Institut for Matematik og Datalogi
Syddansk Universitet, Odense

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MM 508 Topologi I
Ugeseddel 6

The home page of this course can be found on Blackboard or directly on
http://www.imada.sdu.dk/~njn/MM508

In the lectures in week 50 we have finished our discussion on connected topological spaces and have investigated homeomorphisms. This correspond to the pages 58–60 of the notes. Hence we have finished the notes.

In week 51 the will be no lectures. Instead we shall do some exercises.

Important notice In Exam question 2 from June 2011 there is a small misprint which was not corrected in the booklet. Item (iii) should read:

There is a sequence $(z_n) \subseteq S$ so that $f(z_n) \to 1$ for $n \to \infty$.

Exercises for Tuesday in week 51: Jan09: Problem 4, Jun09: Problem 3 and 4, Jan04: Problem 5. EB 37, 38.

Exercises for Wednesday in week 51:

- Remaining exercises from week 50, not covered by the above.
- Jun12: Problem 5
- The following three exercises:

  Problem 1

  Let (X, d_X) and (Y, d_Y) be metric spaces so that X is compact. Further we have a $y \in Y$ and a sequence $(y_n) \subseteq Y$ with $y_n \to y$ for $n \to \infty$ and a continuous function $f : X \to Y$ so that the following condition is satisfied:

There is a sequence $(x_n) \subseteq X$ so that $d_Y(f(x_n), y_n) \to 0$

Prove that there is an $x \in X$ with $f(x) = y$.

Problem 2

Let (X,d) be a compact metric space and let $f : X \to X$ be a continuous function so that there is a sequence $(x_n) \subseteq X$ with $d(x_n, f(x_n)) \to 0$.

Prove that there is an $x \in X$ with $f(x) = x$.

Problem 3

Let (X,d_X) and (Y,d_Y) be metric spaces with X compact. Further let $f,g : X \to Y$ be continuous funtions so that there is a sequence $(x_n) \subseteq X$ so that $d_Y(f(x_n), g(x_n)) \to 0$.

Prove that there is an $x \in X$ so that $f(x) = g(x)$.

Exercises for Friday in week 51 :

- Remaining exercises from above.
- The following exercise:

  Problem 4

  Let $f : [0,1] \to [0,1]$ be a continuous function. We wish to prove that there is an $x \in [0,1]$ with $f(x) = x$. Such an x is called a fixpoint for f. If either $f(0) = 0$ or $f(1) = 1$ we are done, so we only have to consider the case where $f(0) \neq 0$ and $f(1) \neq 1$.

  (i) Show that in this case there is an $x \in]0,1[$ with $f(x) = x$.

  (ii) Can f have more than one fixpoint?

- EB Problem 44.

Greetings Niels Jørgen