

## Korrekthed

Til at bevise korrektheden af en iterativ (eller rekursiv) algoritme kan man bruge en løkke-invariant.

### Sequential Search ( $L, x$ )

$n := L.length$

$i := 1$

While  $*I*$   $i \leq n$  and  $L[i] \neq x$

$i++$

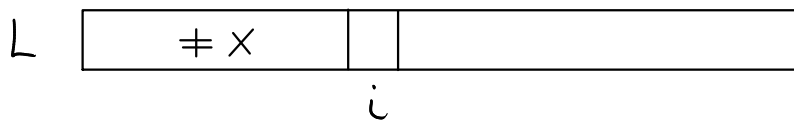
If  $i \leq n$

Return  $i$

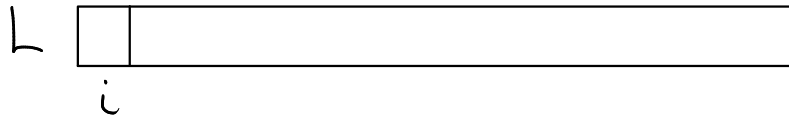
Else

Return "Not found"

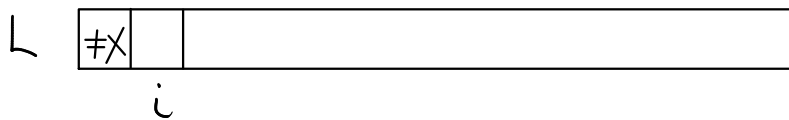
Løkke-invariant  $I$ :  $x \neq L[j]$ , for  $1 \leq j \leq i-1$



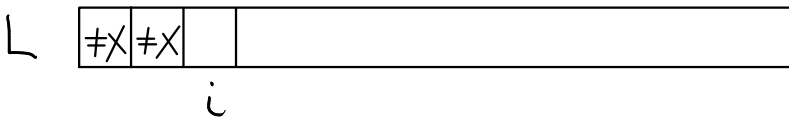
Første gang vi kommer til  $*I*$ :



Anden gang:



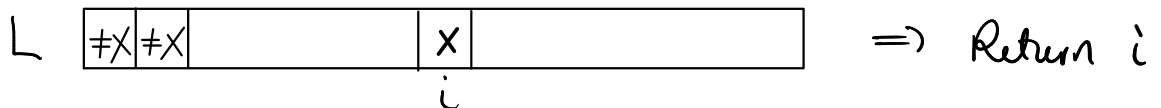
Tredje gang:



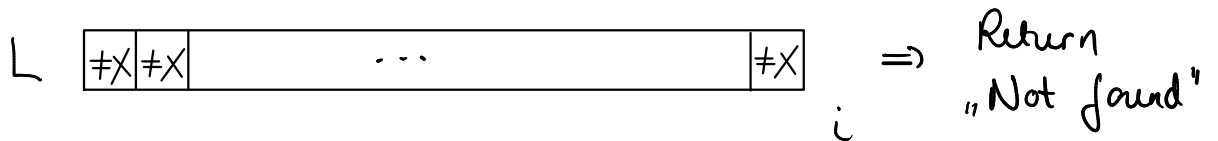
⋮

Sidste gang:

Enter



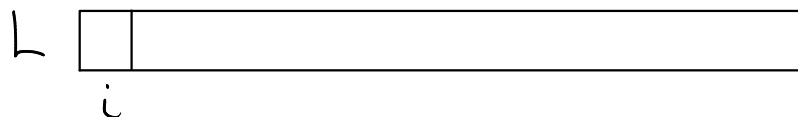
Eller



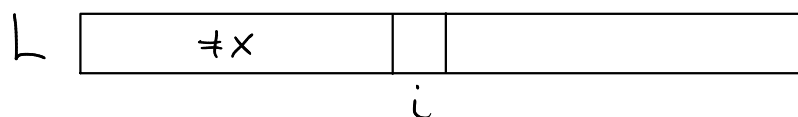
Bevis løkke-inv. v.h.a. induktion over #gennemløb:

Basis (initialization)

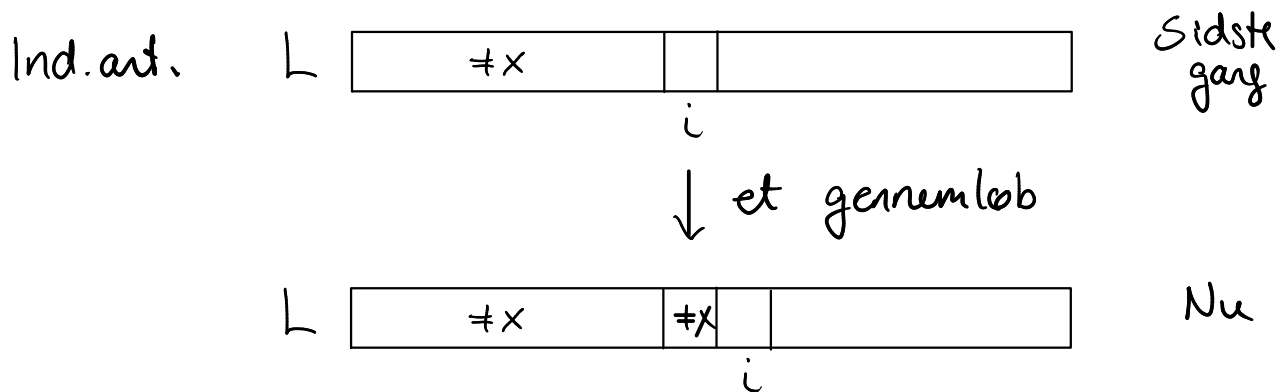
Første gang vi kommer til  $*I*$ :



Induktionsantagelse: Sidst vi var ved  $*I*$ ,  
var løkke-invarianten opfyldt:



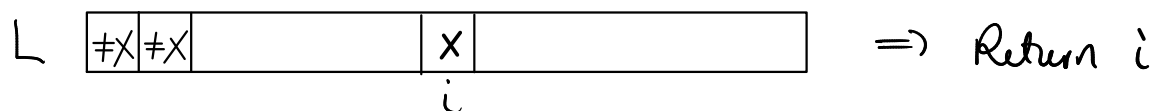
Induktionsstødt (maintenance)



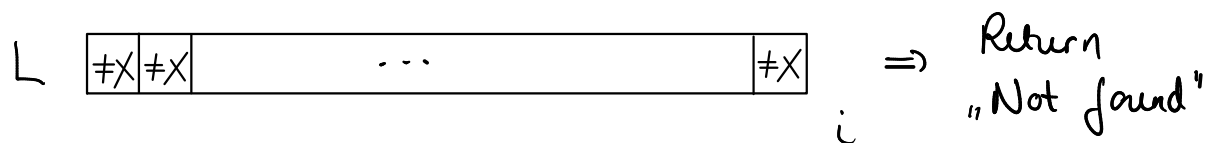
Anvend løkke-inv.:

Afslutning (termination):

Enten



Eller



Binary Search ( $L, x$ )

$l := 1, r := L.length, m := \lfloor \frac{l+r}{2} \rfloor$

While  $\ast I \ast$   $l \leq r$  and  $L[m] \neq x$

If  $x < L[m]$

$r := m - 1$

Else

$l := m + 1$

$m := \lfloor \frac{l+r}{2} \rfloor$

If  $l \leq r$

Return  $m$

Else

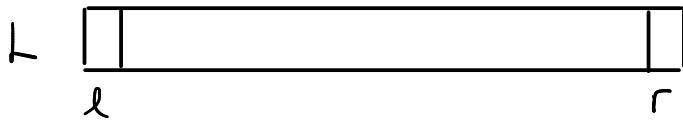
Return „Not found“

Invariant I:

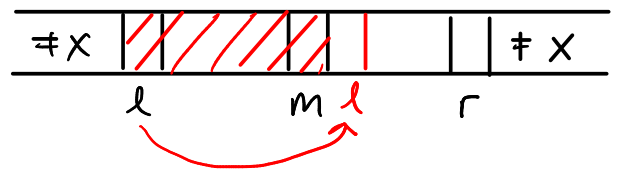
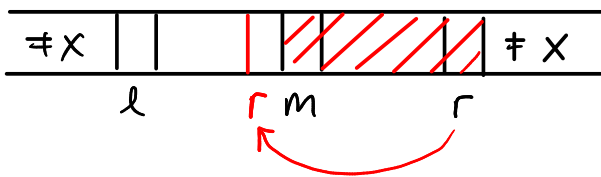
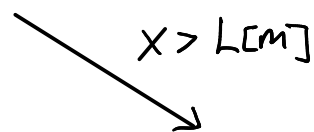
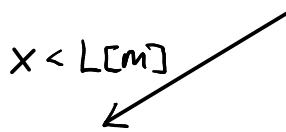
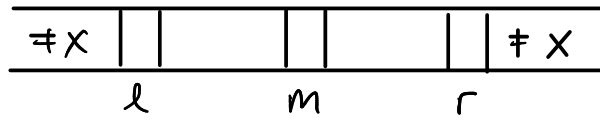
If  $x$  findes in  $L$ , findes den in  $L[l..r]$ :



Basis:

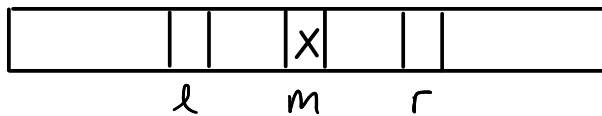


Ind. skridt



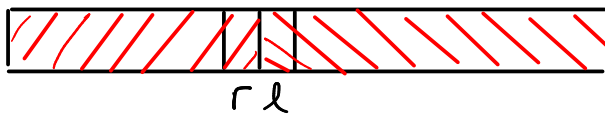
Afslutning:

Enten



⇒ Return m

Eller



⇒ Return „Not found“

Seq Search Rec ( $L, x, l, n$ )

If  $l \leq n$

If  $L[l] = x$

Return  $l$

Else

Return Seq Search Rec ( $L, x, l+1, n$ )

Else

Return "Not found"

Binary Search Rec ( $L, x, l, r$ )

If  $l \leq r$

$m := \lfloor \frac{l+r}{2} \rfloor$

If  $L[m] = x$

Return  $m$

Else if  $x < L[m]$

Return Binary Search Rec ( $L, x, l, m-1$ )

Else

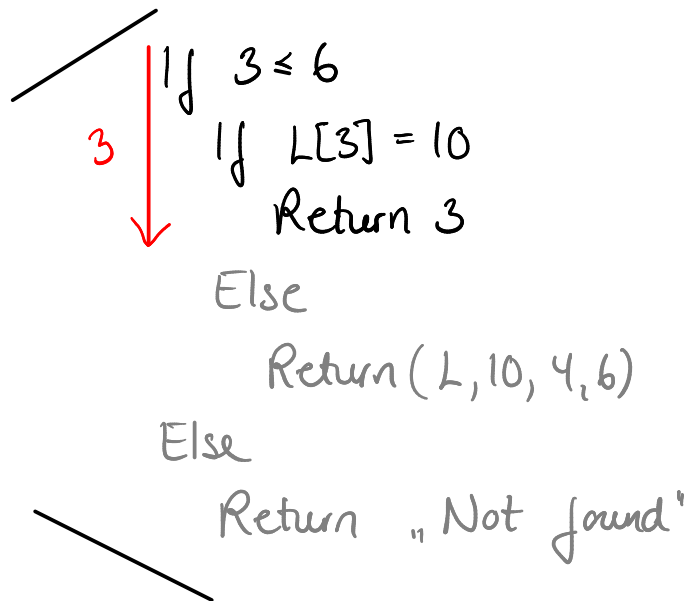
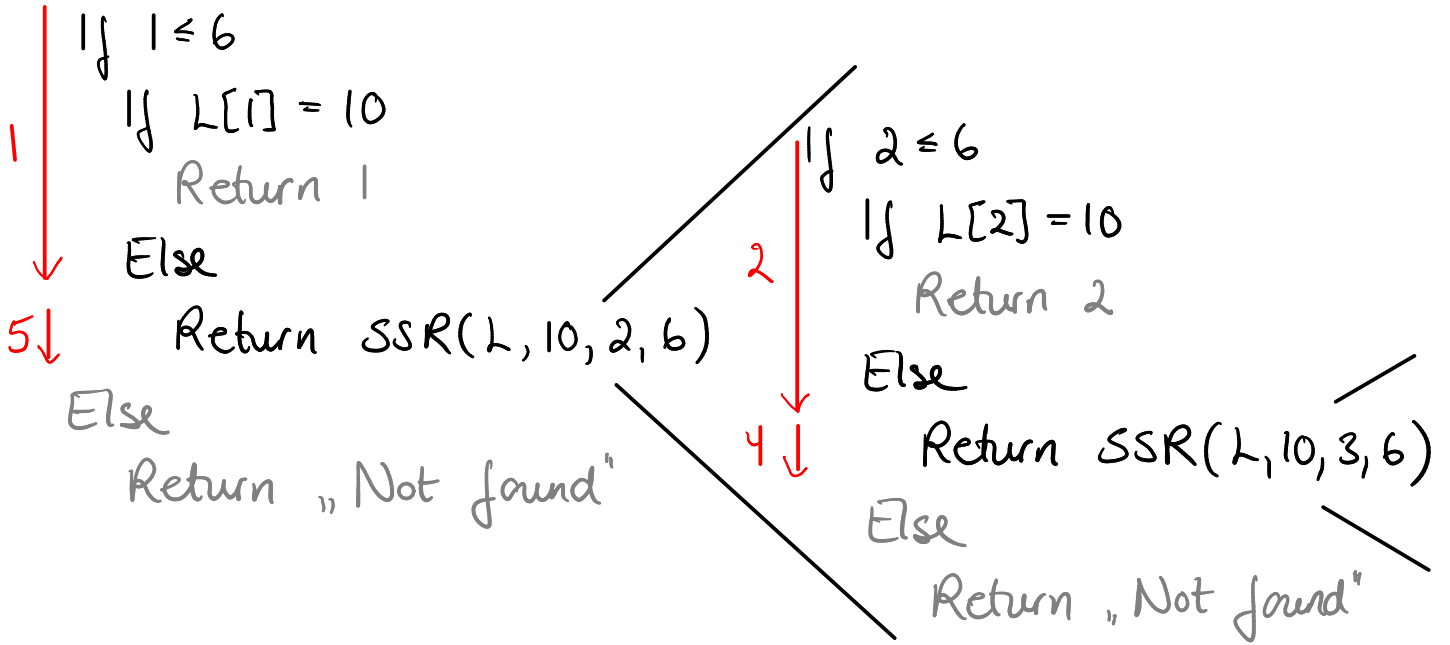
Return Binary Search Rec ( $L, x, m+1, r$ )

Else

Return "Not found"

Eks:  $L = [7, 9, 10, 3, 2, 8]$ ,  $x = 10$

SSR( $L, 10, 1, 6$ )



Eks: Merge Sort

