found on Blackboard or directly on
http://www.imada.sdu.dk/˜njn/MM508

In the lectures in week 48 we have covered continuous functions on topological spaces, proved the Weierstrass accumulation Theorem 6.3, the Heine–Borel Theorem 6.4 and given some consequences. Further we have defined compactness and sequentially compactness and investigated these properties. This corresponds to the pages 34–43 in the lecture notes.

Recall that in the weeks 49–50 there will also be exercises on Tuesdays 14–16. Due to this schedule we have to start on some of the earlier exam problems and therefore I have here to mention some results from the notes **which should be used in some of the exercises in week 49 and week 50.** and which have not been proved yet. The proofs will be given in week 49.

Theorem 6.20

Let K be a subset of a metric space (X,d). The following statementa are equivalent.

(i) K is sequentially compact.

(ii) Every sequence $(x_n) \subseteq K$ has a convergent subsequence with limit in K.

For the definition of a subsequence, consult Definition 6.18.

Theorem 6.21

Let (X,d) be a metric space and let $K \subseteq X$ be a subset. K is compact if and only if it is sequentially compact.

The Heine–Borel Theorem:

Theorem 6.26

Let $n \in \mathbb{N}$ and let $K \subseteq \mathbb{R}^n$ be a subset. K is compact if and only if it is closed and bounded.

Theorem 6.30

If X is a compact topological space and $f : X \to \mathbb{R}$ is a continuous function, then f is bounded and f attains its maximal and minimal values.

Theorem 6.31

Let X and Y be metric spaces so that X is compact. If $f : X \to Y$ is continuous, then it is uniformly continuous.

In the lectures in week 49 we shall prove the theorems mentioned above and give some consequences to the behaviour of continuous functions defined on compact sets.

Exercises for week 49

- Remaining exercises from week 48.
- Exam exercises: 1 (jan10): 1.–4., 1 (jun10): 1.–3., 5 (jun10), 1 (Jan11): 1.–3., 3 (jan11), 1 (Jun11): 1.–3., 4 (Jan12). 1 (jan09): 1.–3., 1 (jun09): 1.–4, 1 (Jan12): 1–4.
- The exercise book: 20 (see Definition 6.13 in the notes), 22.

The exam. Some of you have told me that they have a concentration of exams around the time of the exam for MM508, which is January 8. If this can help you, I am willing to try to move the exam to a later day. It requires however two things: A room should be available (normally not a problem) and the consent of everyone **registered** for the course. Let us discuss it further on Tuesday.

Greetings Niels Jørgen