DM537
Object-Oriented Programming

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RECURSION (REVISITED)

Recursion (Revisited)

- recursive function = a function that calls itself
- Example (meaningless):

```
public static void main(String[] args) {
    if (args.length > 0) {  main(new String[args.length-1]);  }
}
```

- base case = no recursive function call reached
- we say the function call *terminates*

- infinite recursion = no base case is reached
- also called *non-termination*

- Java recursion depth only limited by Java stack size

Comparable Interface

- imposes *total order* on elements, i.e., all elements comparable
- works similar to __cmp__(self, other) method in Python
- classes need to implement interface Comparable:

```
public interface Comparable<T> {
    public int compareTo(T other);
}
```

- has to return 0, if this and other are equal according to equals
- has to return 1, if this is greater than other
- has to return -1, if this is smaller than other

Binary Search

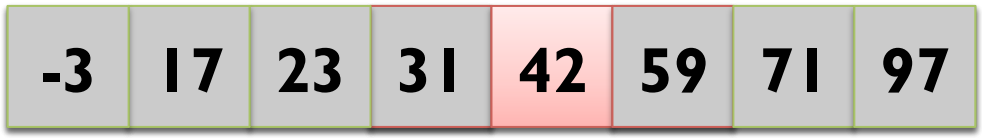
- assume ordered list of elements implementing Comparable
- "Divide et Impera" gives us *binary search* by halving the list
- Example:

```
public int find(Comparable elem) {
    return this.binarySearch(elem, 0, this.getSize()-1);
}
public int binarySearch(Comparable elem, int low, int high) {
    if (low > high) {  return -1;  }        // range is empty; not found
    int middle = (low + high) / 2;
    switch (elem.compareTo(this.get(middle))) { … }
}
```

Binary Search

- Example (continued):

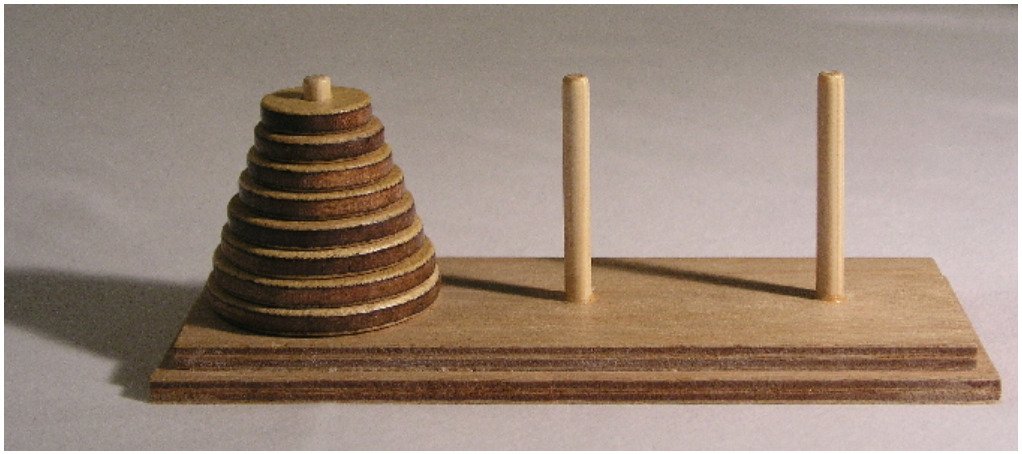
```
public int binarySearch(Comparable elem, int low, int high) {
    if (low > high) {  return -1;  }        // range is empty; not found
    int middle = (low + high) / 2;
    switch (elem.compareTo(this.get(middle))) {
        case 0:    return middle;
        case 1:    return binarySearch(elem, middle+1, high);
        case -1:   return binarySearch(elem, low, middle-1);
    }
    throw new RuntimeException("compareTo error");
}
```



binarySearch(42, 4, 4)

Towers of Hanoi

- game invented by Edouard Lucas in 1883
- three pins with discs of varying diameter



- **Goal:** move all discs from the left to the right pin
- **Rule 1:** only one disc may be moved at a time
- **Rule 2:** a bigger disc may not lie on top of a smaller one
- **Rule 3:** all discs except the one moved are on some pin

Towers of Hanoi

- **Solution:** move discs from left to right using the middle pin
  - move all discs (except the largest) from left pin to the middle pin using the right pin
  - move the largest disc from left to right pin
  - move all discs (except the largest) from middle pin to the right pin using the left pin

Towers of Hanoi

- **Solution:** move discs from left to right using the middle pin
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Towers of Hanoi

- **Solution:** move discs from left to right using the middle pin
  - move all discs (except the largest) from left pin to the middle pin using the right pin
  - move the largest disc from left to right pin
  - move all discs (except the largest) from middle pin to the right pin using the left pin
- this solution is inherently recursive
- can be formulated very simple by specifying individual moves
- to move n discs from pin 1 to pin 3 using pin 2, do:
  - move n-1 discs from pin 1 to pin 2 using pin 3
  - move disc from pin 1 to pin 3
  - move n-1 discs from pin 2 to pin 3 using pin 1

Towers of Hanoi

```
public class Move {  public int from, to;  // simply a pair of ints
    Move(int from, int to) {  this.to = to;  this.from = from;  }
    public String toString() {  return this.from+" -> "+this.to;  }
}
public static List<Move> hanoi(int n, int from, int to, int using) {
    List<Move> moves = new ArrayList<Move>();
    if (n > 1) {  moves.addAll(hanoi(n-1, from, using, to));  }
    moves.add(new Move(from, to));
    if (n > 1) {  moves.addAll(hanoi(n-1, using, to, from));  }
    return moves;
} …
System.out.println(hanoi(3, 1, 3, 2));
```

RECURSIVE DATA STRUCTURES

Recursive Data Structures

- like functions, data structures can be recursive, too
- recursive class = contains a member variable of same class
- Example:

```
public class Student {
    String name;
    Student tutor;
    public String toString() {
        return name + tutor == null ? "" : " (" + tutor.name + ")";
    }
}
```

- useful to implement linked lists, trees, …

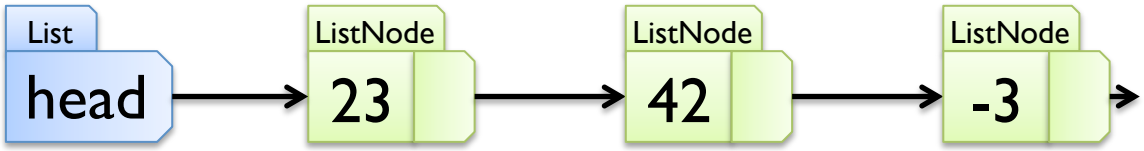
List ADT: Specification (revisited)

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface List<E> {
    public E get(int i);                // get i-th integer (0-based)
    public void set(int i, E elem);     // set i-th element
    public int size();                  // return length of list
    public void add(E elem);            // add element at end
    public void add(int i, E elem);     // insert element at pos. i
    public void remove(int i);          // remove i-th element
}
```

Linked Lists

- arrays require copying when inserting in the middle
- avoid copying by using links from one element to the next
- **Idea:**
  - use a recursive data structure ListNode
  - one instance of this class per element
  - every node (except the last) refers to next element



List ADT: Design & Implementation 4

- Design 4: linked lists
- Implementation 4:

```
public class ListNode<E> {
    private E elem;                          // one list element
    private ListNode<E> next;                // next element
    public ListNode(E elem, ListNode<E> next) {
        this.elem = elem;
        this.next = next;
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public E get(int i) {
        if (i == 0) {  return this.elem;  }
        if (this.next == null) {  throw new Index…Exception();  }
        return this.next.get(i-1);
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public void set(int i, E elem) {
        if (i == 0) {  this.elem = elem;  } else {
            if (this.next == null) {  throw new Index…Exception();  }
            this.next.set(i-1, elem);
        }
    }
    …   }
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public int size() {
        int result = 1;
        if (this.next != null) {  result += this.next.size();  }
        return result;
    }
    …
}
```

List ADT: Implementation 4

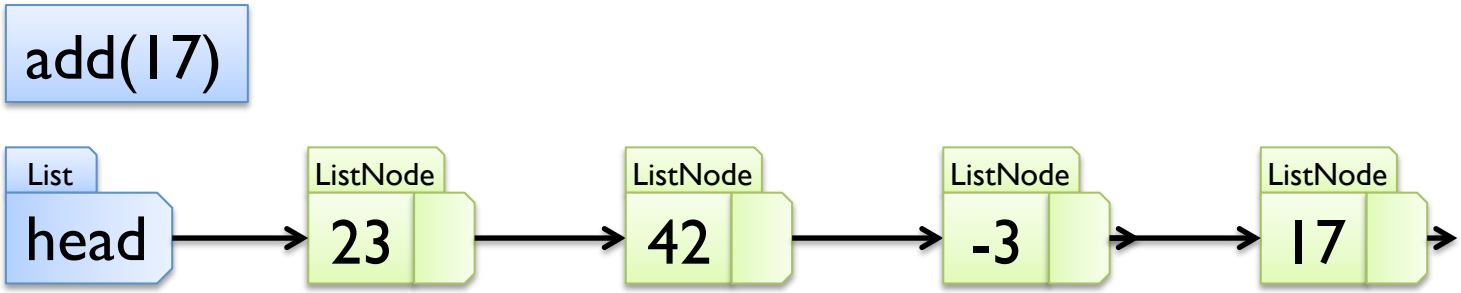
- Implementation 4 (continued)

```
public class ListNode<E> {
    private E elem;                    // one list element
    private ListNode<E> next;          // next element
    …
    public void add(E elem) {
        if (this.next != null) {
            this.next.add(elem);
        } else {
            this.next = new ListNode<E>(elem, null);
        }
    }   …   }
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void add(E elem) {
    if (this.next != null) {
        this.next.add(elem);
    } else {
        this.next = new ListNode<E>(elem, null);
    }
}
```



List ADT: Implementation 4

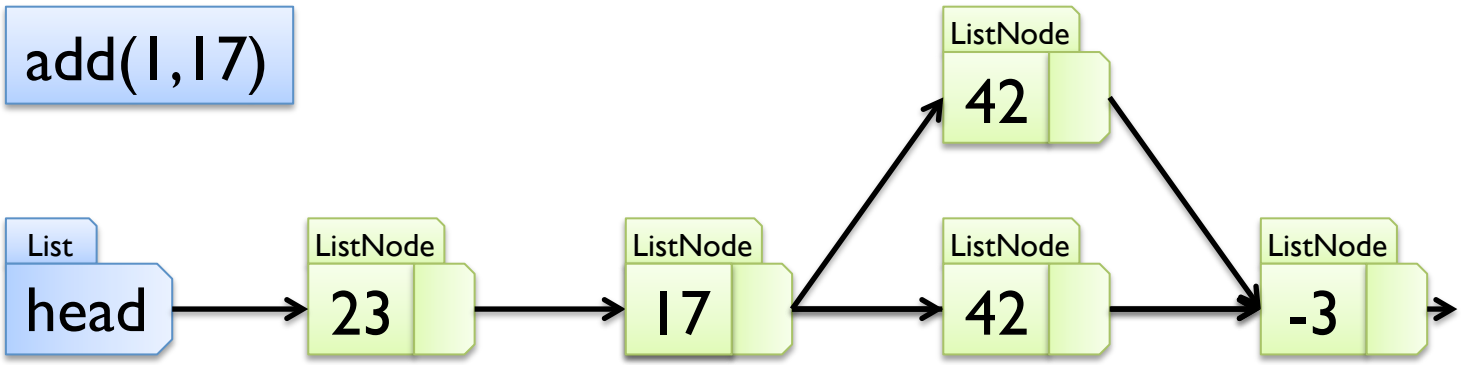
- Implementation 4 (continued)

```
public void add(int i, E elem) {
    if (i == 0) {
        this.next = new ListNode<E>(this.elem, this.next);
        this.elem = elem;
    } else {
        if (this.next == null) {
            if (i == 1) {  this.add(elem);  }       // end of list
            else {  throw new Index…Exception();  }
        }  else {  this.next.add(i-1, elem);  }
    }
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void add(int i, E elem) {
    if (i == 0) {
        this.next = new ListNode<E>(this.elem, this.next);
        this.elem = elem;
    } else {
        …
```



List ADT: Implementation 4

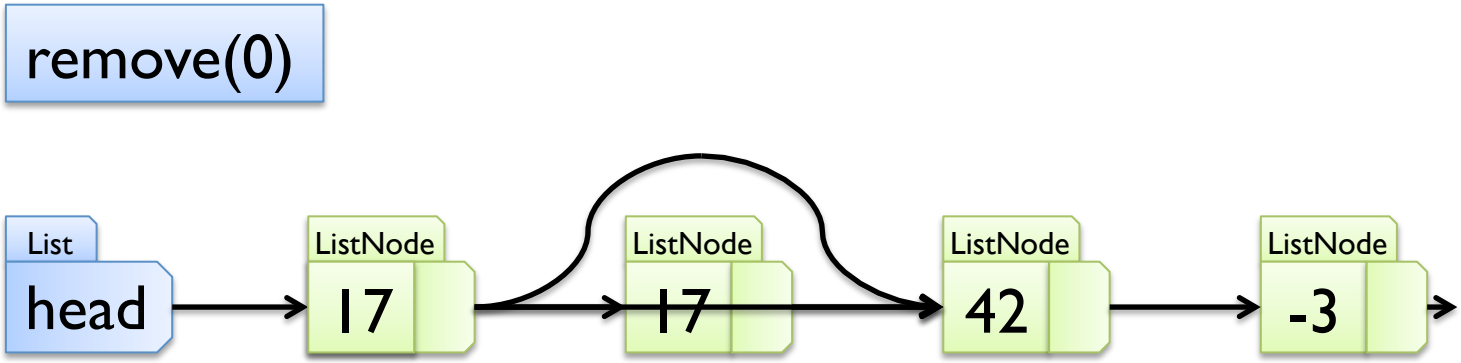
- Implementation 4 (continued)

```
public void remove(int i) {
    if (i == 0) {
        if (this.next == null) {  throw new RuntimeException();  }
        this.elem = this.next.elem;
        this.next = this.next.next;
    } else {
        if (this.next == null) {  throw new Index…Exception();  }
        this.next.remove(i-1);
    }
}
```

List ADT: Implementation 4

- Implementation 4 (continued)

```
public void remove(int i) {
    if (i == 0) {
        if (this.next == null) {  throw new RuntimeException();  }
        this.elem = this.next.elem;
        this.next = this.next.next;
    }   …
```



List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;
    public E get(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        return head.get(i);
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public void set(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        head.set(i, elem);
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public int size() {
        if (head == null) {  return 0;  }
        return head.size();
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {
    private ListNode<E> head = null;   …
    public void add(E elem) {
        if (head == null) {
            head = new ListNode<E>(elem, null);
        } else {
            head.add(elem);
        }
    }
    …
}
```

List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {   …
    public void add(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {
            if (i > 0) {  throw new Index…Exception();  }
            head = new ListNode<E>(elem, null);
        } else {
            head.add(i, elem);
        }
    }   …
}
```

List ADT: Implementation 4

- Implementation 4 (continued):

```
public class LinkedList<E> implements List<E> {   …
    public void remove(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        if (head == null) {  throw new Index…Exception();  }
        else if (head.getNext() == null) {
            if (i > 0) {  throw new Index…Exception();  }
            head = null;
        } else {
            head.remove(i);
        }
    }  }
```

STATIC FUNCTIONS FOR RECURSIVE DATA STRUCTURES

List ADT: Implementation 5

- Implementation 5:

```
public class RecursiveList<E> implements List<E> {
    private ListNode<E> head = null;
    public E get(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        return get(this.head, i);
    }
    public static <E> E get(ListNode<E> node, int i) {
        if (node == null) {  throw new Index…Exception();  }
        if (i == 0) {  return node.getElem();  }
        return get(node.getNext(), i-1);
    }   …
```

List ADT: Implementation 5

- Implementation 5 (continued):

```
public class RecursiveList<E> implements List<E> {   …
    public void set(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        set(this.head, i, elem);
    }
    public static <E> void set(ListNode<E> node, int i, E elem) {
        if (node == null) {  throw new Index…Exception();  }
        if (i == 0) {  node.setElem(elem);  }
        else {  set(node.getNext(), i-1, elem);  }
    }
    …
```

List ADT: Implementation 5

- Implementation 5 (continued):

```
public class RecursiveList<E> implements List<E> {
    …
    public int size() {
        return size(this.head);
    }
    public static <E> int size(ListNode<E> node) {
        if (node == null) {  return 0;  }
        return 1+size(node.getNext());
    }
    …
```

List ADT: Implementation 5

- Implementation 5 (continued):

```
public class RecursiveList<E> implements List<E> {
    …
    public void add(E elem) {
        this.head = add(this.head, elem);
    }
    public static <E> ListNode<E> add(ListNode<E> n, E e) {
        if (n == null) {  return new ListNode<E>(e, null);  }
        n.setNext(add(n.getNext(), e));
        return n;
    }
    …
```

List ADT: Implementation 5

- Implementation 5 (continued):

```
public class RecursiveList<E> implements List<E> {   …
    public void add(int i, E elem) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        this.head = add(this.head, i, elem);
    }
    public static <E> ListNode<E> add(ListNode<E> n, int i, E e) {
        if (i == 0) {  return new ListNode<E>(e, n);  }
        if (n == null) {  throw new Index…Exception();  }
        n.setNext(add(n.getNext(), i-1, e));
        return n;
    }   …
```

List ADT: Implementation 5

- Implementation 5 (continued):

```
public class RecursiveList<E> implements List<E> {   …
    public void remove(int i) {
        if (i < 0) {  throw new IllegalArgumentException();  }
        this.head = remove(this.head, i);
    }
    public static <E> ListNode<E> remove(ListNode<E> n, int i) {
        if (n == null) {  throw new Index…Exception();  }
        if (i == 0) {  return n.getNext();  }
        n.setNext(remove(n.getNext(), i-1));
        return n;
    }   }  // DONE
```

ABSTRACT DATA TYPES FOR STACKS & QUEUES

Stacks

- stacks are special sequences, where elements are only added and removed at one end
- imagine a stack of paper on a desk
- many uses:
  - postfix calculator
  - activation records
  - depth-first tree traversals
  - …
- basic stack operations are
  - looking at the top of the stack
  - removing the top-most element
  - adding an element to the top of the stack

Stack ADT: Specification

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface Stack<E> {
    public boolean isEmpty();          // is stack empty?
    public E peek();                   // look at top element
    public E pop();                    // remove top element
    public void push(E elem);          // add top element
}
```

Stack ADT: Design 1

- Design 1: use dynamic array
  - the top of the stack is the end of the list
  - in other words, num specifies the top position
  - pushing corresponds to adding at the end
  - poping corresponds to removing at the end

Stack ADT: Implementation 1

- Implementation 1:

```
public class DynamicArrayStack<E> implements Stack<E> {
    private int limit;              // maximal number of elements
    private E[] data;               // elements of the list
    private int num = 0;            // current number of elements
    public DynamicArrayStack(int limit) {
        this.limit = limit;
        this.data = (E[]) new Object[limit];
    }
    public boolean isEmpty() {  return this.num == 0;  }
    …
}
```

Stack ADT: Implementation 1

- Implementation 1 (continued):

```
public class DynamicArrayStack<E> implements Stack<E> {   …
    public E peek() {
        if (this.isEmpty()) {  throw new RuntimeException("es");  }
        return this.data[this.num-1];
    }
    public E pop() {
        E result = this.peek();
        num--;
        return result;
    }   …
}
```

Stack ADT: Implementation 1

- Implementation 1 (continued):

```
public class DynamicArrayStack<E> implements Stack<E> {   …
    public void push(E elem) {
        if (this.num >= this.limit) {
            E[] newData = (E[]) new Object[2*this.limit];
            for (int j = 0; j < limit; j++) {  newData[j] = data[j];  }
            this.data = newData;
            this.limit *= 2;
        }
        this.data[num++] = elem;
    }
}
```

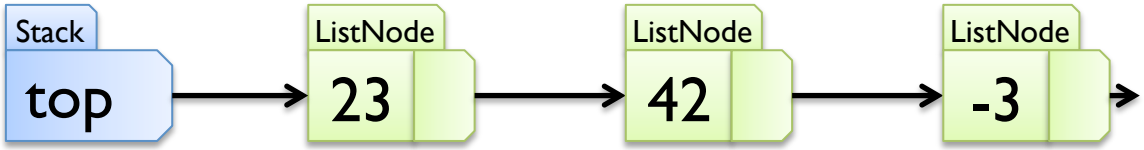
Stack ADT: Design & Implement. 2

- Design 2: reuse dynamic array list (ArrayList<E>)
- Implementation 2:

```
public class ArrayListStack<E> implements Stack<E> {
    private List<E> list = new ArrayList<E>();
    public boolean isEmpty() {  return this.list.isEmpty();  }
    public E peek() {  return this.list.get(this.list.size()-1);  }
    public E pop() {  return this.list.remove(this.list.size()-1);  }
    public void push(E elem) {  this.list.add(elem);  }
}
```

Stack ADT: Design 3

- Design 3: use recursive data structure
  - linked lists have cheap insert and remove operations
  - adding at the end requires running to the end
  - represent top as the beginning of the "list"
- reuse linked list node class (ListNode<E>)
- with dynamic arrays, sometimes need to copy full array
- with linked list, always constant time operations



Stack ADT: Implementation 3

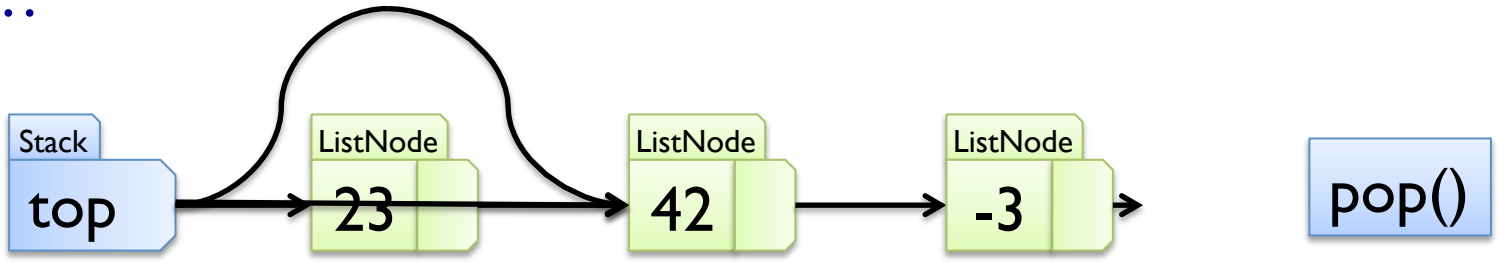
- Implementation 3:

```
public class LinkedStack<E> implements Stack<E> {
    private ListNode<E> top = null;    // top of the stack
    public boolean isEmpty() {  return this.top == null;  }
    public E peek() {
        if (this.isEmpty()) {  throw new RuntimeException("es");  }
        return this.top.get(0);
    }
    …
}
```

Stack ADT: Implementation 3

- Implementation 3 (continued):

```
public class LinkedStack<E> implements Stack<E> {
    …
    public E pop() {
        E result = this.peek();
        this.top = this.top.getNext();
        return result;
    }
    …
}
```



Stack ADT: Implementation 3

- Implementation 3 (continued):

```
public class LinkedStack<E> implements Stack<E> {
    private ListNode<E> top = null;    // top of the stack
    …
    public void push(E elem) {
        this.top = new ListNode<E>(elem, this.top);
    }
}
```