Introduction to Haskell II

Rolf Fagerberg

Spring 2006

Algebraic Types

Beside the simple type synonymes (using the keyword `type`), more advanced user defined types - denoted algebraic types - can be created with the `data` keyword.

The general syntax is:

```
data Typename zero_or_more_type-variables
  = Constructor1 zero_or_more_types |
  = Constructor2 zero_or_more_types |
  ⋮
  = Constructor3 zero_or_more_types
  deriving (list_of_certain_classes)
```

The identifiers for the type name and the constructor names must be capitalized.

Examples

Enumerated types:

```
data Bool = False | True
data Ordering = LT | EQ | GT
data Seasons = Winter | Spring | Summer | Fall
data WeekDays
  = Mon | Tue | Wed | Thu | Fri | Sat | Sun

workDays = [Mon, Tue, Wed, Thu, Fri]
```

Product types (alias tuples, alias records):

```
data DBRecord = DBRec Name Address Age
type Name = String
type Address = String
type Age = Int

person1 = DBRec "Joe Dole" "Main Street 10" 42
```

Examples

Alternatives:

```
data Shape
  = Circle Float | Rectangle Float Float
```

Note: constructors are functions:

```
Circle ::  Float -> Shape
shape1 = Circle 3.0
Rectangle ::  Float -> Float -> Shape
shape2 = Rectangle 45.9 87.6
```

Additionally, they can (like the built-in constructors `[]`, `:`, etc.) be used as patterns in pattern matching:

```
area ::  Shape -> Float
area (Circle r)       = pi*r*r
area (Rectangle w h) = w*h
```

Examples

Algebraic types can be recursive:

```
data IntList = EmptyList | Cons Int IntList

data IntExpr = Literal Int |
               Add IntExpr IntExpr |
               Sub IntExpr IntExpr

data IntTree = IntLeaf |
               IntNode Int IntTree IntTree

tree = IntNode 7 IntLeaf (IntNode 13 IntLeaf IntLeaf)
```

Constructors can be infix operators (identifier must then start with ´:´):

```
data IntList = EmptyList | Int ::: IntList
```

Examples

Algebraic types can be parametric:

```
data List a = EmptyList | Cons a (List a)

data Tree a = Leaf |
              Node a (Tree a) (Tree a)
```

Example functions on trees:

```
depth :: Tree a -> Int
depth Leaf          = 0
depth (Node _ l r) = 1 + max (depth l) (depth r)

inorder :: Tree a -> [a]
inorder Leaf          = []
inorder (Node x l r) = inorder l ++ [x] ++ inorder r
```

Haskell Classes

Class = set of types

Classes defined by giving their signature = the set of functions required to be defined on the types in the class (so signature $\approx$ interface in Java).

```
class Eq a where
(==) ::  a -> a -> Bool
```

Adding types to the class:

```
instance Eq MyBool where
  (==) MyTrue  MyTrue  = True
  (==) MyFalse MyFalse = True
  (==) _       _       = False
```

Context

Classes can be used as context, i.e. requirements on the parametric types used:

```
elem ::  Eq a => a -> [a] -> Bool
elem x [] = False
elem x (y:ys) = (x == y) || (elem x ys)
```

Can also be used in instance declarations:

```
instance Eq a => Eq [a] where
  (==) []     []     = True
  (==) (x:xs) (y:yx) = (x == y) && (xs == ys)
  (==) _      _      = False
```

Note: not all types are in the (built-in) class `Eq`. E.g. function types are not (it seems difficult to give an operational feasible definition of function equality).

Overloading vs. Polymorphism

Polymorphism

One definition of function works for many types.

Overloading

Several definitions of the same function (i.e. same identifier), one for each type.

OO languages like Java normally have overloading but not polymorphism.

In Haskell, overloading eases coding (imagine naming a version of `==` for each type) and makes the notion of polymorphism stronger (more functions can be defined with the same code).

Default Definitions

Class declarations can contain default definitions:

```
class Eq a where
  (==), (/=) ::  a -> a -> Bool
  x /= y     = not (x==y)
  x == y     = not (x/=y)
```

Now, instance declarations only need to define `/=` or `==`.
Defining (overriding) both is OK.

Derived Classes

Classes can be derived from other classes (again using the context notation):

```
class (Eq a) => Ord a where
  (<), (<=), (>), (>=) ::  a -> a -> Bool
  max, min             ::  a -> a -> a
  compare              ::  a -> a -> Ordering
```

When declaring a type an instance of `Ord`, the methods of `Eq` are inherited.

Thus, type classes form a hierarchy rather like the class hierarchy in OO languages.

Some Built-In Classes and Types

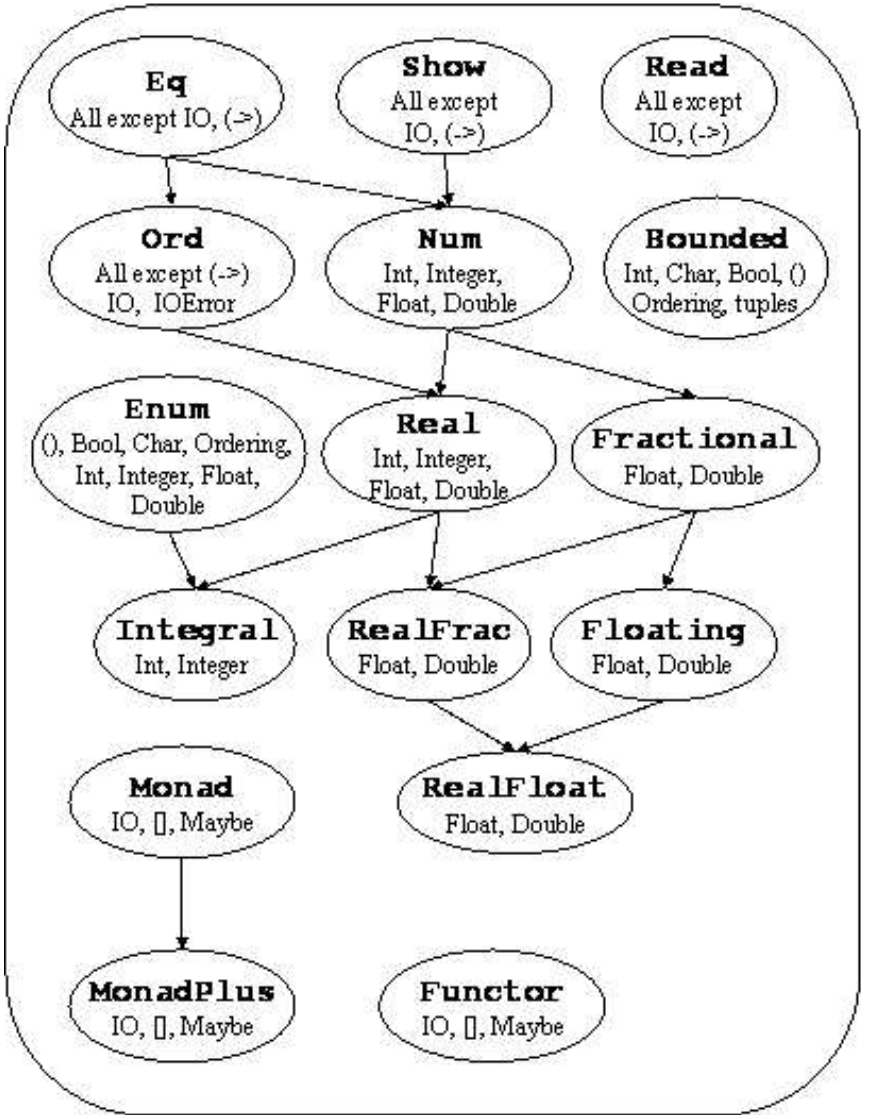
The standard prelude contains many predefined type classes.

E.g. for equality (`Eq`), ordering (`Ord`), enumeration(`Enum`), serialization (`Show`, `Read`), numeric types (`Int`, `Integer`, `Float`, `Double`, `Rational`, `Complex`).

Literals may be overloaded, which can lead to ambiguities for Haskell. Of what type is e.g. `2+3`? It may be necessary to resolve explicitly:

```
(2+3)::Int
```

All Built-In Classes



See `www.haskell.org/onlinereport/`

Deriving Membership of Classes

Membership of certain standard type classes can be generated automatically in Haskell:

```
data WeekDays
  = Mon | Tue | Wed | Thu | Fri | Sat | Sun
  deriving (Eq, Ord, Enum, Show, Read)
```

The operations of the classes are automatically defined using obvious (recursive) definitions (with ordering going from left to right, and using analogy with lexicographic ordering for recursive structures). The derivation of `Enum` can only be done for enumeration types (nullary constructors only). More on `Enum` in later slides.

```
[Mon,Wed .. Sat]  ⇝  [Mon,Wed,Fri]
```

Type Synonyms and Copies

Type Synonyms

```
type String = [Char]
type Coordinate2D = (Float,Float)
```

Not a new type, just another (more informative) name.

Type Copying

```
type MyString = MString String
```

A new type (copy of the old). Class memberships may be independent from old type.

More Haskell Syntax

List comprehensions

Math: $\{x \in S \mid x \geq 1,\ x \text{ even}\}$

Haskell: `[ x | x <- S, x >= 1, isEven x ]`

General form: `[ exp | generators, guards]`

Examples:

`[ x+y | (x,y) <- [(1,2),(7,8)], y > 5 ]` $\leadsto$ `[15]`

`[ (i,j) | i<-[1,2,3,4], j<-[8,9], isEven i ]`
$\leadsto$ `[(2,8),(2,9),(4,8),(4,9)]`

`[ j^2 | i<-[[1,2],[10,20]], j<-i ]` $\leadsto$ `[1,4,100,400]`

`[ [j^2|j<-i] | i<-[[1,2],[10,20]] ]` $\leadsto$ `[[1,4],[100,400]]`

More Haskell Syntax

Lambda definitions

Nameless functions defined inline:

```
zipWith (\x y -> x^2 + y^2) [1,2,3] [2,3,4]
```

$\leadsto$ `[5,13,25]`

```
compose2 f g = \x y -> g (f x) (f y)
```

Enumeration expression

Easy generation of lists of certain types (types in the `Enum` class).

```
[3 .. 10]  ⇝  [3,4,5,6,7,8,9,10]
[3, 3.3 .. 4]  ⇝  [3.0,3.3,3.6,3.9]
['a', 'c' .. 'i']  ⇝  "acegi"
[False ..]  ⇝  [False,True]
```

More Haskell Syntax

Local definitions

`where`: (often used)

```
f x y
  | x  < 0 = -(sqx*sqy + sqx + sqy) + g y
  | x >= 0 =   sqx*sqy + sqx + sqy
    where
      sqx = x*x
      sqy = y*y
      g z = (max x z) + t
        where t = x*y*z
```

`let`: (rarely used)

```
f x = let y=x^3; z=log x in y*z + z^2
```

More Haskell Syntax

Choice

`case`: (rarely used)

```
isOdd x
  = case (x `mod` 2) of
    0 -> False
    1 -> True
```

or (not using “layout”):

```
isOdd x = case (x `mod` 2) of {0 -> False; 1 -> True}
```

`if then else`: (somewhat used, especially in textbook)

```
isOdd x = if (x `mod` 2)==0 then "Even" else "Odd"
```

Lists

A very useful type. Many powerful and generic functions in standard Prelude for working with lists, including (see Section 8.1 in the Haskell Report):

```
map, ++, filter, concat, concatMap, head, last,
tail, init, null, length, !!, foldl, foldl1,
scanl, scanl1, foldr, foldr1, scanr, scanr1,
iterate, repeat, replicate, cycle, take, drop,
splitAt, takeWhile, dropWhile, span, break,
lines, words, unlines, unwords, reverse, and,
or, any, all, elem, notElem, lookup, sum,
product, maximum, minimum, zip, zip3, zipWith,
zipWith3, unzip, unzip3
```

Textbook covers the most important of these in Chapter 4 (not necessarily with the same implementations). Even more functions can be found in the standard library List.