Superphase Alg.

Mål : reducér V til $\frac{E}{B}$ og brug så $O(V + Sort(E))$ alg.

[$\Rightarrow O(\text{reduktionstid} + Sort(E))$.]

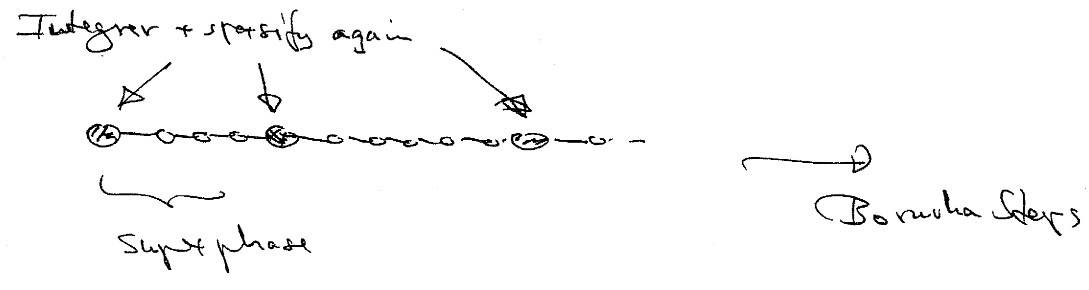
Max $\log_2\left(\frac{VB}{E}\right)$ Boruvka-steps will do $\left(\frac{V}{2^i} = \frac{E}{B} \Leftrightarrow i = \log_2\left(\frac{VB}{E}\right) \right)$

Straightforward use: $O(\underbrace{Sort(E)}_{\text{pris pr. fulde Boruvka steps}} \cdot \log_2\left(\frac{VB}{E}\right))$

$\boxed{\text{Idé}}$: + Gem kun de x mindste kanter for hver knude ($\Rightarrow$ mindre graf) (gør sparse)

+ Kør så længe vi kan garantere er korrekt (de mindste kan blive interne i cc'er undervejs $\Rightarrow$ forsvinder)

+ Integrér de udeladte fra E og gør sparse igen



$\boxed{\text{Mål}}$: hver superphase $O(Sort(E))$

færre superphases end $\log_2\left(\frac{VB}{E}\right)$

via Inv. A opnås $\log_{3/2}\log_2\left(\frac{VB}{E}\right)$ $\boxed{= O\left(Sort(E) \cdot \log\log\left(\frac{VB}{E}\right)\right)}$

$\Rightarrow 2\frac{V}{N_i} \le \frac{E}{V} \Leftrightarrow N_i \ge 2\frac{V^2}{E} \Leftrightarrow 2^{(3/2)^i} \ge 2\frac{V^2}{E}$

$2^{(3/2)^i} \ge x \Leftrightarrow (3/2)^i \ge \log_2 x \Leftrightarrow i \ge \log_{3/2}\log_2(x)$

Def : $N_i = 2^{(3/2)^i}$ $\left(\Rightarrow N_{i+1} = N_i^{3/2} \right)$

Superphase i = $\log_2(N_i^{1/2})$ phases

Invariant A: at start of superphase i, ~~#nodes~~ $|V_i| = 2\frac{|V|}{N_i}$
(graph $G_i = (V_i, E_i)$)

[OK ved start af superphase 0]

Sparsification at start of superphase i $(E_i \to E'_i)$:

For $\forall v$, keep $N_i^{1/2}$ lightes edges incident to v. (at most)

Blocking value for v = vægt af ~~[illegible]~~ største vægte

NB: for v med færre end $N_i^{1/2}$ kanter: no blocking val. [Not stated in paper].

Sparsification (+ blocking value) done by sort + skan.

$\Rightarrow O(\mathrm{Sort}(E_i))$.

Borůvka steps during superphase :

During contraction, new (super) node gets blocking val. = minimum of old nodes block. Val. (scan of kanter under udlægning [efter transfer of root-ID])). [if any B.Val exists.]

$\Rightarrow$ maintain invariant : all edges in original graph (at start of superphase) with value at most B.Val. (if any) are still present. [cut edges in supernode]

Only include node in Boruvka step if min edge out is (at most) blocking val. (if any B.Val present, else just include).

(then we know is smallest in real graph (un-sparsified), by invariant).

$\Rightarrow$ correctness OK (as Boruvka steps no need in to have all nodes choose an edge).

Proof of maintenance of invariant A

Case 1: en (super) node udgår af kontraktioner pga blocking value.

B.Val. kom fra knude u.

Alle u's $N_i^{1/2}$ naboer med kant vægt $\leq$ B.Val $\rightarrow$ så er i supernode

$\Rightarrow$ super node contains $\geq N_i^{1/2}$ contracted nodes

Case 2: ~~ELSE~~ : Size $= 2^{\log_2(N_i^{1/2})}$ at end of superphase.

So $V_{i+1} \leq \frac{V_i}{N_i^{1/2}} = 2\frac{V}{N_i \cdot N_i^{1/2}} = 2 \cdot \frac{V}{N_{i+1}}$ □

indo hyp. (invariant A)

Note:

Price of superphase : $V E_i' \le V_i \cdot N_i^{1/2}$

{error in paper: factor 2 appears}

$\le 2 \cdot \frac{V}{N_i} \cdot N_i^{1/2} = 2 \frac{V}{N_i^{1/2}}$

[invariant A]

Price: $O\Big(\mathrm{Sort}(E) + \log(N_i^{1/2}) \cdot \overbrace{\mathrm{Sort}\Big(\frac{V}{N_i^{1/2}}\Big)}^{\text{Boruvka steps in superstep.}}\Big)$

(sparsify, add removed edges back at end of superphase)

$= O(\mathrm{Sort}(E))$.

We must exchange/update ID's of endpoints of nodes:
During superphase, maintain translation ~~[illegible]~~ tuples (Old ID, current ID) for nodes, update in $O(\mathrm{Sort}(V_i))$ after each phase of superphase i.

This takes $O\Big(\log(N_i^{1/2}) \cdot \underbrace{\mathrm{Sort}(V_i)}_{\le \frac{2V}{N_i}}\Big)$ [Inv. A]

$= O(\mathrm{Sort}(V)) = O(\mathrm{Sort}(E))$

in total for superstep. $\square$.