

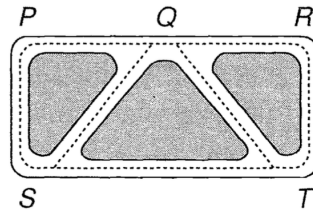


DM534 - Introduction to Computer Science

Exercises Week 45, Graph Theory

November 5, 2016

Exercise 1



Draw the graph representing the road system in the figure above, and write down the number of vertices, the number of edges and the degree of each vertex.

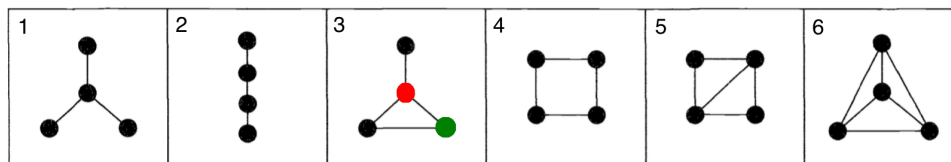
Exercise 2

On Twitter:

- John follows Joan, Jean and Jane; Joe follows Jane and Joan; Jean and Joan follow each other. Draw a digraph illustrating these follow-relationships between John, Joan, Jean, Jane and Joe.
- Twitter has ≈ 313 million active users (June 2016, based on Twitter Inc.). Imagine you would like to store the digraph for the follow-relationships in an adjacency matrix that uses 4 bytes per entry on your new laptop which has 64 GB of RAM. Is this feasible?
- The municipality of Odense has a population of ≈ 200000 people. Let G be the graph where the meaning of an edge from vertex i to j is “*person i is friends with person j* ”. Imagine you would like to store the adjacency matrix for this graph for the relationships in a matrix representation that uses 4 bytes per entry on your new laptop which has 64 GB of RAM. Is this feasible?

Exercise 3

Given are the following 6 graphs. (Note, that the nodes do not have labels).



- How many walks of length 3 from the red vertex to the green vertex are there in graph 3?
- How many paths from the red vertex to the green vertex are there in graph 3?
- How many shortest paths from the red vertex to the green vertex are there in graph 3?
- For each of the graphs: what is the longest of all pairwise shortest paths?
- Give an adjacency matrix for graph 1. Can there be different adjacency matrices for the same graph? If so, name a second adjacency matrix for graph 1. Can you find two different adjacency matrices for graph 6?

Exercise 4

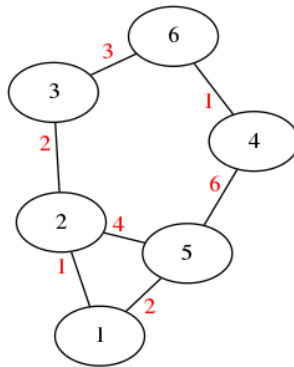
Let A be an adjacency matrix. In the lecture you learned that the ij -entry of A^k is the number of different walks from vertex i to vertex j using exactly k edges.

- What is the interpretation of the ij -entry of the matrix $A^1 + A^2 + A^3$?
- Complete the following sentence with the missing expression: In a graph G with adjacency matrix A , vertex i and j are connected if and only if $\dots > 0$.



Exercise 5

Given the following weighted graph (from the lecture slides, weights are depicted in red)



with the already computed distance matrix D

$$D = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} 0 & 1 & 3 & 7 & 2 & 6 \\ 1 & 0 & 2 & 6 & 3 & 5 \\ 3 & 2 & 0 & 4 & 5 & 3 \\ 7 & 6 & 4 & 0 & 6 & 1 \\ 2 & 3 & 5 & 6 & 0 & 7 \\ 6 & 5 & 3 & 1 & 7 & 0 \end{pmatrix} \end{matrix}$$

- How many shortest path in G are of length 6? Name them.
- How long is the longest of all pairwise shortest paths in the graph? Are there several longest shortest paths?
- How many paths in G are of length 6? (Note: a path does not necessarily need to be a shortest path.) Name them.

Exercise 6

Assume in this exercise that all weights on edges are non-negative values.

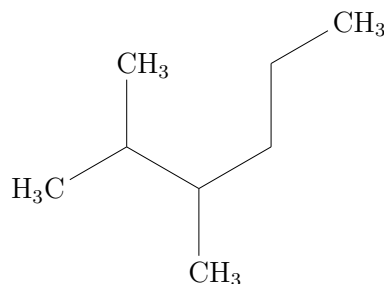
- In a graph G with $n = 6$ vertices, how many matrix-matrix multiplication operations are needed in the worst case in order to compute the distance matrix D , when the method of repeated squaring is used to compute D ?



- b) In a graph G with $n = 200$ vertices, how many matrix-matrix multiplications are needed in the worst case in order to compute the distance matrix D , when the method of repeated squaring is used to compute D ?
- c) What is the computational runtime (in O -notation, as a function of n) in order to compute the distance matrix D for a graph G with n vertices if the method of repeated squaring is used to compute D ?
- d) Can you find a graph G with $n = 6$ vertices, for which $W^4 \neq W^5$? If so, depict it.
- e) Can you find a graph G with $n = 6$ vertices, for which $W^5 \neq W^6$? If so, depict it.
- f) Can you find a graph G with $n = 6$ vertices, for which $W^1 = W^2$? If so, depict it.

Exercise 7

Given is the following molecule (it's called 2,3-Dimethylhexane, see <https://en.wikipedia.org/wiki/2,3-Dimethylhexane>):



- a) How many carbon atoms does this molecule have?
- b) Draw the graph G corresponding to the carbon backbone of the molecule.
- c) Give the edge weight matrix W for the graph G .
- d) Use your brain or the python program `shortestPaths.py` to infer the distance matrix (Hint: the graph is rather simple, you won't need a program for that.)
- e) What is the Wiener Index $\mathcal{W}(G)$?
- f) How many shortest paths of length 3 $i \rightarrow \dots \rightarrow j$ with $i < j$ are in G ?
- g) Using Wiener's method for predicting the boiling point, what is your prediction for 2,3-Dimethylhexane?