DM537
Object-Oriented Programming

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ABSTRACT DATA TYPES FOR (BINARY) TREES

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Trees

- trees store elements non-sequentially

- every node in a tree has 0 or more children

- imagine a tree with root in the air ☺

- many uses:

  - decision tree

  - binary sort trees

  - data base indices

  - …

- no consensus on what basic binary tree operations are ☹

- set of operations depends on application

- **here:** keeping elements sorted, combinatorics
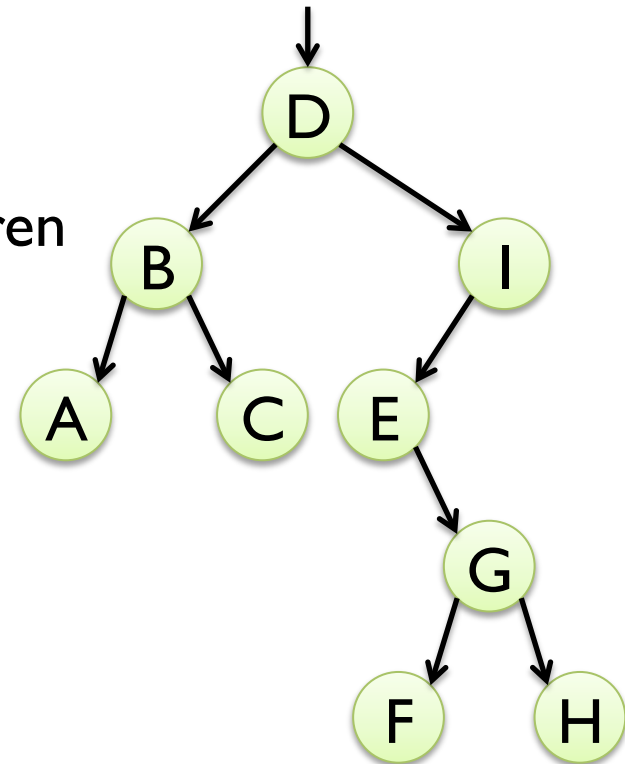
Binary Trees

- special case of general trees

- every node in a tree has 0, 1 or 2 children

- tree on the right is an example

- notation:
  - first node is called "root"
  - other nodes either in "left subtree"
  - … or in "right subtree"

- every node is root in its own subtree!

- for example, look at node B

- node A is the "left child" of B

- node C is the "right child" of B

- node B is the "parent" of both A and C

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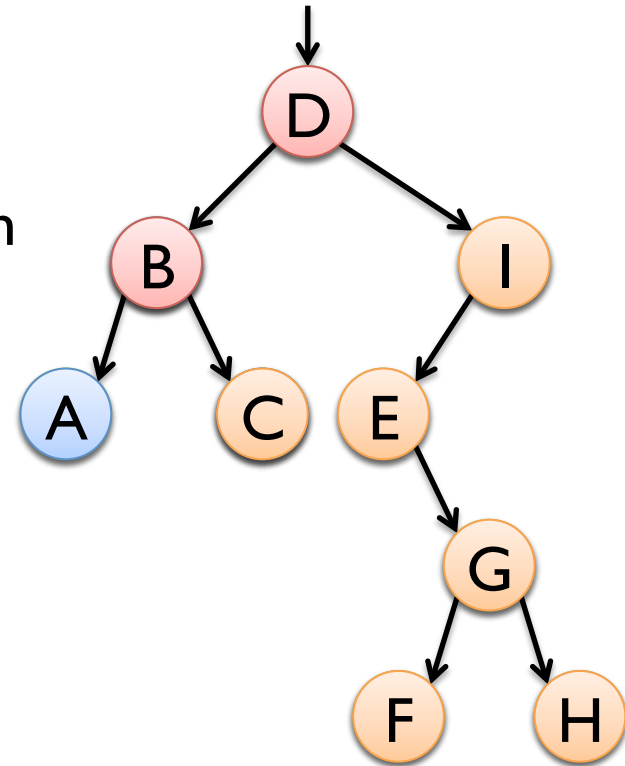
BinTree ADT: Specification

- data are arbitrary objects of class E
- operations are defined by the following interface

```
public interface BinTree<E> {
    public boolean isEmpty();           // is tree empty?
    public int size();                  // number of elements
    public int height();                // maximal depth
    public List<E> preOrder();          // pre-order traversal
    public List<E> inOrder();           // in-order traversal
    public List<E> postOrder();         // post-order traversal
}
```

BinTree ADT: Design & Implement. I

- Design 1:   use recursive data structure
  - based on representing tree nodes by BinTreeNode<E>
- Implementation 1:

```
public class BinTreeNode<E> {
    public E elem;
    public BinTreeNode<E> left, right;
    public BinTreeNode(E elem, BinTreeNode<E> left,
                                BinTreeNode<E> right) {
        this.elem = elem;
        this.left = left;  this.right = right;
    }
}
```
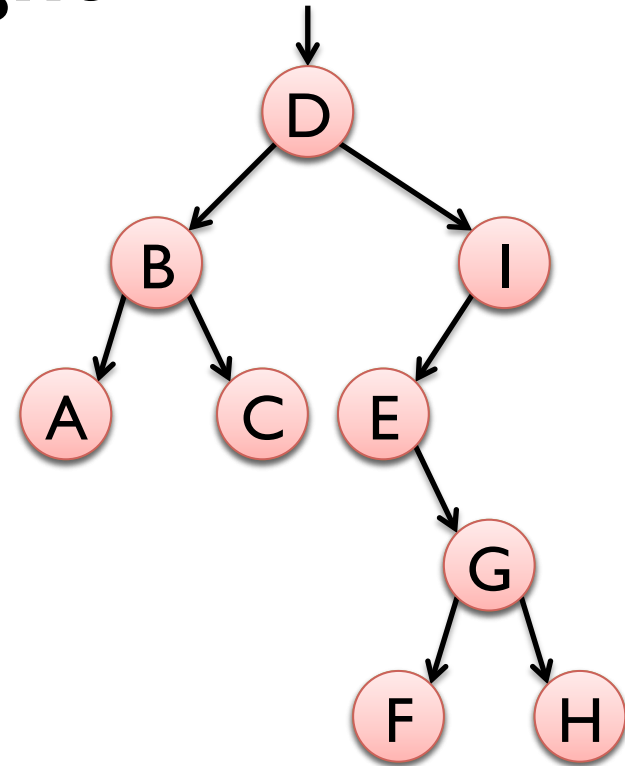
BinTree ADT: Implementation 1

- Implementation 1 (continued):

```
public class RecursiveBinTree<E> {
    private BinTreeNode<E> root = null;
    public boolean isEmpty() {  return this.root == null;  }
    public int size() {  return size(this.root);  }
    private static <E> int size(BinTreeNode<E> node) {
        if (node == null) {  return 0;  }
        return 1 + size(node.left) + size(node.right);
    }
    …
}
```

Depth and Height

- depth of the root is 0
- depth of other nodes is 1+depth(parent)
- Example:
  - 0
  - 1
  - 2
  - 3
  - 4
- height of a subtree is maximal depth of any of its nodes
- Example: height of tree (=subtree starting in D) is 4

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BinTree ADT: Implementation 1

- Implementation 1 (continued):

```
public class RecursiveBinTree<E> {
    private BinTreeNode<E> root = null;

    …

    public int height() {  return height(this.root);  }
    private static <E> int height(BinTreeNode<E> node) {
        if (node == null) {  return -1;  }
        return 1 + max(height(node.left), height(node.right));
    }
    private static int max(int a, int b) {  return a > b ? a : b;  }

    …
}
```

Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
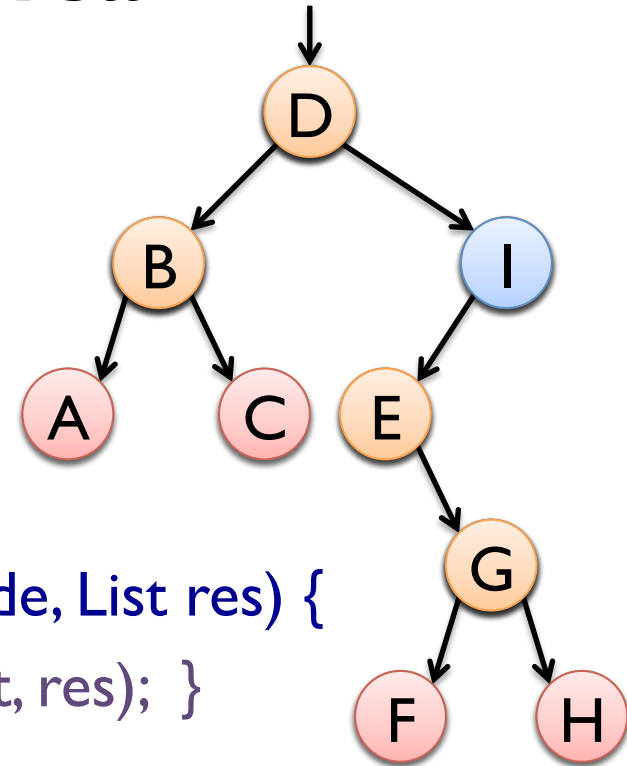 1. pre-order
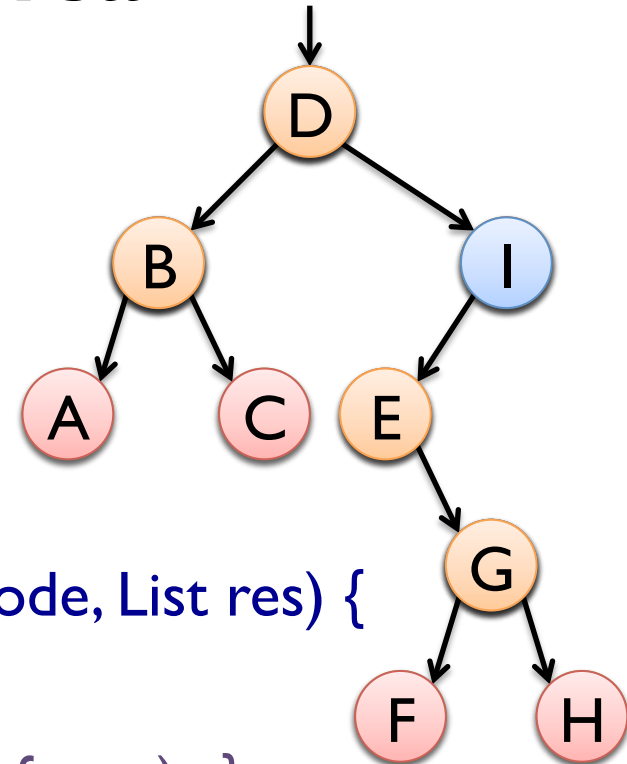
```
public static void preOrder(BinTreeNode node, List res) {
    res.add(node.elem);
    if (node.left != null)  {  preOrder(node.left, res);  }
    if (node.right != null) {  preOrder(node.right, res);  }
}
```

D	B	A	C	I	E	G	F	H

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Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
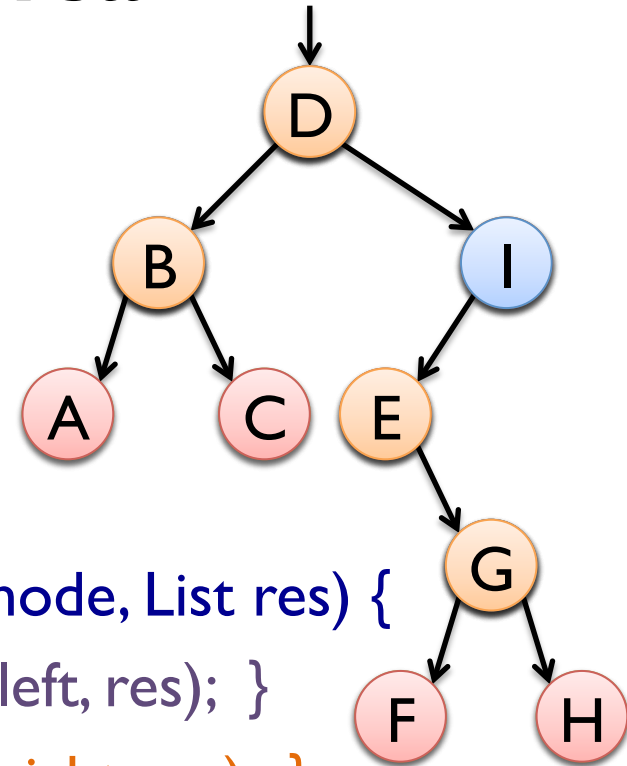 2. in-order

```
public static void inOrder(BinTreeNode node, List res) {
    if (node.left != null)   {  inOrder(node.left, res);  }
    res.add(node.elem);
    if (node.right != null) {  inOrder(node.right, res);  }
}
```
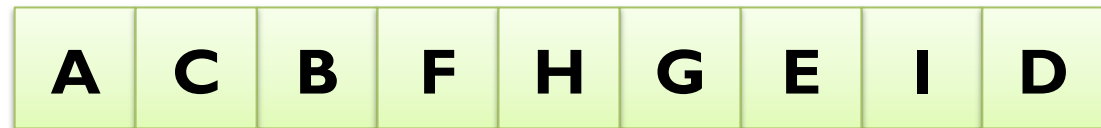
A	B	C	D	E	F	G	H	I

Binary Tree Traversal

- traversal can be either
  - depth-first
  - breadth-first
- three standard depth-first traversals
  3. post-order

public static void postOrder(BinTreeNode node, List res) {

    if (node.left != null)　{　postOrder(node.left, res);　}

    if (node.right != null) {　postOrder(node.right, res);　}

    res.add(node.elem);

}

A	C	B	F	H	G	E	I	D

BinTree ADT: Implementation 1

- Implementation 1 (continued):

```
public class RecursiveBinTree<E> {
    private BinTreeNode<E> root = null;

    …

    public List<E> preOrder() {
        List<E> res = new ArrayList<E>();
        if (this.root != null) {  preOrder(this.root, res);  }
        return res;
    }

    …   // the same for inOrder, postOrder
}
```
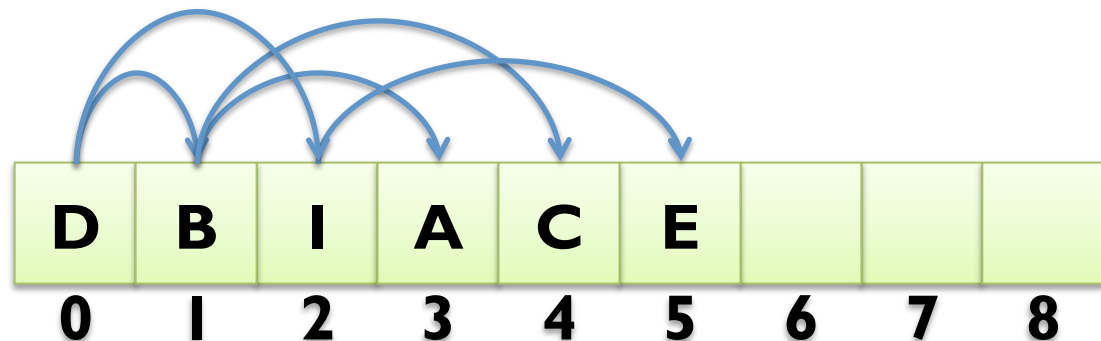
BinTree ADT: Design & Implement. 2

- Design 2:   use array (list)
  - root stored at position 0
  - left child of position n stored at 2n+1
  - right child of position n stored at 2n+2
  - null where position stores no element
- Implementation 2:

public class ArrayBinTree<E> {

private List<E> data = new ArrayList<E>();

…

}

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BinTree ADT: Implementation 2

- Implementation 2 (continued):

```java
public class ArrayBinTree<E> {
    private List<E> data = new ArrayList<E>();

    …

    public boolean isEmpty() {  return this.data.get(0) == null;  }
    public int size() {
        int counter = 0;
        for (int i = 0; i < this.data.size(); i++) {
            if (this.data.get(i) != null) {  counter++;  }
        }
        return counter;
    }  …  }
```

BinTree ADT: Implementation 2

- Implementation 2 (continued):

```
public class ArrayBinTree<E> {
    private List<E> data = new ArrayList<E>();

    …

    public int height() {  return height(0);  }
    private int height(int index) {
        if (this.data.get(index) == null) {  return -1;  }
        return 1 + max(height(2*index+1), height(2*index+2));
    }
    private static in max(int a, int b) {  return a > b ? a : b;  }

    …

}
```

BinTree ADT: Implementation 2

- Implementation 2 (continued):

```
public List<E> preOrder() {
    return preOrder(0, new ArrayList<E>());
}
private List<E> preOrder(int index, java.util.List<E> res) {
    E elem = this.data.get(index);
    if (elem != null) {
        res.add(elem);
        preOrder(2*index+1, res);   preOrder(2*index+2, res);
    }
    return res;
}   … /* same for inOrder, postOrder */   }
```

BINARY SORTING TREES

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Binary Sort Tree

- special binary tree
- invariant for all subtrees:
  - all elements smaller than root in left subtree
  - all elements bigger than root in right subtree
- elements need to be comparable
- interface Comparable
- use method compareTo

- our example tree is a sort tree
- nodes are of class Character

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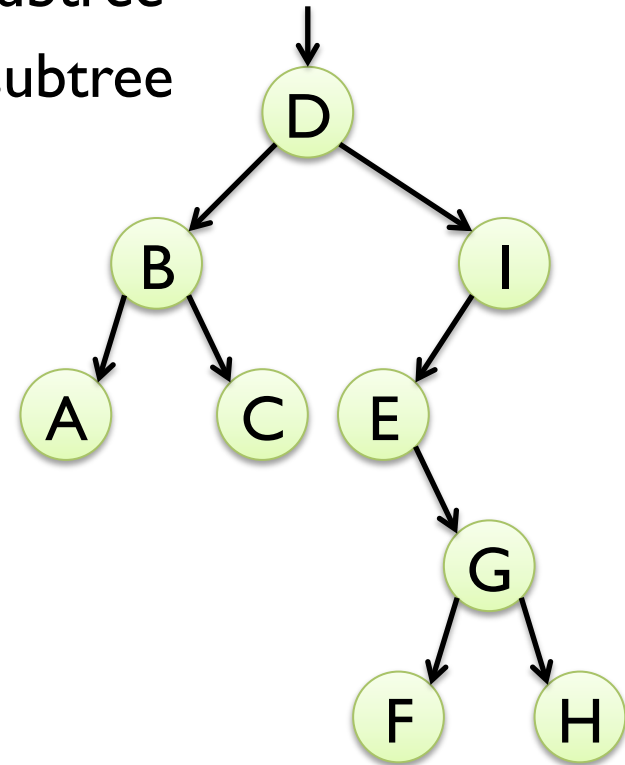
SortTree ADT: Specification

- data are objects of some class E that extends Comparable
- operations are defined by the following interface

```
public interface SortTree<E extends Comparable<E>> {
    public int size();                    // number of elements
    public boolean contains(E elem);      // true, if elem in tree
    public void add(E elem);              // add elem to tree
    public void remove(E elem);           // tremove elem from tree
    public List<E> traverse();            // in-order traversal
    public void balance();                // balance the tree
}
```

SortTree ADT: Design & Implement.

- Design: use recursive data structure
  - reuse class BinTreeNode<E>
- Implementation 1:

```
public class BinSortTree<E extends Comparable<E>>
    implements SortTree<E> {
    private BinTreeNode<E> root = null;
    public int size() {  return size(this.root);  }
    private static <E extends Comparable<E>> int
        size(BinTreeNode<E> node) {
        if (node == null) {  return 0;  }
        return 1 + size(node.left) + size(node.right);
    }  …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

contains('G')

```
public boolean contains(E elem) {
    return contains(this.root, elem);
}
private static <E extends Comparable<E>> boolean
    contains(BinTreeNode<E> node, E elem) {
    if (node == null) {  return false;  }
    switch (signum(elem.compareTo(node.elem))) {
    case -1:  return contains(node.left, elem);
    case 0:   return true;
    case 1:   return contains(node.right, elem);
    }  throw new RuntimeException("compareTo error");  }  …
```
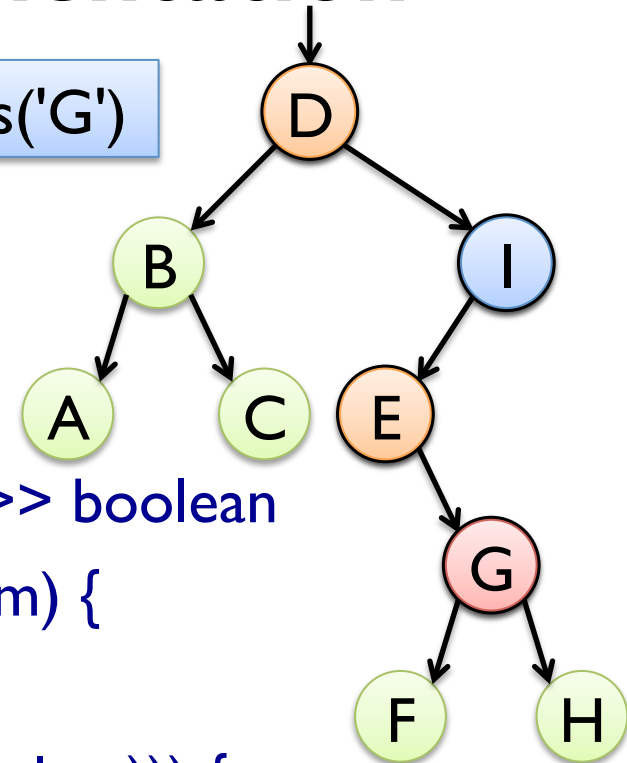
D
B
I
A
C
E
G
F
H

SortTree ADT: Implementation

- Implementation 1 (continued):

```
public void add(E elem) {  this.root = add(this.root, elem);  }
private static int signum(int x) {
    return x == 0 ? 0 : (x > 0 ? 1 : -1);
}
private static <E extends Comparable<E>> BinTreeNode<E>
    add(BinTreeNode<E> node, E elem) {
    if (node == null) {
        return new BinTreeNode<E>(elem, null, null);
    }
    switch (signum(elem.compareTo(node.elem))) {
    …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

      case -1:

          node.left = add(node.left, elem);

          return node;

      case 0:   return node;

      case 1:

          node.right = add(node.right, elem);

          return node;
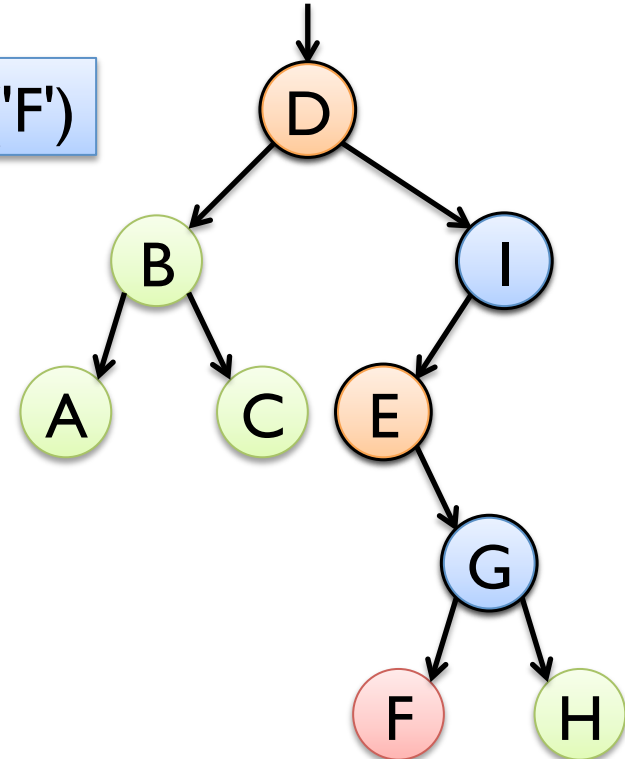
      }

      throw new RuntimeException("compareTo error");

      }

  …

add('F')

Binary Sort Tree: Removal
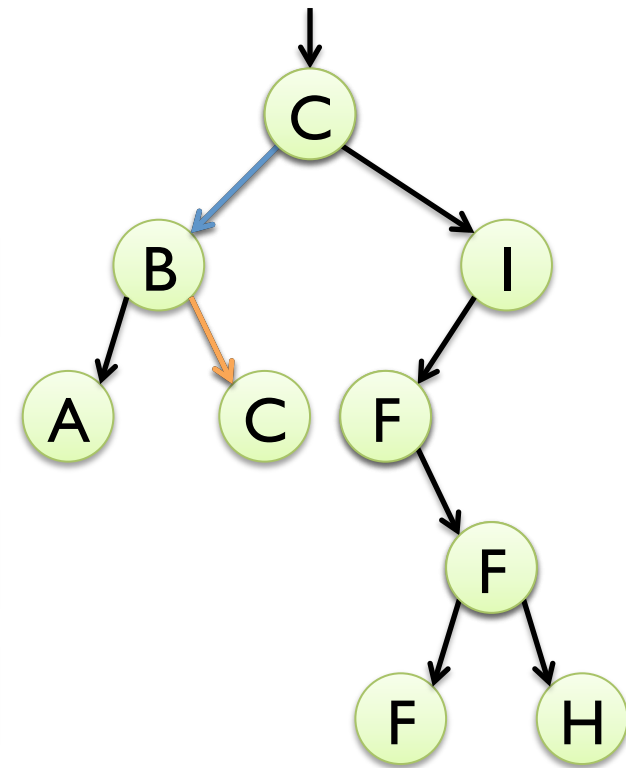
- first, find the node (as contains or add)
- four cases for node to be removed
  1. no children
  2. only a left child
  3. only a right child
  4. both left and right child
- easy to handle 1-3:
  1. delete node
  2. replace node by left child
  3. replace node by right child
  4. replace node by largest element in left subtree

delete('H')

delete('G')

delete('E')

delete('D')

SortTree ADT: Implementation

- Implementation 1 (continued):

```
public void remove(E elem) {
    this.root = remove(this.root, elem);
}
private static <E extends Comparable<E>> E
    max(BinTreeNode<E> node) {
    return node.right == null ? node.elem : max(node.right);
}
private static <E extends Comparable<E>> BinTreeNode<E>
    remove(BinTreeNode<E> node, E elem) {
    if (node == null) {  return null;  }
    switch (signum(elem.compareTo(node.elem))) {   …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

```
    case -1:  return new BinTreeNode<E>(node.elem,
                    remove(node.left, elem), node.right);
    case 0:   if (node.left == null) {  return node.right;  }
              if (node.right == null) {  return node.left;  }
              E max = max(node.left);
              return new BinTreeNode<E>(max,
                    remove(node.left, max), node.right);
    case 1:   return new BinTreeNode<E>(node.elem,
                    node.left, remove(node.right, elem));
    }
    throw new RuntimeException("compareTo error");  }  …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

```java
public java.util.List<E> traverse() {
    return traverse(this.root, new java.util.ArrayList<E>());
}
private static <E extends Comparable<E>> List<E>
    traverse(BinTreeNode<E> node, List<E> result) {
    if (node != null) {
        traverse(node.left, result);
        result.add(node.elem);
        traverse(node.right, result);
    }
    return result;  }  …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

```
public void balance() {  this.root = balance(this.root);  }
private static <E extends Comparable<E>> E
    min(BinTreeNode<E> node) {
    return node.left == null ? node.elem : min(node.left);
}
private static <E extends Comparable<E>> BinTreeNode<E>
    balance(BinTreeNode<E> node) {
    if (node == null) {  return null;  }
    int lSize = size(node.left);
    int rSize = size(node.right);
    …
```

SortTree ADT: Implementation

- Implementation 1 (continued):

```
    while (lSize > rSize+1) {
        E max = max(node.left);  lSize--;  rSize++;
        node = new BinTreeNode<E>(max,
            remove(node.left, max), add(node.right, node.elem));  }
    while (rSize > lSize+1) {
        E min = min(node.right);  rSize--;  lSize++;
        node = new BinTreeNode<E>(min,
            add(node.left, node.elem), remove(node.right, min));  }
    return new BinTreeNode<E>(node.elem,
        balance(node.left), balance(node.right));
} } // DONE!
```

MULTIVARIATE TREES

Multivariate Trees

- general class of trees

- nodes can have any number of children

- our application:
  - k-permutations of n
  - all sequences without repetition
  - total of n elements
  - sequences of length k < n

- Example:
  - total of n = 3 elements
  - sequences of length k = 2
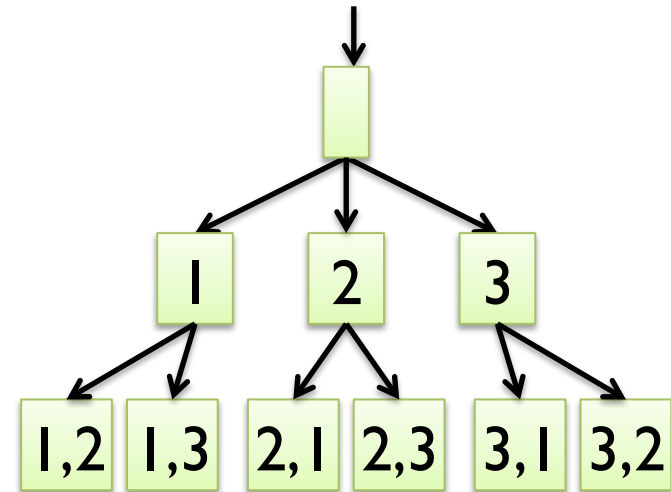
MTree ADT: Specification

- data are sequences of integers (List<Integer>)
- operations are defined by the following interface

```
public interface MTreeADT extends javax.swing.tree.TreeModel {
    public Object getRoot();
    public boolean isLeaf(Object node);
    public int getChildCount(Object node);
    public Object getChild(Object parent, int index);
    public int getIndexOfChild(Object parent, Object child);
}
```

- TreeModel specifies additional operations needed for JTree

MTree ADT: Design & Implement.

- Design: use recursive data structure
- Implementation:

```
public class MTreeNode {
    private List<Integer> seq;  // sequence of integers
    private List<MTreeNode> children =
        new ArrayList<MTreeNode>();
    public MTreeNode(List<Integer> seq) {  this.seq = seq;  }
    public List<Integer> getSeq() {  return this.seq;  }
    public List<MTreeNode> getChildren() {  return this.children;  }
    public String toString() {  return this.seq.toString();  }
}
```

MTree ADT: Implementation

- Implementation (continued):

```
public class MTree implements MTreeADT {
    private MTreeNode root = new MTreeNode(
        new ArrayList<Integer>());
    public Object getRoot() {  return this.root;  }
    public boolean isLeaf(Object node) {
        return this.getChildCount(node) == 0;
    }
    public int getChildCount(Object node) {
        return ((MTreeNode)node).getChildren().size();
    }
    …
```

MTree ADT: Implementation

- Implementation (continued)

```
public class MTree implements MTreeADT {   …
    public Object getChild(Object parent, int index) {
        return ((MTreeNode)parent).getChildren().get(index);
    }
    public int getIndexOfChild(Object parent, Object child) {
        return ((MTreeNode)parent).getChildren().indexOf(child);
    }
    public void addTreeModelListener(TreeModelListener l) {}
    public void removeTreeModelListener(TreeModelListener l) {}
    public void valueForPathChanged(TreePath p, Object o) {}
    …
```

MTree ADT: Implementation

- Implementation (continued)

```
public class MTree implements MTreeADT {   …
    private static MTree makeTree(String title) {
        MTree tree = new MTree();
        JScrollPane content = new JScrollPane(new JTree(tree));
        JFrame window = new JFrame(title);
        window.setContentPane(content);
        window.setDefaultCloseOperation(JFrame.EXIT_ON_CLOSE);
        window.setSize(400,400);
        window.setVisible(true);
        return tree;
    }   …
```

MTree ADT: Implementation

- Implementation (continued)

```
public class MTree implements MTreeADT {   …
    public static void main(String[] args) {
        int n = 4;
        int k = 3;
        MTree tree1 = makeTree("MTree queue");
        tree1.expandQueue(n,k);
        MTree tree2 = makeTree("MTree stack");
        tree2.expandStack(n,k);
        MTree tree3 = makeTree("MTree recursive");
        tree3.expandRecursively(n,k);
    }   …
```

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MTree ADT: Implementation

- Implementation (continued)

```java
public class MTree implements MTreeADT {

    …

    private void expandRecursively(int n, int k) {
        expandRecursively(this.root, n, k);
    }

    private List<Integer> copyAdd(List<Integer> seq, int i) {
        List<Integer> copySeq = new ArrayList<Integer>(seq);
        copySeq.add(i);
        return copySeq;
    }

    …
```

MTree ADT: Implementation

- Implementation (continued):

```
private void expandRecursively(MTreeNode node, int n, int k) {
    List<Integer> seq = node.getSeq();
    if (seq.size() < k) {
        for (int i = 1; i <= n; i++) {
            if (!seq.contains(i)) {
                List<Integer> newSeq = copyAdd(seq, i);
                MTreeNode child = new MTreeNode(newSeq);
                node.getChildren().add(child);
                expandRecursively(child, n, k);
} } } }
…
```

MTree ADT: Implementation

- Implementation (continued):

```
private void expandStack(int n, int k) {
    Stack<MTreeNode> stack = new LinkedStack<MTreeNode>();
    stack.push(this.root);
    while (!stack.isEmpty()) {
        MTreeNode node = stack.pop();
        List<Integer> seq = node.getSeq();
        if (seq.size()<k) {  for (int i=1; i<=n; i++) {  if (!seq.contains(i)) {
            MTreeNode child = new MTreeNode(copyAdd(seq, i));
            node.getChildren().add(child);
            stack.push(child);
        }}}   }   }   …
```

MTree ADT: Implementation

- Implementation (continued):

```
private void expandQueue(int n, int k) {
    Queue<MTreeNode> queue = new LinkedQueue<MTreeNode>();
    queue.offer(this.root);
    while (!queue.isEmpty()) {
        MTreeNode node = queue.poll();
        List<Integer> seq = node.getSeq();
        if (seq.size()<k) {  for (int i=1; i<=n; i++) {  if (!seq.contains(i)) {
            MTreeNode child = new MTreeNode(copyAdd(seq, i));
            node.getChildren().add(child);
            queue.offer(child);
        }}}   }   }   } // DONE!
```