Institut for Matematik og Datalogi
Syddansk Universitet, Odense

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MM513 Sandsynlighedsteori II
Ugeseddel 2

The home page of this course can be found through Blackboard or directly on

http://www.imada.sdu.dk/∼njn/MM513/

In the lectures on 24.4 we have covered the following material: Jensen's inequality, preliminary results on martingales and submartingales, and stopping times. This corresponds to the notes 8–10 and JP 201, 207–208, and Definition 25.1

In the lectures 25.4 and week 18 we shall prove some sampling theorems for martingales, prove some martingale inequalities and prepare for the martingale convergence theorem.

Exercises for week 18

- Remaining exercises from week 17.
- Note that a new version of the Exercises for MM513 has appeared on the home page. Some misprints and omissions have been corrected. The most important is that in Problem 6 (ii) (X_n) is assumed to be a martingale.
- Exercises for MM513: 6,7,8,9 and the following two problems:

  Problem

  Let $(X_n)_{n\geq O} \subseteq L_1(P)$ be a sequence of independent, identically distributed stochastic variables and put for every $n \geq 0$ $S_n = \sum_{k=0}^{n} X_k$ og $\mathcal{F}_n = \sigma(X_0, X_1, \cdots, X_n)$.

  1. Show that if $E(X_0) = 0$, then (S_n) is a martingale.
  2. Show that if $E(X_0) > 0$, then (S_n) is a submartingale.
  3. Guess the next question yourselves!!

Problem

Let$(X_n)_{n\geq 0}$ be a martingale with respect to the filtration $(\mathcal{F}_n)$ and let τ be a stopping time with $P(\tau < \infty) = 1$. Assume in addition that there exists an M, so that $|X_n| 1_{(n\leq\tau)} \leq M$, hence that (X_n) is bounded up to the time τ.

1. Show that $|X_\tau| \leq M$ and conclude that $E(|X_\tau|) < \infty$.
2. Show that $E(X_{\tau\wedge n} \to E(X_\tau$ for $n \to \infty$, and use JP Theorem 24.2 to conclude that $E(X_\tau) = E(X_0)$.

   (Hint: Write $X_{\tau\wedge n} = X_\tau 1_{(\tau\leq n)} + X_n 1_{(n<\tau)}$.)

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