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# MM513 Sandsynlighedsteori II

## Ugeseddel 1

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The home page of this course can be found through Blackboard or directly on

<http://www.imada.sdu.dk/~njn/MM513/>

**The connection between Resumé of the lectures and Probability Essentials (JP)** The notes are meant to be an extended supplement to the book. They contain proofs of the theorems in the book as I have done them in the lectures, either more detailed or differently from the book. The notes also contain some results I feel are missing in the book.

**In the lectures in week 15** we have covered the following material:

Hilbert spaces, the projection theorem, conditional expectations and their properties and convergence theorems for these. This corresponds to the notes on Hilbert spaces, pages 15 and 21–24, Resumé of the lectures 1–6 and 7. I can recommend that you read Chapter 22 of JP, which contains what we have done on Hilbert spaces and in addition some results on  $L_2$  proved in the course “Measure and Integration Theory”.

In chapter 23 of JP you can also find the results on conditional expectations we have proved in the lectures. I can also recommend that you read the motivating Definition 23.1 and Theorem 23.1 which you should compare to Problems 2 and 3 of the exercises and the problem below.

**In the lectures in week 17** we will prove Jensen’s inequality for conditional expectations and then start on the theory of martingales.

### Exercises for week 17

- Problem: Let  $(\Omega, \mathcal{F}, P)$  be a probability spaces, let  $A, B \in \mathcal{F}$  with  $0 < P(B) < 1$ , and let  $\mathcal{G} = \{B, \Omega \setminus B, \Omega, \emptyset\}$ . Show that

$$E(1_A | \mathcal{G}) = \frac{P(A \cap B)}{P(B)} 1_B + \frac{P(A \cap (\Omega \setminus B))}{P(\Omega \setminus B)} 1_{\Omega \setminus B}$$

- Exercises for MM 513: 1–4.
- Discuss JP Definition 23.1 and Theorem 23.1 and show that if  $X$  and  $Y$  are as in definition 23.2, then the  $E(Y | X)$  there is the same as we get using the definition in exercise 4.
- JP: Exercises 23.2 and 23.11.

### **Teaching material on the home page**

- Noter om Hilbertrumsteori
- Exercises for MM 513 1–25
- Resumé of the lectures (will be edited during the course)
- Lecture Notes on Metric and Topological Spaces (brugt i MM508)

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