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Exercise 1

Consider the sequence $s = (a_0, a_1, \ldots, a_{n-1})$, where n is a power of 2. In the following cases, prove that the sequences s_2 and s_2 obtained by performing the bitonic split operation described in Section 9.2.1, on the sequence s, satisfy the properties that (1) s_1 and s_2 are bitonic sequences, and (2) the elements of s_1 are smaller than the elements of s_2 for the three following cases

- a) s is a bitonic sequence such that $a_0 \leq a_1 \leq \ldots \leq a_{n/2-1}$ and $a_{n/2} \geq a_{n/2+1} \geq \ldots \geq a_{n-1}$
- b) s is a bitonic sequence such that $a_0 \le a_1 \le \ldots \le a_i$ and $a_{i+1} \ge a_{i+1} \ge \ldots \ge a_{n-1}$ for some $0 \le i \le n-1$
- c) s is a bitonic sequence that becomes increasing-decreasing after shifting its elements.

Exercise 2

Assume the following hypothetical overhead function for an algorithm (as usual W denotes the problem size).

$$T_O = p^2 \cdot \sqrt{W} + p \cdot \sqrt{W}$$

Assume that maximal degree of concurrency of the algorithm is $2^{\sqrt{W}}$.

- a) Determine the parallel computation time T_P (as a function in W and p).
- b) Determine the isoefficiency function due to the overheads T_O .
- c) Determine the isoefficiency function due to the maximal concurrency.
- d1) Determine the number of processes p', for which the parallel runtime is minimal.
- d2) Determine the runtime when using p' processes (cmp. d1).
- d3) Determine the asymptotic efficiency (as a function in W) when using p' processes. What is the efficiency for arbitrary large problem sizes W when using p' processes?

Bitonic Sort

Scalability