## DM534 - Øvelser Uge 44

## Introduktion til Datalogi, Efterår 2021

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## 1 I

Start med at repetere følgende definitioner fra slides:

- En Deterministic Finite Automaton (DFA).
- At en DFA accepterer en streng.
- Sproget bestemt af en DFA.
- En Context-Free Grammar (CFG).
- At en CFG udleder (derives) en streng.
- Sproget bestemt af en CFG.


## 1.1

What is the language of the following DFA?


SVAR: 0 og så nul eller flere 3'er.

## 1.2

What is the language of the following DFA?


SVAR: 1 mindst en gang, hvis der er et 2 , så skal det efterfølges af mindst et 1 .

## 1.3

Define a DFA that recognises the following language: All strings of 0s and 1s that contain an odd number of 1 s and any number of 0 s .

SVAR:


## 1.4

Define a DFA that recognises the following language: All strings of 0s and 1s that contain the string 010.

## SVAR:



## 1.5

Define a DFA that recognises the following language: All strings of 0s and 1s that contain at least two occurrences of 10 and an even number of 0 s.

## SVAR:



## 1.6

What is the language of the following CFG?

$$
\begin{gathered}
S \rightarrow a b \\
S \rightarrow S S
\end{gathered}
$$

SVAR: $(a b)^{+}$. At least one repeat of the string $a b$.

## 1.7

Write two different derivations for the string 0001111 with the following CFG. (Same end result, but some different intermediate steps.)

$$
\begin{gathered}
S \rightarrow 0 M 1 \\
M \rightarrow M 1 \\
M \rightarrow 0 M \\
M \rightarrow 0 \\
M \rightarrow 1
\end{gathered}
$$

## SVAR:

$$
\begin{aligned}
& S \rightarrow 0 M 1 \rightarrow 00 M 1 \rightarrow 000 M 1 \rightarrow 000 M 11 \rightarrow 000 M 111 \rightarrow 0001111 \\
& S \rightarrow 0 M 1 \rightarrow 0 M 11 \rightarrow 0 M 111 \rightarrow 0 M 1111 \rightarrow 00 M 1111 \rightarrow 0001111
\end{aligned}
$$

## 1.8

What is the language of the following CFG?

$$
\begin{gathered}
S \rightarrow 0 M M 1 \\
M \rightarrow 0 M \\
M \rightarrow 1 M \\
M \rightarrow 0 \\
M \rightarrow 1
\end{gathered}
$$

SVAR: Any $0-1$ string that starts with 0 and ends with 1 of at least length 4.

## 2 II

## 2.1

Define a CFG that recognizes the following language: All strings of 0 s and 1 s consisting of n 0s followed by n 1s. Examples: 0011 is OK, 1100 is not OK, 011 is not OK.

## SVAR:

$$
\begin{gathered}
S \rightarrow 0 S 1 \\
S \rightarrow 01
\end{gathered}
$$

## 2.2

Define a DFA that recognises the same language of this CFG:

$$
\begin{gathered}
S \rightarrow 0 M \\
S \rightarrow 1 \\
M \rightarrow 0 S \\
M \rightarrow 1 T \\
T \rightarrow 0 M \\
T \rightarrow 1 T
\end{gathered}
$$

SVAR:


## 2.3

Define a CFG that recognises the following language: All strings of arithmetic additions that contain numbers, the + sign, and (balanced) parentheses. Examples: $(0+1)$ is OK, $(2+(3))+4$ is $\mathrm{OK}, 2+3(2)$ is not OK .

## SVAR:

$$
\begin{gathered}
S \rightarrow E \\
E \rightarrow(E) \\
E \rightarrow E+E \\
E \rightarrow N \\
N \rightarrow N N \\
N \rightarrow 0|1| 2|3| 4|5| 6|7| 8 \mid 9
\end{gathered}
$$

