

The “short reciprocal division” procedure is similar to hand computed long division, and will be illustrated by a decimal example employing a  $3 \times 6$  digit integer multiplier where the quotient is developed digit serially in the higher radix  $10^2 = 100$ .

**Example 5.6.1** (Short Reciprocal Division)

*Problem:* Find the 6-digit integer quotient  $q$  and 6-digit integer non-negative remainder  $r$  with  $0 \leq r \leq y - 1$  such that  $x = q \times y + r$ , where

$$\begin{aligned} x &= 365748000000 && (12 \text{ digit dividend}) \\ y &= 784731 && (6 \text{ digit divisor}) \end{aligned}$$

*Solution:* The result is determined digit serially as 3 radix 100 quotient digits in the redundant digit range  $[-100, 100]$ , and then converted to standard decimal as follows. Note that the arithmetic employed here is sign-magnitude decimal.

*Step 1:* Determine the 3 digit short reciprocal

$$\overline{y} = \left\lceil \frac{10^8}{784731} \right\rceil = 128.$$

*Step 2:* Perform digit serial division radix 100. Digit selection is obtained iteratively by multiplying the 3-digit short reciprocal by the 6-digit remainder, with the initial quotient digit obtained by multiplying the short reciprocal by the leading 6 digits of the dividend, and then selecting the signed leading 3 digits of the 9-digit product as the next redundant radix 100 digit. The digit selection is underlined in the following, and each high radix digit is expressed in decimal, with negative digits overlined.

$$\begin{array}{rcll} & & 46 \ 61 \ \overline{19}. & \\ 78 \ 47 \ 31 \ \overline{) 36 \ 57 \ 48 \ 00 \ 00 \ 00.} & & & \\ \quad 128 \times \quad 36 \ 57 \ 48 & = & \underline{46} \ 81 \ 57 \ 44 & \text{first digit selection} \\ 46 \times 78 \ 47 \ 31 = \quad 36 \ 09 \ 76 \ 26 & & & \\ \quad 128 \times \quad 47 \ 71 \ 74 & = & \underline{61} \ 07 \ 82 \ 72 & \text{second digit selection} \\ 61 \times 78 \ 47 \ 31 = \quad 47 \ 86 \ 85 \ 91 & & & \\ \quad 128 \times \quad -15 \ 11 \ 91 & = & \underline{-19} \ 35 \ 24 \ 48 & \text{third digit selection} \\ -19 \times 78 \ 47 \ 31 = \quad -14 \ 90 \ 98 \ 89 & & & \\ & & -20 \ 92 \ 11 & \end{array}$$

*Step 3:* Determine the positive remainder:  $r = 784731 - 209211 = 575520$ . Decrement and convert the redundant quotient to standard decimal form:

$$q = 46 \ 61 \ \overline{19}_{100} - 1 = 466080. \quad \square$$

The digit selection procedure illustrated in Step 2 of the example is justified as follows. In the first digit selection we seek to estimate a leading radix 100 digit (2 decimal digits) of  $\frac{365748}{784731}$ . Employing the exact divisor reciprocal  $\frac{1}{784731} = 127.4322 \cdots \times 10^{-4}$ , note that  $(127.4322 \cdots \times 10^{-4}) \times 365748 = 46.60807 \cdots$ , so “46” should be our leading radix 100 digit. Multiplying by the short reciprocal yields  $128 \times 365748 = 046815744$ , where the comparably normalized value is larger by less than 1% over the infinitely precise quotient. We select “46” knowing that it is either the correct radix 100 digit (leading to a positive