



## Mergesort

Del op  $X$  og  $Y$

$A = \text{Mergesort}(X)$

$B = \text{Mergesort}(Y)$

Merge  $(A, B)$

$T(n)$ : tiden brugt på at løse et problem af str.  $n$   
Eks: tiden brugt på at sortere  $n$  tal.

$$\begin{aligned}\text{Mergesort: } T(n) &= O(1) + 2 \cdot T\left(\frac{n}{2}\right) + O(n) \\ &= 2 \cdot T\left(\frac{n}{2}\right) + O(n)\end{aligned}$$

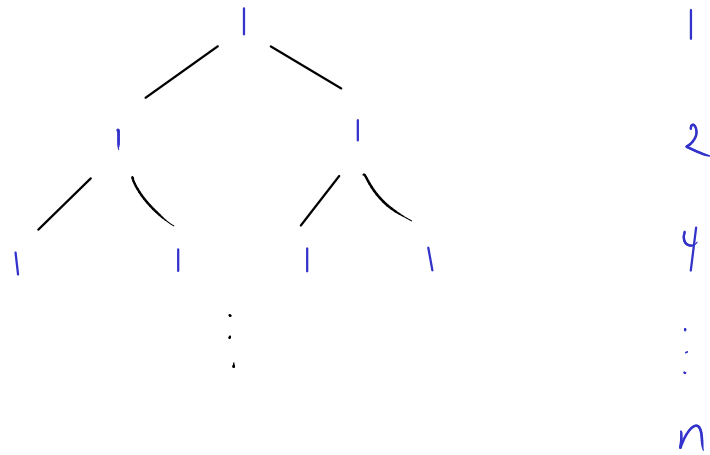
$$\text{Slide 18: } T(n) = 3T\left(\frac{n}{4}\right) + O(n^2)$$

$\approx \log_4 n$  niveauer

$$\begin{aligned}T(n) &= \sum_{i=0}^{\log_4 n} \left(\frac{3}{16}\right)^i cn^2 = cn^2 \sum_{i=0}^{\log_4 n} \left(\frac{3}{16}\right)^i \\ &< cn^2 \sum_{i=0}^{\infty} \left(\frac{3}{16}\right)^i \stackrel{(A.6)}{=} cn^2 \frac{1}{1-\frac{3}{16}} \\ &= \frac{16}{13} cn^2 \in \Theta(n^2)\end{aligned}$$

$\Rightarrow$  øverste lag er dominerende  
(d.v.s. afgør den asymp. køretid)

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + 1$$



$$T(n) = \sum_{i=0}^{\log_2 n} 2^i = 2^{\log_2 n + 1} - 1 = 2n - 1$$

$\Rightarrow$  nedvste by dominerer

## Master-sætningen (s. 94)

$$T(n) = aT\left(\frac{n}{b}\right) + f(n), \quad a \geq 1, \quad b > 1$$

1.  $f(n) \in O(n^{\log_b a - \epsilon}), \quad \epsilon > 0$

$$\Rightarrow T(n) \in \Theta(n^{\log_b a})$$

2.  $f(n) \in \Theta(n^{\log_b a})$

$$\Rightarrow T(n) \in \Theta(n^{\log_b a} \log n)$$

3.  $f(n) \in \Omega(n^{\log_b a + \epsilon}), \quad \epsilon > 0,$  og  
 $\exists c < 1, n_0 : \forall n \geq n_0 : f\left(\frac{n}{b}\right) \leq c \cdot f(n)$

$$\Rightarrow T(n) \in \Theta(f(n))$$

Eks:  $T(n) = 2T\left(\frac{n}{2}\right) + n$

$$a = b = 2$$

$$f(n) = n$$

$$\log_b a = \log_2 2 = 1$$

$$\Rightarrow n^{\log_b a} = n = f(n)$$

Tilfælde 2  
 $\Rightarrow T(n) \in \Theta(n \log n)$

Eks:  $T(n) = 2T(\frac{n}{2}) + n \log n$

$$a=b=2$$

$$f(n) = n \log n$$

$$\Rightarrow f(n) = n^{\log_2 2} \cdot \log n \in \omega(n^{\log_2 2}) \quad \text{og}$$

$$f(n) \in o(n^{\log_2 2 + \epsilon}), \quad \text{for alle } \epsilon > 0.$$

$$(\log n \in o(n^\epsilon) \quad \text{for alle } \epsilon > 0)$$

$\Rightarrow$  Master-sætningen kan ikke bruges

Eks:  $T(n) = 2T(\frac{n}{2}) + 1$

$$a=b=2$$

$$f(n) = 1$$

$$\Rightarrow f(n) \in O(n^{\log_2 2 - 1})$$

Tilfælde 1  
 $\Rightarrow T(n) \in \Theta(n)$

Eks:  $T(n) = 3T(\frac{n}{4}) + n^2$

$$a=3, \quad b=4$$

$$\log_b a = \log_4 3 < 1$$

$$f(n) = n^2$$

$$\Rightarrow f(n) \in \Omega(n^{\log_4 3 + 1}) \Rightarrow \text{Tilfælde 3?}$$

$$f(\frac{n}{b}) = f(\frac{n}{4}) \leq c \cdot f(n), \quad c < 1 \Leftrightarrow$$

$$\frac{n^2}{16} \leq c \cdot \frac{n^2}{4}, \quad c < 1$$

$$\text{OK: } \frac{n^2}{16} \leq \frac{1}{2} \cdot \frac{n^2}{4}$$

Tilfælde 3

$$\Rightarrow f(n) \in \Theta(n^2)$$