# DM573 Introduction to Computer Science Exercises on Satisfiability

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# PART I

# **Exercise I-1: Check Satisfiability**

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

Note: in all exercises, logical negation (NOT) is denoted by "-".

- a) A ^ B
- b) A v B
- c) A → B
- d) A ^ –A
- e) Av-A

# **Exercise I-2: Equivalent Formulas**

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- a) –A ∧ B
- b) –A v B
- c) A → B
- d)  $(A \rightarrow B) \land (-B \rightarrow A)$
- e)  $(-A \rightarrow B) \land (-B \rightarrow -A)$

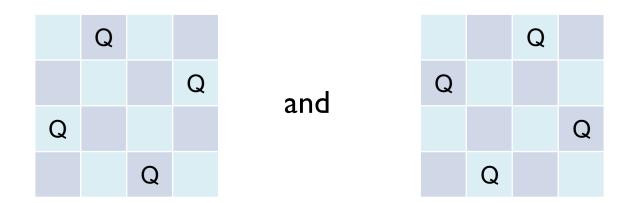
### **Exercise I-3: Convert to CNF**

Convert the following formulas into CNF:

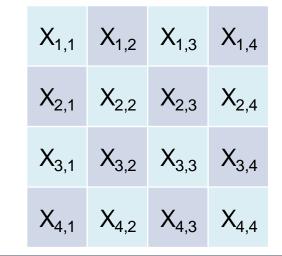
- a) –A ∧ B
- b) –A v B
- c) A → B
- d)  $(A \rightarrow B) \land (-B \rightarrow A)$

# **Exercise I-4: Breaking Symmetry**

Solutions to N-Towers and N-Queens are symmetric:



Write two clauses that forbid solutions where there is a queen in the right half of the first row.



### **Exercise I-5: Preparation**

- Install lingeling or another compatible SAT solver
- Alternatively, use a Javascript SAT solver, e.g.:
  - https://www.msoos.org/2013/09/minisat-in-your-browser/
- Test it using the following input saved as test.cnf
- p cnf 4 6
- -1 -2 0
- -1 -3 0
- -2 -4 0
- -3 -4 0
- 1 2 0
- 3 4 0

## **Exercise I-6: Removing Redundancies**

The formula from Slide 22 contains redundant information. For example,  $X_{1,1} \rightarrow -X_{1,2}$  and  $X_{1,2} \rightarrow -X_{1,1}$  are equivalent. Understand and remove these redundancies:

- a) Why do these redundancies occur?
- b) Identify all such redundancies!
- c) Write down a simplified formula without redundancies!
- d) Convert the simplified formula into CNF!
- e) Write the formula in DIMACS format!
- f) Run the lingeling or another SAT solver on it and interpret the result!

# PART II

## **Exercise II-1: Check Satisfiability**

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

- a)  $(A \rightarrow B) \land (B \rightarrow A)$
- b)  $(A \rightarrow B) \land (B \rightarrow A) \land A$
- c)  $(A \rightarrow B) \land (B \rightarrow A) \land -A$
- d)  $(A \rightarrow B) \land (B \rightarrow -A) \land (-A \rightarrow -B) \land (-B \rightarrow A)$

# Exercise II-2: Equivalent Formulas

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- a)  $(A \rightarrow B) \land (-B \rightarrow A)$
- b)  $(A \rightarrow -B) \land (B \rightarrow A)$
- c)  $(-A \vee -B) \wedge (A \vee -B)$
- d)  $(B \lor A) \land (-A \lor B)$

#### **Exercise II-3: Convert to CNF**

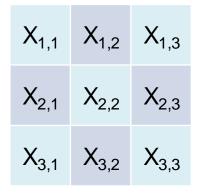
Convert the following formulas into CNF:

- a)  $(-A \rightarrow B) \land (-B \rightarrow -A)$
- b) A → (– (B ∧ D))
- c) A → (– (B ∨ D))
- d)  $A \rightarrow (-(B \rightarrow (C \land D)))$

### **Exercise II-4: 3-Towers**

Write a Python program that generates the input for a SAT solver to solve the 3-Towers problem:

- a) Write a function pair2int(r,c) which maps (1,1), (1,2), ..., (3,3) to 1 to 9 using the formula 3\*(r-1)+C.
- b) Write nested for-loops that go through all positions on the board from (1,1) to (3,3) and produces clauses that represent attacks.
- c) Write a for-loop that produces clauses that specify that all 3 rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.

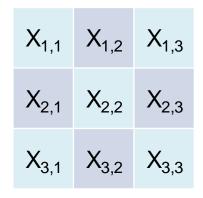


1	2	3
4	5	6
7	8	9

# **Exercise II-5: N-Towers**

Generalize your Python program from Exercise II-4 to generate the input for a SAT solver to solve the N-Towers problem:

- a) Write a function pair2int (n,r,c)
   which maps pairs (r,C) to the integers 1 to n<sup>2</sup>
   using the formula n\*(r-1)+C.
- b) Write nested for-loops that go through all positions on the board from (1,1) to (n,n) and produces clauses that represent attacks.
- c) Write a for-loop that produces clauses that specify that all rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.

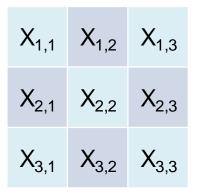


1	2	3
4	5	6
7	8	9

# **Exercise II-6: N-Queens**

Extend your Python program from Exercise II-5 to generate the input for a SAT solver to solve the N-Queens problem:

- a) Reuse your function pair2int(n,r,c) from Exercise II-5.
- b) Adapt your for-loops from Exercise II-5 to produce also clauses for the diagonals.
- c) Reuse the for-loop from Exercise II-5 that produces clauses that specify that all rows contain a tower.
- d) Using (a)–(c), write a DIMACS file and test it using lingeling or another SAT solver.



1	2	3
4	5	6
7	8	9