DM503
Programming B

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RECURSIVE DATA STRUCTURES

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Recursive Data Structures

- like functions, data structures can be recursive, too
- recursive class = contains a member variable of same class
- Example:

```
public class Student {
  String name;
  Student tutor;
  public String toString() {
    return name + tutor == null ? "" : " (" + tutor.name + ")";
  }
}
```

- useful to implement linked lists, trees, …

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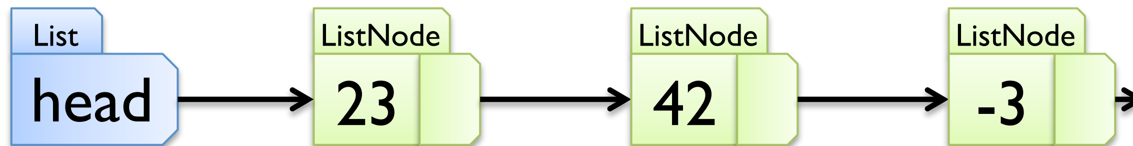
List ADT: Specification (revisited)

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface List<E> {
    public E get(int i);                // get i-th integer (0-based)
    public void set(int i, E elem);     // set i-th element
    public int size();                  // return length of list
    public void add(E elem);            // add element at end
    public void add(int i, E elem);     // insert element at pos. i
    public void remove(int i);          // remove i-th element
}
```

Linked Lists

- arrays require copying when inserting in the middle
- avoid copying by using links from one element to the next
- **Idea:**
  - use a recursive data structure ListNode
  - one instance of this class per element
  - every node (except the last) refers to next element



List ADT: Design & Implementation 4

- Design 4: linked lists
- Implementation 4:

```
public class ListNode<E> {
    private E elem;                  // one list element
    private ListNode<E> next;        // next element
    public ListNode(E elem, ListNode<E> next) {
        this.elem = elem;
        this.next = next;
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public E get(int i) {
        if (i == 0) {  return this.elem;  }
        if (this.next == null) {  throw new Index…Exception();  }
        return this.next.get(i-1);
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                        // one list element
    private ListNode<E> next;              // next element
    …
    public void set(int i, E elem) {
        if (i == 0) {  this.elem = elem;  } else {
            if (this.next == null) {  throw new Index…Exception();  }
            this.next.set(i-1, elem);
        }
    }
    …   }
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public int size() {
        int result = 1;
        if (this.next != null) {  result += this.next.size();  }
        return result;
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public void add(E elem) {
        if (this.next != null) {
            this.next.add(elem);
        } else {
            this.next = new ListNode<E>(elem, null);
        }
    }   …   }
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void add(E elem) {
    if (this.next != null) {
        this.next.add(elem);
    } else {
        this.next = new ListNode<E>(elem, null);
    }
}
```

add(17)

List ADT: Implementation 4

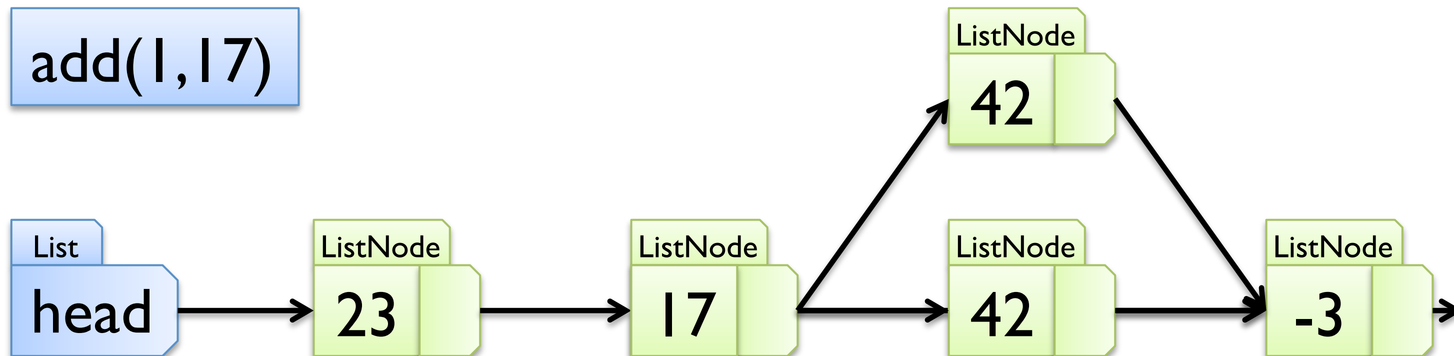
- Implementation 4 (continued)

```
public void add(int i, E elem) {
    if (i == 0) {
        this.next = new ListNode<E>(this.elem, this.next);
        this.elem = elem;
    } else {
        if (this.next == null) {
            if (i == 1) {  this.add(elem);  }      // end of list
            else {  throw new Index…Exception();  }
        }  else {  this.next.add(i-1, elem);  }
    }
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void add(int i, E elem) {
    if (i == 0) {
        this.next = new ListNode<E>(this.elem, this.next);
        this.elem = elem;
    } else {
        …
```



List ADT: Implementation 4

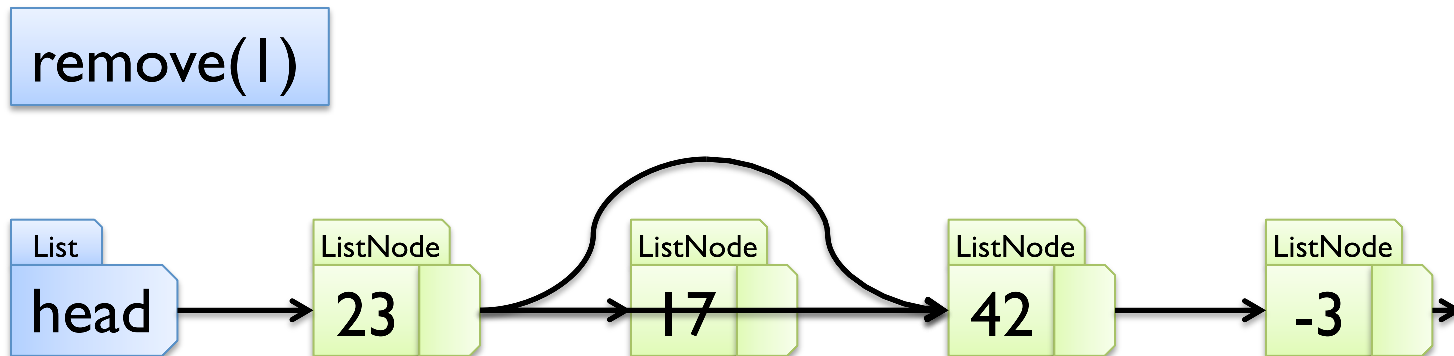
- Implementation 4 (continued)

```
public void remove(int i) {
    if (this.next == null) {  throw new Index…Exception();  }
    if (i == 1) {
        this.next = this.next.next;
    } else {
        this.next.remove(i-1);
    }
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void remove(int i) {
    if (this.next == null) {  throw new Index…Exception();  }
    if (i == 0) {
        this.next = this.next.next;
    }   …
```



List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;
    public E get(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        return head.get(i);
    }
    …
}
```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List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public void set(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        head.set(i, elem);
    }
    …
}
```


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List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public int size() {
        if (head == null) {  return 0;  }
        return head.size();
    }
    …
}
```


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List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public void add(E elem) {
        if (head == null) {
            head = new ListNode<E>(elem, null);
        } else {
            head.add(elem);
        }
    }
    …
}
```


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List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {   …
    public void add(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {
            if (i > 0) {  throw new Index…Exception();  }
            head = new ListNode<E>(elem, null);
        } else {
            head.add(i, elem);
        }
    }   …
}
```


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List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {   …
    public void remove(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        if (head.getNext() == null) {
            if (i > 0) {  throw new Index…Exception();  }
            head = null;
        } else if (i == 0) {  head = head.getNext();  }
        else {  head.remove(i);  }
    }
}
```


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ABSTRACT DATA TYPES FOR STACKS & QUEUES

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Stacks

- stacks are special sequences, where elements are only added and removed at one end
- imagine a stack of paper on a desk
- many uses:
  - postfix calculator
  - activation records
  - depth-first tree traversals
  - …
- basic stack operations are
  - looking at the top of the stack
  - removing the top-most element
  - adding an element to the top of the stack

Stack ADT: Specification

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface Stack<E> {
    public boolean isEmpty();          // is stack empty?
    public E peek();                   // look at top element
    public E pop();                    // remove top element
    public void push(E elem);          // add top element
}
```


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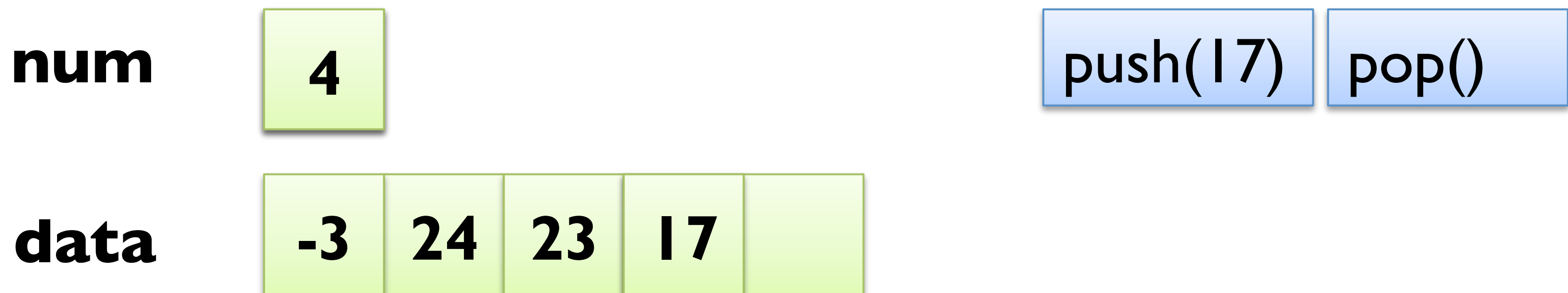
Stack ADT: Design 1

- Design 1: use dynamic array
  - the top of the stack is the end of the list
  - in other words, num specifies the top position
  - pushing corresponds to adding at the end
  - poping corresponds to removing at the end



Stack ADT: Implementation 1

- Implementation 1:

```
public class DynamicArrayStack<E> implements Stack<E> {
    private int limit;              // maximal number of elements
    private E[] data;               // elements of the list
    private int num = 0;            // current number of elements
    public DynamicArrayStack(int limit) {
        this.limit = limit;
        this.data = (E[]) new Object[limit];
    }
    public boolean isEmpty() {  return this.num == 0;  }
    …
}
```

Stack ADT: Implementation 1

- Implementation 1 (continued):

```
public class DynamicArrayStack<E> implements Stack<E> {   …
    public E peek() {
        if (this.isEmpty()) {  throw new RuntimeException("es");  }
        return this.data[this.num-1];
    }
    public E pop() {
        E result = this.peek();
        num--;
        return result;
    }   …
}
```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Stack ADT: Implementation 1

- Implementation 1 (continued):

```
public class DynamicArrayStack<E> implements Stack<E> {   …
    public void push(E elem) {
        if (this.num >= this.limit) {
            E[] newData = (E[]) new Object[2*this.limit];
            for (int j = 0; j < limit; j++) {  newData[j] = data[j];  }
            this.data = newData;
            this.limit *= 2;
        }
        this.data[num++] = elem;
    }
}
```

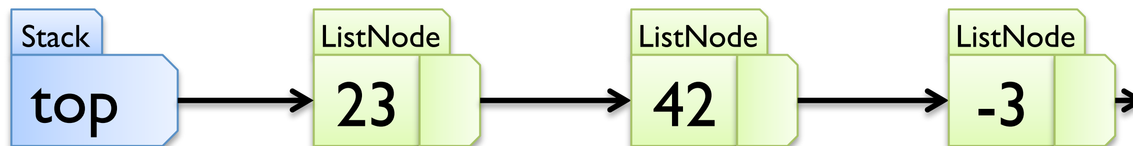
Stack ADT: Design & Implement. 2

- Design 2: reuse dynamic array list (ArrayList<E>)
- Implementation 2:

```
public class ArrayListStack<E> implements Stack<E> {
    private List<E> list = new ArrayList<E>();
    public boolean isEmpty() {  return this.list.isEmpty();  }
    public E peek() {  return this.list.get(this.list.size()-1);  }
    public E pop() {  return this.list.remove(this.list.size()-1);  }
    public void push(E elem) {  this.list.add(elem);  }
}
```

Stack ADT: Design 3

- Design 3: use recursive data structure
  - linked lists have cheap insert and remove operations
  - adding at the end requires running to the end
  - represent top as the beginning of the "list"
- reuse linked list node class (ListNode<E>)
- with dynamic arrays, sometimes need to copy full array
- with linked list, always constant time operations



Stack ADT: Implementation 3

- Implementation 3:

```
public class LinkedStack<E> implements Stack<E> {
    private ListNode<E> top = null;   // top of the stack
    public boolean isEmpty() {  return this.top == null;  }
    public E peek() {
        if (this.isEmpty()) {  throw new RuntimeException("es");  }
        return this.top.get(0);
    }
    …
}
```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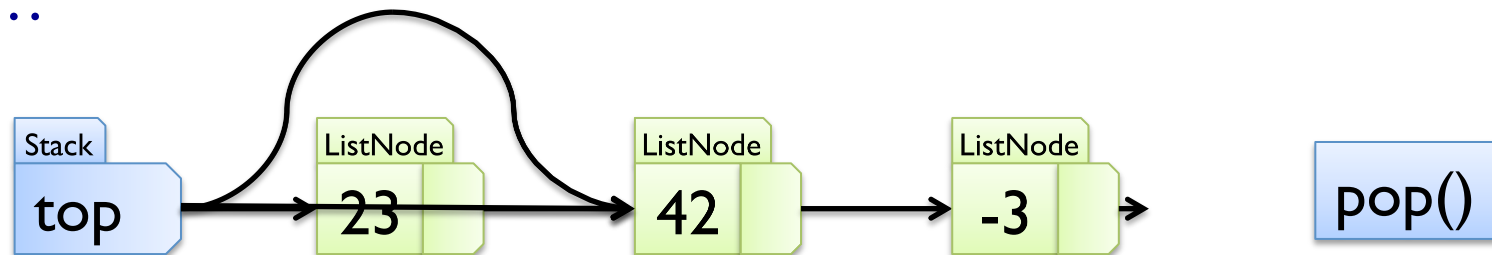
Stack ADT: Implementation 3

- Implementation 3 (continued):

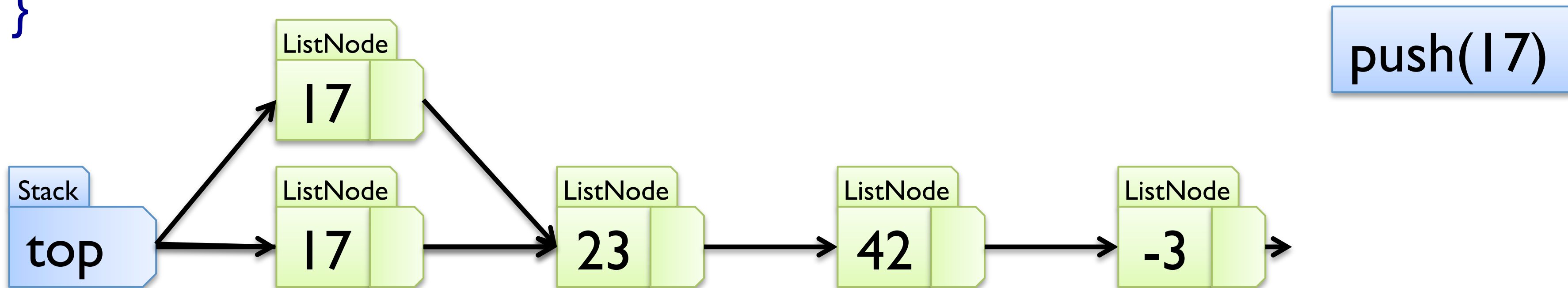
```
public class LinkedStack<E> implements Stack<E> {
    …
    public E pop() {
        E result = this.peek();
        this.top = this.top.getNext();
        return result;
    }
    …
}
```



Stack ADT: Implementation 3

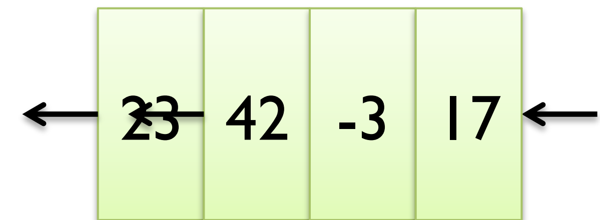
- Implementation 3 (continued):

```
public class LinkedStack<E> implements Stack<E> {
    private ListNode<E> top = null;    // top of the stack
    …
    public void push(E elem) {
        this.top = new ListNode<E>(elem, this.top);
    }
}
```



Queues

- queues are special sequences, where elements are added on one and removed at the other end
- imagine a waiting line in the supermarket
- many uses:
  - network send/receive buffers
  - process scheduling
  - breadth-first tree traversals
  - …
- basic queue operations are
  - looking at the beginning of the queue
  - removing the first element
  - adding an element to the end of the queue



Queue ADT: Specification

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface Queue<E> {
    public boolean isEmpty();          // is queue empty?
    public E peek();                   // look at first element
    public E poll();                   // remove first element
    public boolean offer(E elem);      // true, if element added
        // at end of queue; false, if queue is full
}
```

Queue ADT: Design & Implement. I

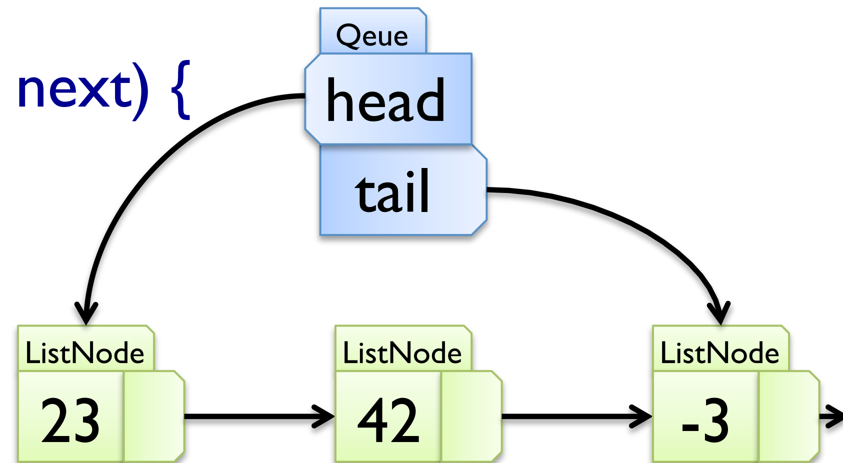
- Design I: reuse dynamic array list (ArrayList<E>)
- Implementation I:

```
public class ArrayListQueue<E> implements Queue<E> {
    private List<E> list = new ArrayList<E>();
    public boolean isEmpty() {  return this.list.isEmpty();  }
    public E peek() {  return this.list.get(0);  }
    public E poll() {  return this.list.remove(0);  }
    public boolean offer(E elem) {
        this.list.add(elem);
        return true;
    }
}
```

Queue ADT: Design & Implement. 2

- Design 2: use recursive data structure
  - use two references instead of one
  - one reference to end of queue
  - one reference to beginning of queue
- reuse & extend linked list node class (ListNode<E>)
- Implementation 2:

```
public class ListNode<E> {   …
    public void setNext(ListNode<E> next) {
        this.next = next;
    }
}
```



Queue ADT: Implementation 2

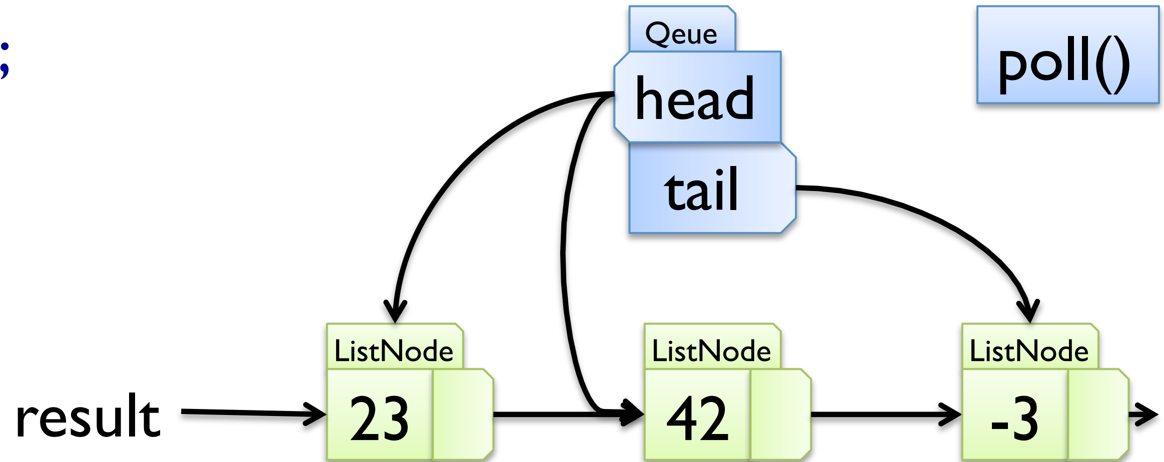
- Implementation 2 (continued):

```
public class LinkedQueue<E> implements Queue<E> {
    private ListNode<E> head = null;          // beginning
    private ListNode<E> tail = null;          // end
    public boolean isEmpty() {
        return this.head == null;
    }
    public E peek() {
        return this.head.get(0);
    }
    …
}
```

Queue ADT: Implementation 2

- Implementation 2 (continued):

```
public class LinkedQueue<E> implements Queue<E> {   …
    public E poll() {
        E result = this.peek();
        this.head = this.head.getNext();
        if (this.head == null) {
            this.tail = null;
        }
        return result;
    }
    …
}
```



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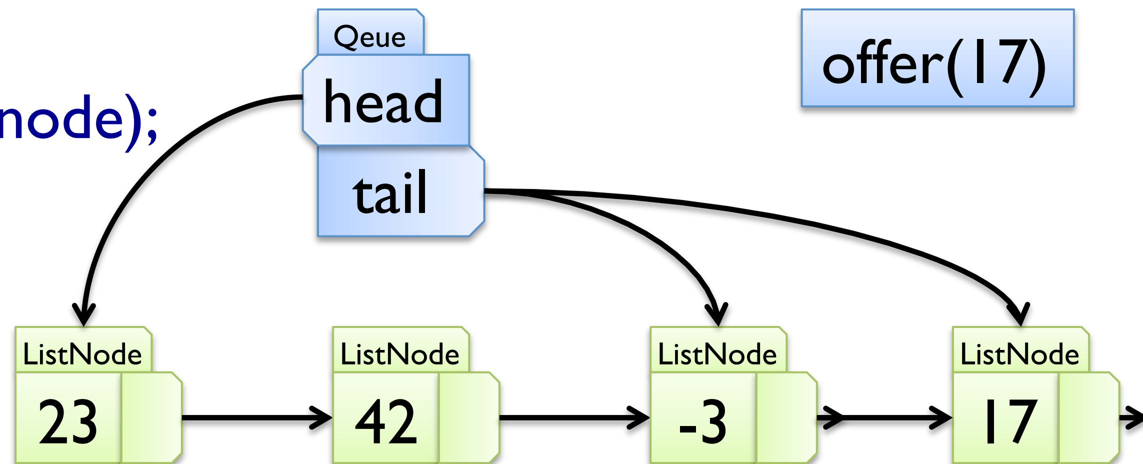
Queue ADT: Implementation 2

- Implementation 2 (continued):

```
public class LinkedQueue<E> implements Queue<E> {   …
    public boolean offer(E elem) {
        ListNode<E> node = new ListNode<E>(elem, null);
        if (this.head == null) {
            this.head = this.tail = node;
        } else {
            this.tail.setNext(node);
            this.tail = node;
        }
        return true;
} }
```



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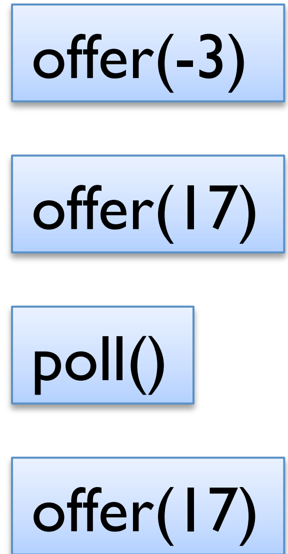
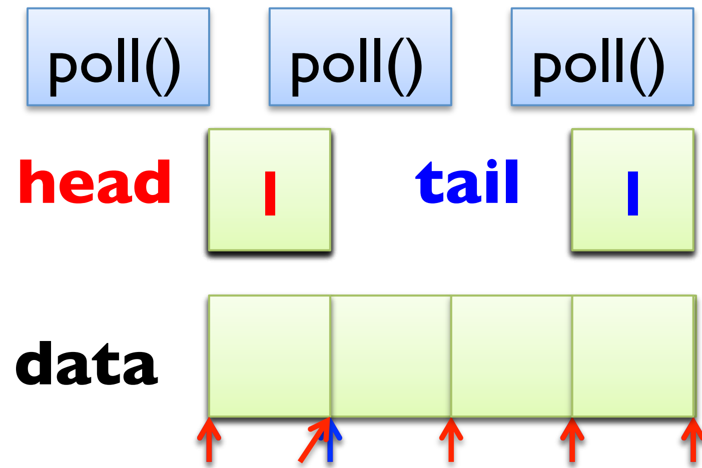
Queue ADT: Design & Implement. 3

- Design 3: use a fixed length array
  - use two indices denoting beginning and end
  - wrap around end of array
- very efficient (memory and runtime – no objects!)
- Implementation 3:

```
public class RingQueue<E> implements Queue<E> {
    private int limit;
    private int head = 0;        // beginning
    private int tail = 0;        // end
    private E[] data;
    …
}
```





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Queue ADT: Implementation 3

- Implementation 3 (continued):

```
public class RingQueue<E> implements Queue<E> {   …
    private int head = 0;       // beginning
    private int tail = 0;       // end
    private E[] data;
    public boolean isEmpty() {  return this.head == this.tail;  }
    public E peek() {
        if (this.isEmpty()) {  throw new RuntimeException("eq");  }
        return this.data[this.head];
    }
    …
}
```


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Queue ADT: Implementation 3

- Implementation 3 (continued):

```
public class RingQueue<E> implements Queue<E> {
    …
    public E poll() {
        E result = this.peek();
        this.head = (this.head+1) % this.limit;
        return result;
    }
    …
}
```


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Queue ADT: Implementation 3

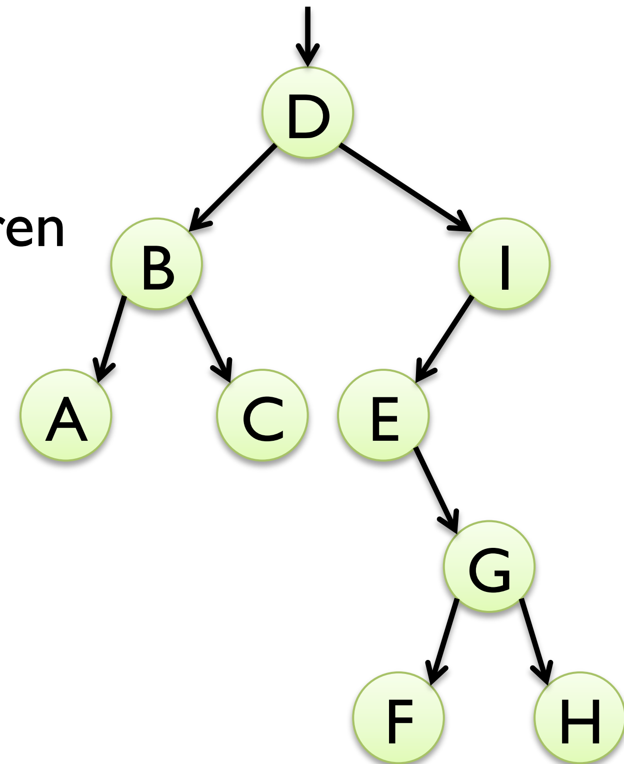
- Implementation 3 (continued):

```
public class RingQueue<E> implements Queue<E> {   …
    public boolean offer(E elem) {
        int newTail = (this.tail+1) % this.limit;
        if (newTail == this.head) {
            return false;                // full
        }
        this.data[this.tail] = elem;
        this.tail = newTail;
        return true;
    }   …
}
```

ABSTRACT DATA TYPES FOR (BINARY) TREES

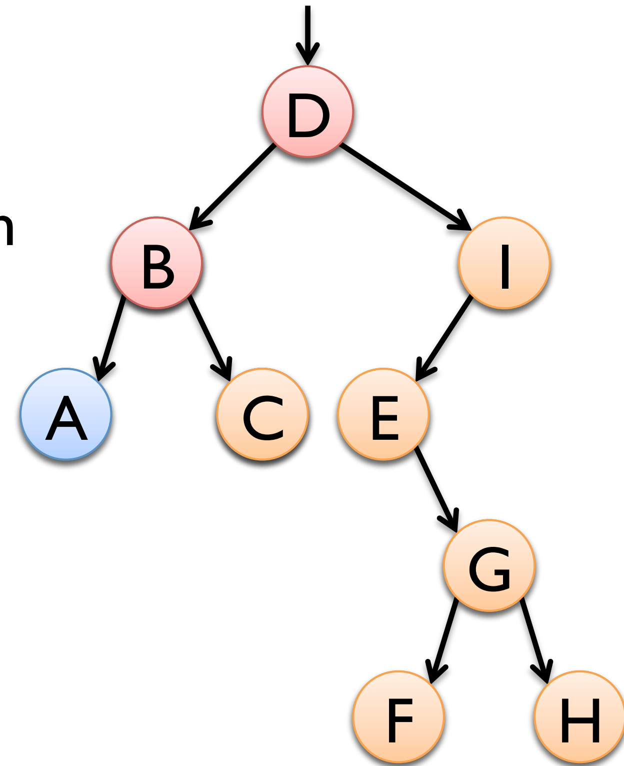
Trees

- trees store elements non-sequentially
- every node in a tree has 0 or more children
- imagine a tree with root in the air ☺
- many uses:
  - decision tree
  - binary sort trees
  - data base indices
  - …
- no consensus on what basic binary tree operations are ☹
- set of operations depends on application
- **here:** keeping elements sorted, indexing a database



Binary Trees

- special case of general trees
- every node in a tree has 0, 1 or 2 children
- tree on the right is an example
- notation:
  - first node is called "root"
  - other nodes either in "left subtree"
  - … or in "right subtree"
- every node is root in its own subtree!
- for example, look at node B
- node A is the "left child" of B
- node C is the "right child" of B
- node B is the "parent" of both A and C



BinTree ADT: Specification 1

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface BinTree<E> {
    public boolean isEmpty();               // is tree empty?
    public int size();                      // number of elements
    public int height();                    // maximal depth
    public boolean contains(E elem);        // true, if elem in tree
    public List<E> preOrder();              // pre-order traversal
    public List<E> inOrder();               // in-order traversal
    public List<E> postOrder();             // post-order traversal
}
```

BinTree ADT: Design & Implement. I

- Design 1: use recursive data structure
  - based on representing tree nodes by BinTreeNode<E>
- Implementation 1:

```
public class BinTreeNode<E> {
    public E elem;
    public BinTreeNode<E> left, right;
    public BinTreeNode(E elem, BinTreeNode<E> left,
                                   BinTreeNode<E> right) {
        this.elem = elem;
        this.left = left;  this.right = right;
    }
}
```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BinTree ADT: Implementation 1

- Implementation 1 (continued):

```
public class RecursiveBinTree<E> {
    private BinTreeNode<E> root = null;
    public int size() {  return size(this.root);  }
    private static int size(BinTreeNode<E> node) {
        if (node == null) {  return 0;  }
        return 1 + size(node.left) + size(node.right);
    }
    …
}
```


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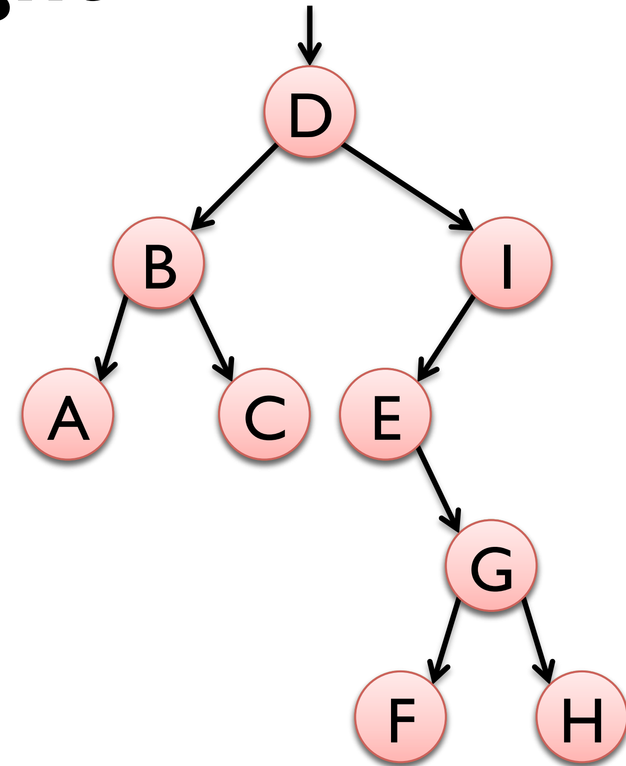
Depth and Height

- depth of the root is 0
- depth of other nodes is 1+depth(parent)
- Example:
  - 0
  - 1
  - 2
  - 3
  - 4
- height of a subtree is maximal depth of any of its nodes
- Example: height of tree (=subtree starting in D) is 4



BinTree ADT: Implementation 1

- Implementation 1 (continued):

```
public class RecursiveBinTree<E> {
    private BinTreeNode<E> root = null;
    …
    public int height() {  return height(this.root);  }
    private static int height(BinTreeNode<E> node) {
        if (node == null) {  return -1;  }
        return 1 + max(height(node.left), height(node.right));
    }
    private static in max(int a, int b) {  return a > b ? a : b;  }
    …
}
```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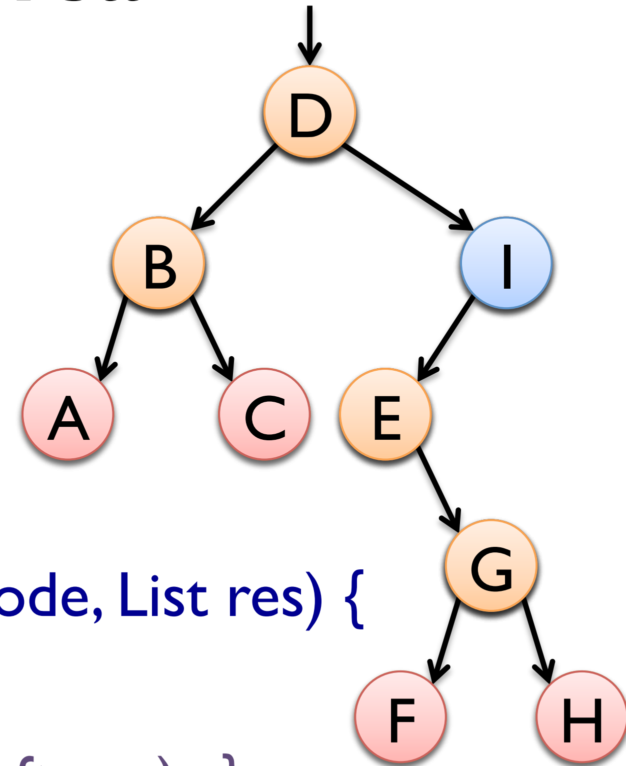
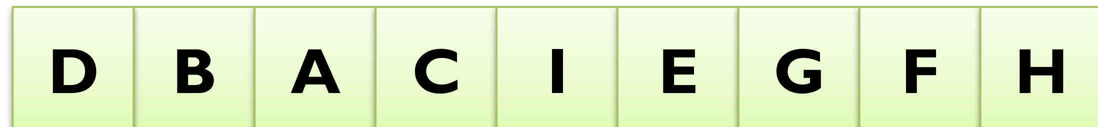
Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
  1. pre-order

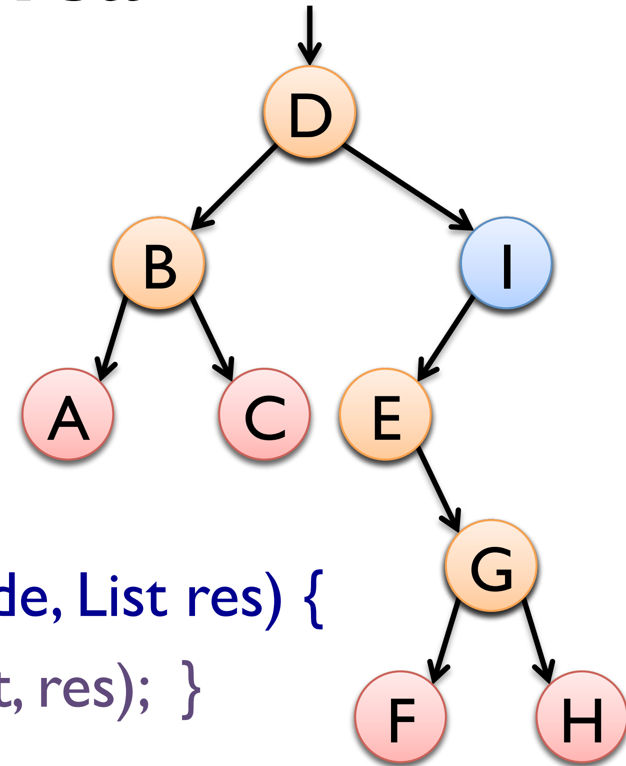
```
public static void preOrder(BinTreeNode node, List res) {
    res.add(node.elem);
    if (node.left != null)   {  preOrder(node.left, res);  }
    if (node.right != null) {  preOrder(node.right, res);  }
}
```



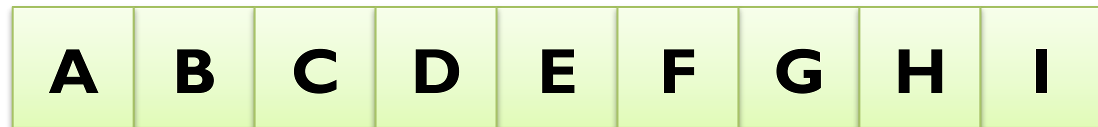


Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
  2. in-order

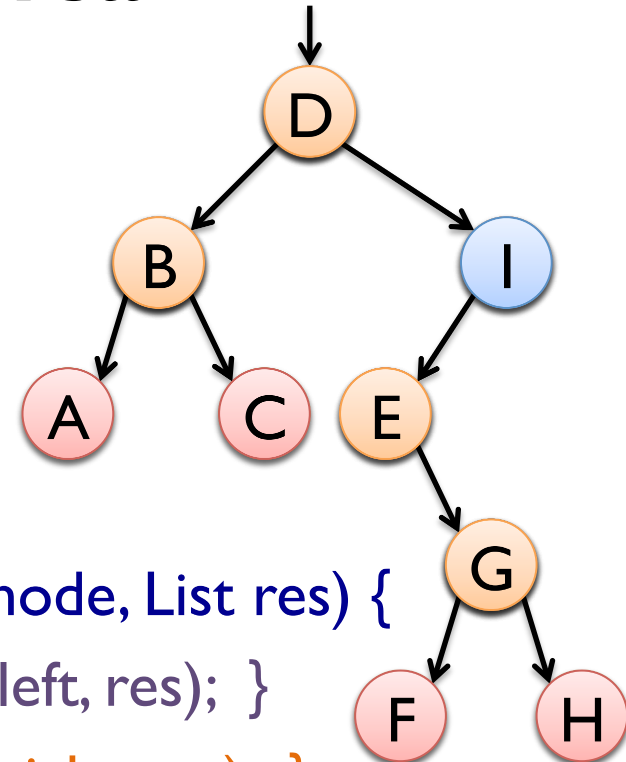



```
public static void inOrder(BinTreeNode node, List res) {
    if (node.left != null)   {  inOrder(node.left, res);  }
    res.add(node.elem);
    if (node.right != null) {  inOrder(node.right, res);  }
}
```



Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
  3. post-order




```
public static void postOrder(BinTreeNode node, List res) {
    if (node.left != null)   {  postOrder(node.left, res);  }
    if (node.right != null) {  postOrder(node.right, res);  }
    res.add(node.elem);
}
```