DM534
Introduction to Computer Science
Exercises on Satisfiability

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Exercise 1: Check Satisfiability

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

a) $A \wedge B$

b) $A \vee B$

c) $A \rightarrow B$

d) $A \wedge \neg A$

e) $A \vee \neg A$

f) $(A \rightarrow B) \wedge (B \rightarrow A)$

g) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge A$

h) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge \neg A$

i) $(A \rightarrow B) \wedge (B \rightarrow \neg A) \wedge (\neg A \rightarrow \neg B) \wedge (\neg B \rightarrow A)$

Exercise 2: Equivalent Formulas

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

a) $\neg A \wedge B$

b) $\neg A \vee B$

c) $A \rightarrow B$

d) $(A \rightarrow B) \wedge (\neg B \rightarrow A)$

e) $(\neg A \rightarrow B) \wedge (\neg B \rightarrow \neg A)$

Exercise 3: Convert to CNF

Convert the following formulas into CNF:

a) $\neg A \wedge B$

b) $\neg A \vee B$

c) $A \rightarrow B$

d) $(A \rightarrow B) \wedge (\neg B \rightarrow A)$

e) $(\neg A \rightarrow B) \wedge (\neg B \rightarrow \neg A)$

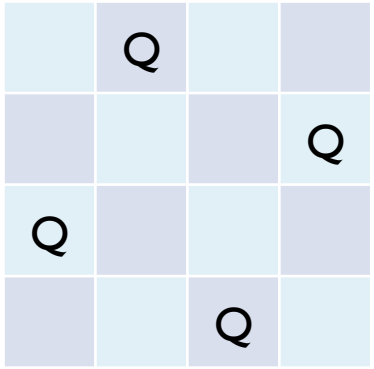
f) $A \rightarrow (\neg (B \wedge D))$

g) $A \rightarrow (\neg (B \vee D))$

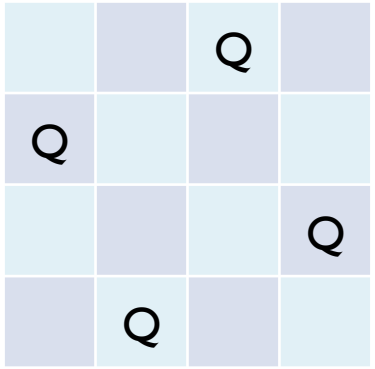
h) $A \rightarrow (\neg (B \rightarrow (C \wedge D)))$

Exercise 4: Breaking Symmetry

Solutions to N-Towers and N-Queens are symmetric:



and



a) Write two clauses that forbid solutions where there is a queen in the right half of the first row.

b) Instead of adding two clauses, change an existing clause.

$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$
$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$
$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$
$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$

Exercise 5: Preparation

- Install **lingeling** or another compatible SAT solver
- Alternatively, use a Javascript SAT solver, e.g.:
  - https://www.msoos.org/2013/09/minisat-in-your-browser/
- Test it using the following input saved as `test.cnf`

```
p cnf 4 6
-1 -2 0
-1 -3 0
-2 -4 0
-3 -4 0
1 2 0
3 4 0
```

Exercise 6: Removing Redundancies

The formula from Slide 11 contains redundant information. For example, $X_{1,1} \rightarrow \neg X_{1,2}$ and $X_{1,2} \rightarrow \neg X_{1,1}$ are equivalent.
Understand and remove these redundancies:

a) Why do these redundancies occur?
b) Identify all such redundancies!
c) Write down a simplified formula without redundancies!
d) Convert the simplified formula into CNF!
e) Write the formula in DIMACS format!
f) Run the lingeling solver on it and interpret the result!