

# DM534 — Øvelser Uge 46

## Introduktion til Datalogi, Efterår 2021

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### 1 I

#### 1.1

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

- (a)  $A \wedge B$ . **SVAR:** Satisfiable,  $A=t$   $B=t$ .
- (b)  $A \vee B$ . **SVAR:** Satisfiable,  $A=t$   $B=t$ .
- (c)  $A \Rightarrow B$ . **SVAR:** Satisfiable,  $A=t$   $B=t$ .
- (d)  $A \wedge \neg A$ . **SVAR:** Unsatisfiable, as any assignment has either  $A=t$  or  $A=f$ , and the expression is false in either case.
- (e)  $A \vee \neg A$ . **SVAR:** Satisfiable,  $A=t$ .

#### 1.2

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- (a)  $\neg A \wedge B$ .
- (b)  $\neg A \vee B$ .
- (c)  $A \Rightarrow B$ .
- (d)  $(A \Rightarrow B) \wedge (\neg B \Rightarrow A)$ .
- (e)  $(\neg A \Rightarrow B) \wedge (\neg B \Rightarrow \neg A)$ .

**SVAR:** (b) is equivalent to (c), (d) is equivalent to (e).

#### 1.3

Convert the following formulas into CNF:

- (a)  $\neg A \wedge B$ . **SVAR:**  $(\neg A) \wedge (B)$ .
- (b)  $\neg A \vee B$ . **SVAR:**  $(\neg A \vee B)$ .
- (c)  $A \Rightarrow B$ . **SVAR:**  $(\neg A \vee B)$ .
- (d)  $(A \Rightarrow B) \wedge (\neg B \Rightarrow A)$ . **SVAR:**  $(\neg A \vee B) \wedge (B \vee A)$ .

## 1.4

- Write two clauses that forbid solutions where there is a queen in the right half of the first row. **SVAR:**  $(\neg X_{1,1}) \wedge (\neg X_{1,2})$
- Instead of adding two clauses, change an existing clause. **SVAR:** Change  $(X_{1,1} \vee X_{1,2} \vee X_{1,3} \vee X_{1,4})$  to  $(X_{1,3} \vee X_{1,4})$ .

## 1.5

Run SAT solver.

## 1.6

The formula from Slide 11 contains redundant information. For example,  $X_{1,1} \Rightarrow \neg X_{1,2}$  and  $X_{1,2} \Rightarrow \neg X_{1,1}$  are equivalent. Understand and remove these redundancies:

- Why do these redundancies occur?
- Identify all such redundancies!
- Write down a simplified formula without redundancies!
- Convert the simplified formula into CNF!
- Write the formula in DIMACS format!
- Run the lingeling solver on it and interpret the result!

**SVAR a:** A tower in spot x might exclude a tower in spot y, and the same is true reverse, but both of these statements are equivalent to stating, that a tower can't be in both x and y.

**SVAR b:**

- $X_{1,1} \Rightarrow \neg X_{1,2}$  og  $X_{1,2} \Rightarrow \neg X_{1,1}$ .
- $X_{1,1} \Rightarrow \neg X_{2,1}$  og  $X_{2,1} \Rightarrow \neg X_{1,1}$ .
- $X_{1,2} \Rightarrow \neg X_{2,2}$  og  $X_{2,2} \Rightarrow \neg X_{1,2}$ .
- $X_{2,1} \Rightarrow \neg X_{2,2}$  og  $X_{2,2} \Rightarrow \neg X_{2,1}$ .

**SVAR c:**

$$(X_{1,1} \Rightarrow \neg X_{1,2}) \wedge (X_{1,1} \Rightarrow \neg X_{2,1}) \wedge (X_{1,2} \Rightarrow \neg X_{2,2}) \wedge (X_{2,1} \Rightarrow \neg X_{2,2}) \\ \wedge (X_{1,1} \vee X_{1,2}) \wedge (X_{2,1} \vee X_{2,2})$$

**SVAR d:**

$$(\neg X_{1,1} \vee \neg X_{1,2}) \wedge (\neg X_{1,1} \vee \neg X_{2,1}) \wedge (\neg X_{1,2} \vee \neg X_{2,2}) \wedge (\neg X_{2,1} \vee \neg X_{2,2}) \\ \wedge (X_{1,1} \vee X_{1,2}) \wedge (X_{2,1} \vee X_{2,2})$$

**SVAR e:** p cnf 4 6 -1 -2 0 -1 -3 0 -2 -4 0 -3 -4 0 1 2 0 3 4 0

**SVAR f:** Did in 1.5. Got 1, -2, -3, 4. This translates to a tower on  $X_{1,1}$  and  $X_{2,2}$ .

## 2 II

### 2.1

Which of the following formulas are satisfiable (give a satisfying assignment)? Which are not (give reasons)?

- (a)  $(A \Rightarrow B) \wedge (B \Rightarrow A)$ . **SVAR:** Satisfiable,  $A=t$   $B=t$ .
- (b)  $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge A$ . **SVAR:** Satisfiable,  $A=t$   $B=t$ .
- (c)  $(A \Rightarrow B) \wedge (B \Rightarrow A) \wedge \neg A$ . **SVAR:** Satisfiable,  $A=f$   $B=f$ .
- (d)  $(A \Rightarrow B) \wedge (B \Rightarrow \neg A) \wedge (\neg A \Rightarrow \neg B) \wedge (\neg B \Rightarrow A)$ . **SVAR:** Unsatisfiable. Argument: The first part  $(A \Rightarrow B)$  must be true, which excludes the assignment  $A=t$   $B=f$ . Similarly, the next three parts exclude the assignments  $A=t$   $B=t$ ,  $A=f$   $B=t$ , and  $A=f$   $B=f$ , respectively. Hence, no assignment of  $A$  og  $B$  can make the formula true.

### 2.2

Two formulas are equivalent, if the same assignments satisfy both of them.

Which of the following formulas are equivalent?

- (a)  $(A \Rightarrow B) \wedge (\neg B \Rightarrow A)$ .
- (b)  $(A \Rightarrow \neg B) \wedge (B \Rightarrow A)$ .
- (c)  $(\neg A \vee \neg B) \wedge (A \vee \neg B)$ .
- (d)  $(B \vee A) \wedge (\neg A \vee B)$ .

**SVAR:** (a) is equivalent to (d) and (b) is equivalent to (c).

### 2.3

Convert the following formulas into CNF:

- (a)  $(\neg A \Rightarrow B) \wedge (\neg B \Rightarrow \neg A)$ . **SVAR:**  $(A \vee B) \wedge (B \vee \neg A)$ .
- (b)  $A \Rightarrow (\neg(B \wedge D))$ . **SVAR:**  $(\neg A \vee \neg B \vee \neg D)$
- (c)  $A \Rightarrow (\neg(B \vee D))$ . **SVAR:**  $(\neg A \vee \neg B) \wedge (\neg A \vee \neg D)$
- (d)  $A \Rightarrow (\neg(B \Rightarrow (C \wedge D)))$ . **SVAR:**  $(\neg A \vee B) \wedge (\neg A \vee \neg C \vee \neg D)$ .

### 2.4

Write a Java program that generates the input for a SAT solver to solve the 3-Towers problem:

- (a) Write a method `pair2int(int r, int c)`. Should map  $(1,1), (1,2), \dots, (3,3)$  to 1 to 9 using the formula  $3 \cdot (r - 1) + c$ .
- (b) Write nested for-loops that go through all positions on the board from  $(1,1)$  to  $(3,3)$  and produces clauses that represent attacks.
- (c) Write a for-loop that produces clauses that specify that all 3 rows contain a tower.

- (d) Using (a)–(c), write a DIMACS file and test it using `lingeling`.

## 2.5

Generalize your Java program from Exercise II-4 to generate the input for a SAT solver to solve the N-Towers problem:

- (a) The method `pair2int(int n,int r,int c)` should map pairs  $(r,c)$  to the integers 1 to  $n^2$  using the formula  $n \cdot (r - 1) + c$ .
- (b) Write nested for-loops that go through all positions on the board from  $(1,1)$  to  $(n,n)$  and produces clauses that represent attacks.
- (c) Write a for-loop that produces clauses that specify that all rows contain a tower.
- (d) Using (a)–(c), write a DIMACS file and test it using `lingeling`.

## 2.6

Extend your Java program from Exercise II-5 to generate the input for a SAT solver to solve the N-Queens problem:

- (a) Reuse your function `pair2int(n,r,c)` from Exercise II-5.
- (b) Adapt your for-loops from Exercise II-5 to produce also clauses for the diagonals.
- (c) Reuse the for-loop from Exercise II-5 that produces clauses that specify that all rows contain a tower.
- (d) Using (a)–(c), write a DIMACS file and test it using `lingeling`.