DM840 Algorithms in Cheminformatics

Daniel Merkle

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Subject overview

- Representation of Molecular Structures
- Combinatorial Structures (Counting, Generating Functions, ...)
- Structure Representation (Canonicalization Algorithms)
 Structure Invariant (e.g. Topological Indices)
- Graph Grammars ("Formal Languages" for Graphs)
- Synthesis Planning (e.g., Shortest Paths in Hypergraphs)
- Enzymatic Design (Discrete Optimization, ILP)
- Concurrency Theory and Causality (e.g. Petri Nets, Category Theory)
- Artificial Chemistries (e.g. "Lattices")
- Quantitative Structure Activity Relationship

Principal Component Analysis Algorithms for Minimum Cycle Basis

- Organization Theory
- Stoichiometric Models
- Metabolic Networks and Metabolic Pathways
- (Flux Balance Analysis)
- . . .

Representation of Molecular Structures



Figure: From the peyote cactus (Lophophora williamsii)

Representation of Molecular Structures

$$\begin{array}{c} NH_2 \\ NH$$

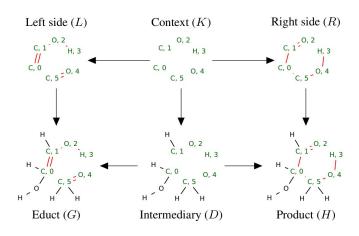
Figure: L-Glutathione oxidized

 ${\tt https://www.sigmaaldrich.com/catalog/product/sigma/g4376?lang=en}$

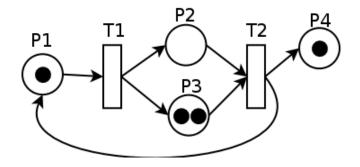
Representation of Molecular Structures

Figure: Thalidomide enantiomers (Contergan)

Graph Grammars



Petri Nets



Metabolic Networks and Metabolic Pathways

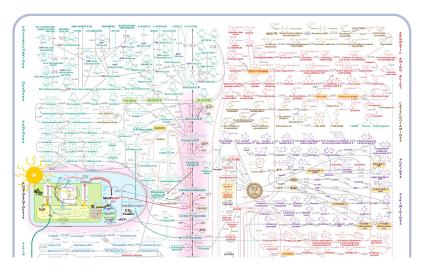
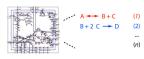


Figure: Typical metabolic network of a cell; (click here for the pdf)

Flux Balance Analysis

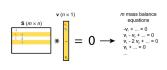
a Curate metabolic reactions



b Formulate **S** matrix



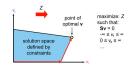
c Apply mass balance constraints



d Define objective function Z



e Optimize Z using linear progamming

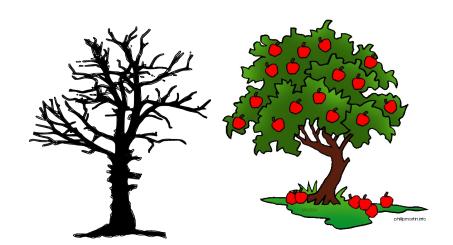


Motivation

Why is the sky blue?



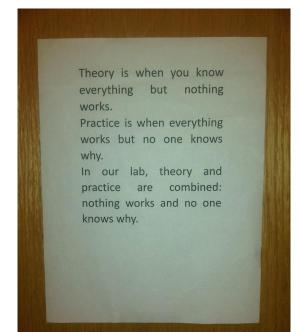
The research situation



Money



Theory - Practice



The End

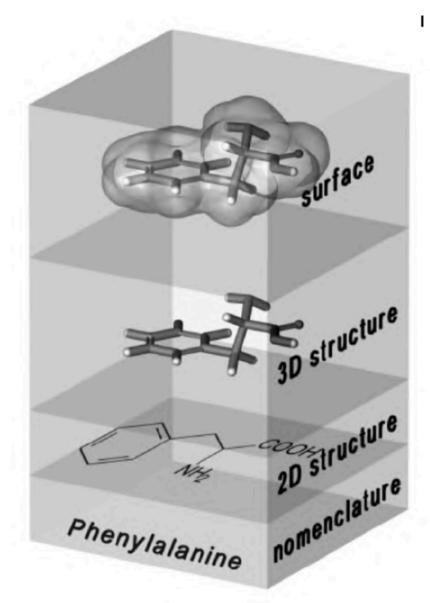
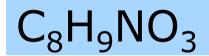


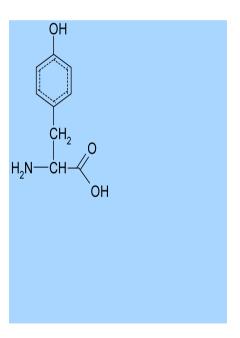
Figure 2-1. Hierarchical scheme for representations of a molecule with different contents of structural information.

- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms



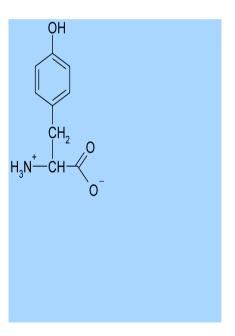
- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms

- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types (aromatic ring identification)
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms

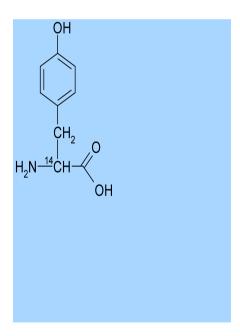


- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms

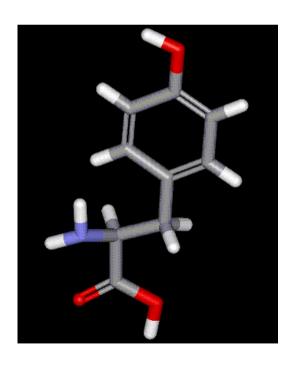
- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms



- How much information do you want to include?
 - atoms present
 - connections between atoms
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- How much information do you want to include?
 - atoms present
 - connections between atoms
 - bond types
 - stereochemical configuration
 - charges
 - isotopes
 - 3D-coordinates for atoms



Store Substance as Graphics or by Name

- Pederin
- **2H-Pyran-2-glycolamide**, N-((6-(2,3-dimethoxypropyl) tetrahydro-4-hydroxy-5,5-dimethyl-2H-pyran-2-yl)methoxymethyl) tetrahydro-2-methoxy-5,6-dimethyl-4- methylene-
- D-manno-Nonitol, 2,6-anhydro-3,5,7-trideoxy-1-C-(((2S)-hydroxy((2R,5R,6R)-tetrahydro-2-methoxy-5,6-dimethyl-4-methylene-2H-pyran-2-yl)acetyl)amino)-5,5-dimethyl-1,8,9-tri-O-methyl-,(1S)-

Elements of Formal Grammars

- **1 Terminal Symbols** *T* (represented by lowercase letters).
- **2 Nonterminals Symbols** *N* (represented by uppercase letters).
- **3 Production Rules** with a left- and a right-hand side consisting of strings of these symbols.
- 4 Start Symbol (also called Axiom)

The example grammar defines the language of all strings of the form $\{ax_1x_2...x_kb \mid k \geq 0 \land x_i \in \{a,b\}\}$. (A is the Axiom).

members: ab, abab, aaabbb non-members: a, b, ababa

Chomsky Hierarchy

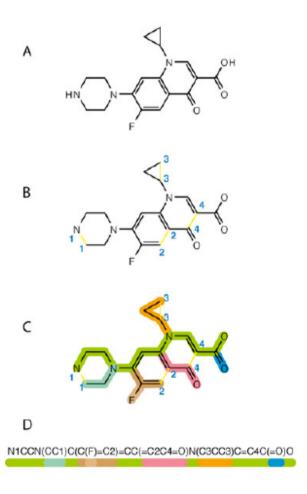
Type No	Type Name	Rule Pattern	
0	unrestricted	$x \rightarrow y$,	$x \in (N \cup T)_+$
			$y \in (N \cup T)*$
1	context	$x \rightarrow y$,	$x \in (N \cup T)_+$
	sensitive		$y \in (N \cup T)_+$
			$ x \leq y $
2	context free	$x \rightarrow y$,	$x \in N$
			$y \in (N \cup T)*$
3	regular	$w \rightarrow x \mid yz$	$w, x, z \in N$
			$y \in T$

BNF grammar of Daylight's SMILES

```
smiles
                 ::= chain terminator
chain
                 ::= branched_atom | chain branched_atom
                  | chain bond branched atom | chain '.' branched atom
branched atom
                ::= atom ringbond* branch*
                 ::= bracket_atom | aliphatic_organic | aromatic_organic
atom
                    ,*,
ringbond ::= bond? DIGIT | bond? '\%' DIGIT DIGIT
           ::= '(' chain ')' | '(' bond chain ')' | '(' '.' chain ')'
branch
bracket_atom ::= '[' isotope? symbol chiral? hcount? charge? class? ']'
              ::= NUMBER
isotope
symbol ::= element_symbols | aromatic_symbols | '*'
chiral
             ::= '0' | '00'
hcount ::= 'H' DIGIT?
charge ::= '-' DIGIT? | '+' DIGIT?
class
               ::= ':' NUMBER
aliphatic_organic ::= 'B' | 'C' | 'N' | '0' | 'S' | 'P' | 'F' | 'Cl' | 'Br' | 'I'
aromatic_organic ::= 'b' | 'c' | 'n' | 'o' | 's' | 'p'
element_symbols ::= 'H' | 'He' | 'Li' | 'Be' | 'B' | 'C' | 'N' | 'O' | etc
aromatic_symbols ::= 'c' | 'n' | 'o' | 'p' | 's' | 'se' | 'as'
             ::= '-' | '=' | '#' | '\$' | ':' | '/' | '\'
bond
terminator ::= SPACE | TAB | '\n' | '\0'
```

Generation of SMILES

SMILES (Simplified Molecular Input Line Entry System)



Six basic rules:

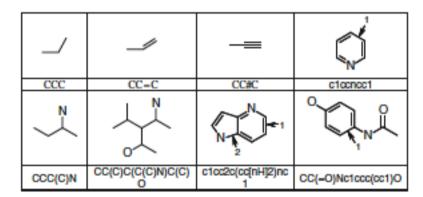
- 1) Atoms by atomic symbol
- Hydrogen atoms added to saturate free valences
- 3) Neighboring atoms stand next to one another
- Double and triple bonds by = and #
- 5) Branching shown by parentheses
- 6) Rings shown by digit at ring closures

Canonical SMILES: unique for each structure

Isomeric SMILES: describe isotopism, configuration around double bonds and tetrahedral centers, chirality

SMILES

Illustrative SMILES: molecular structures and the corresponding SMILES strings are paired vertically. The numbered arrows on the three cyclic molecular structures are not part of the molecules. They are used to indicate the break points for deriving the corresponding SMILES strings (see text)



J.Z. Zhou (ed.), Chemical Library Design, Methods in Molecular Biology 685, 2011

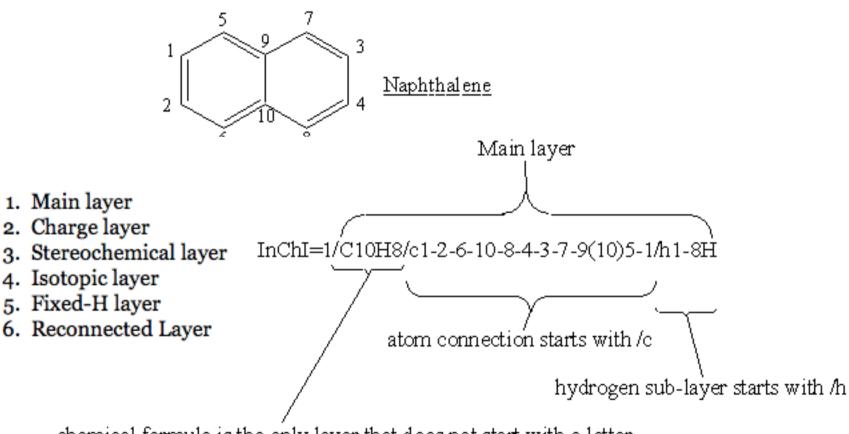
SLN (SYBYL Line Notation)

Table 2-3. Basic SLN syntax without description of attributes and macro atoms.

SLN	Chemical structure	Compound name
Atoms: Atoms are represented by their letter symbols the second letter is low		
CH4	CH ₄	methane
NH2	$-NH_2$	amine
Bonds: Single bonds are omitted; doub " = ", " # " and ": ", respectively. In c but a property of bonds. A period ind	ontrast to SMILES, aromatic	city is not an atomic property,
HC(=O)OH	НСООН	formic acid
Na.OH	NaOH	sodium hydroxide
Branches: Branches are indicated by p	arentheses.	
CH3C(=O)OH	ОН	acetic acid
	сн₃—с′	
Cyclic structures: Ring closures are des specified by a unique ID number. The the atom. An " @ " indicates a ring of	ID is a positive integer plac	
C[15]H2CH2CH2CH2CH2@15	\bigcirc	cyclohexane
O[6]:CH:CH:CH:@6	\bigcirc	furan

InChI (IUPAC International Chemical Identifier)

InChI=1/C10H8/c1-2-6-10-8-4-3-7-9(10)5-1/h1-8H



chemical formula is the only layer that does not start with a letter

Adjacency and Distance matrices

Acetaldehyde: CH3CH=O

Adjacency Matrix Distance Matrix	
C1	H ₄ C ₁ C ₆ H ₆

1: atoms i j are bonded

0: atoms i j are not bonded

Length of shortest path between atoms i j

Bond matrix

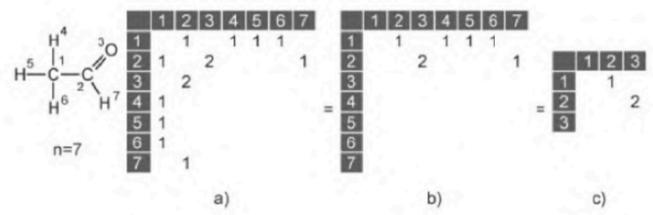


Figure 2-17. a) The redundant bond matrix of ethanal with the zero values omitted. b) It can be compressed by reduction to the top right triangle. c) Omitting the hydrogen atoms provides the simplest non-redundant matrix representation.

Connection Table

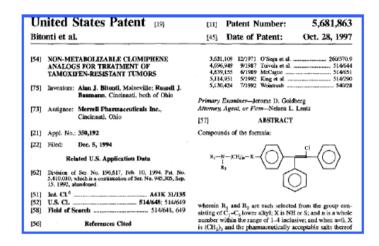
Bond list				
1 st atom	2 nd atom	bond order		
1	2	1		
2	3	2		
2	7	1		
1	4	1		
1	5	1		
1	6	1		

Figure 2-20. A connection table: the structure diagram of ethanal, with the atoms arbitrarily labeled, is defined by a list of atoms and a list of bonds.

Н

Special notations of chemical structures

Markush structures



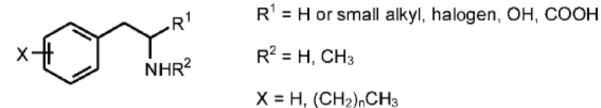


Figure 2-62. The substituted phenyl derivative is an example of a typical Markush structure. Herein, a number of compounds are described in one structure diagram by fill-ins. Phenylalanine is one of these structures when R^1 is COOH, R^2 is H, and X is H.

Special notations of chemical structures

• Fingerprints

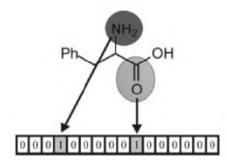


Figure 2-64. How an excerpt from a binary code could appear, if only $-NH_2$ and C=O are available in the fragment library.

MACCS fingerprints: 166 structural keys that answer questions of the type:

- Is there a ring of size 4?
- Is at least one F, Br, Cl, or I present?

where the answer is either TRUE (1) or FALSE (0)

SMILES .smi file

```
N12CCC36C1CC(C(C2)=CCOC4CC5=0)C4C3N5c7ccccc76 Strychnine
c1cccc1C(=0)OC2CC(N3C)CCC3C2C(=0)OC cocaine
COc1cc2c(ccnc2cc1)C(0)C4CC(CC3)C(C=C)CN34 quinine
OC(=0)C1CN(C)C2CC3=CCNc(ccc4)c3c4C2=C1 lyseric acid
CCN(CC)C(=0)C1CN(C)C2CC3=CNc(ccc4)c3c4C2=C1 LSD
C123C5C(0)C=CC2C(N(C)CC1)Cc(ccc40)c3c4O5 morphine
C123C5C(OC(=0)C)C=CC2C(N(C)CC1)Cc(ccc4OC(=0)C)c3c4O5 heroin
c1ncccc1C1CCCN1C nicotine
CN1C(=0)N(C)C(=0)C(N(C)C=N2)=C12 caffeine
C1C(C)=C(C=CC(C)=CC=CC(C)=CCO)C(C)(C)(C)(C) vitamin a
```

MOLfile

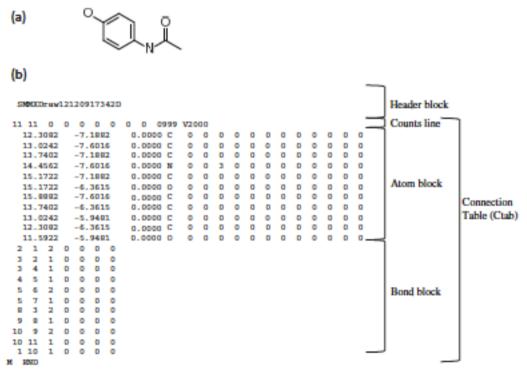


Fig. 2.1. Illustrative example of a MOLFile for acetaminophen (also known as paracetamol). (a) Molecular structure of acetaminophen, commonly known as Tylenol. Tylenol is a widely used medicine for reducing fever and pain. (b) MOLFile for acetaminophen.

Aspirin in SDF Format

```
2244
  -OEChem-03090904423D
 21 21 0
               0 0
    1.8152
             -0.9382
    5.1920
              -2.1043
    3.9623
              -2.6855
    2.9441
              1.1113
    1.8509
              -0.9767
    2.9180
              -1.5734
    0.8008
              -0.4105
    2.9348
              -1.6038
    0.8176
              -0.4410
    1.8846
              -1.0376
    4.0236
              -2.1714
    2.4171
               0.2017
    2.3265
               0.1503
   -0.0345
               0.0550
    3.7445
              -2.0729
   -0.0005
              -0.0011
    1.8958
              -1.0640
    1.2777
               0.1264
    2.7936
               1.0443
    2.8566
              -0.7302
              -2.5075
 13 20
M END
```

3.8563 O

2.6799 C

2.0082 C

1.9570 C

0.6137 C

0.5626 C

2.7435 C

4.5978 C

6.0942 C

0.0609 H

-0.0002 H

-1.1947 H

6.3998 н

6.5170 H

6.4654 H

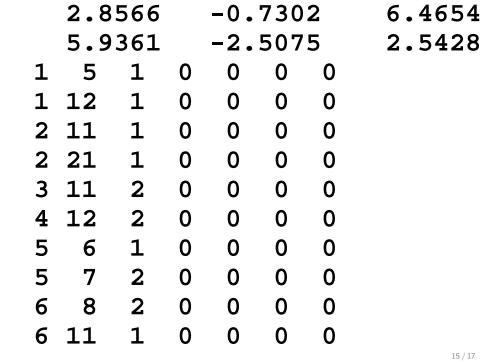
2.5428 H

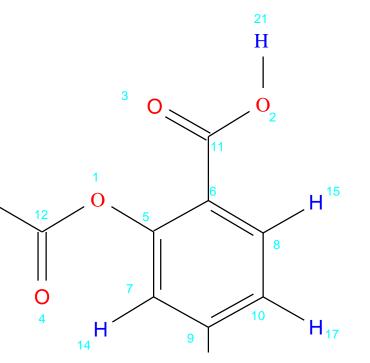
-0.1090 C

2244

-OEChem-03090904423D

0.1	01	•	•	_	^	^	_	~ ~			~ ~	
21	21	0	0	0	0	0	0	09	99	V20	00	
	1.8	152	-0.	938	2	4.	.041	L9	0	0	0	0
	5.1	920	-2.	104	3	2.	.046	57	0	0	0	0
	3.9	623	-2.	685	5	3.	.856	53	0	0	0	0
	2.9	441	1.	111	3	3.	971	L 2	0	0	0	0
	1.8	509	-0.	976	7	2.	679	9	C	0	0	0
	2.9	180	-1.	573	4	2.	.008	32	C	0	0	0
	0.8	800	-0.	410	5	1.	.957	70	C	0	0	0
	2.9	348	-1.	603	8	0.	613	37	C	0	0	0
	0.8	176	-0.	441	0	0.	. 562	26	C	0	0	0
	1.8	846	-1.	037	6	-0.	.109	90	C	0	0	0
	4.0	236	-2.	171	4	2.	.743	35	C	0	0	0
	2.4	171	0.	201	7	4.	. 597	78	C	0	0	0
	2.3	265	0.	150	3	6.	.094	<u> 1</u> 2	C	0	0	0
												15 / 17





SDfile

```
*c *d *I [Data Header] ...... > <data field name> (record number)

[Data] ......numerical or text data field entry

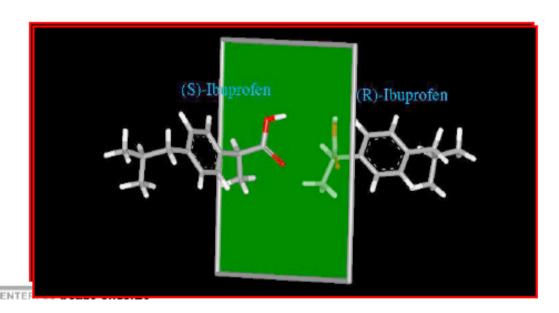
[Blank line] $$$$ ......delimiter terminates record
```

```
jscorina 12209406473DS
LongName
 7 6
  -0.0187
            1.5258
                     0.0104 C
   0.0021
           -0.0041
                     0.0020 C
                               0
   1.6831
            2.1537
                               0
  -1.4333
           -0.5336
                     0.0129 C
                               0 0
   2.0692
            1.9811
  -1.4126
           -2.0635
                     0.0045 C
                               0 0 0
   1.4620
            3.1542
                    -2.5386 C 0 0 0 0 0
  2 1 1 0 0 0
  3 1 1 0 0 0
         0 0 0
  6 4 1 0 0 0
> <NSC>
19
> <CAS_RN>
638-46-0
2222
```

Importance of stereochemistry

Enantiomers (mirror image molecules) have

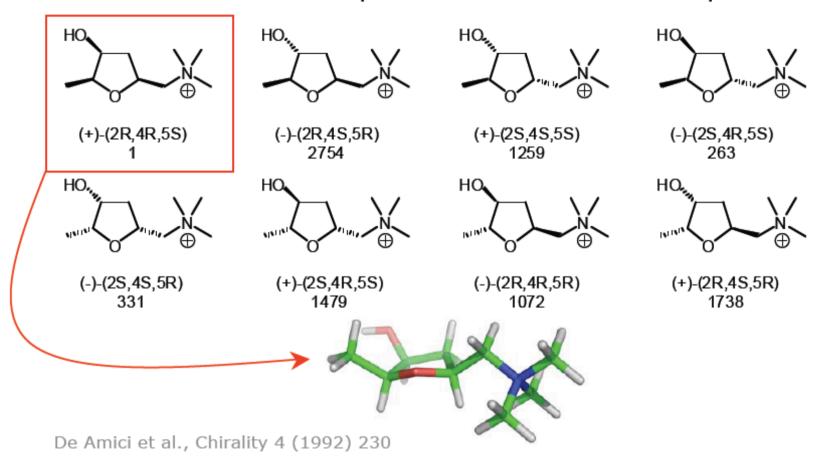
- ☆ identical physical and chemical properties
- ☆ different optical rotation
- ☆ often different biological activities





Importance of stereochemistry

- Muscarine (muscarinic agonist): Natural product most potent
- 7 other stereoisomers possible: Considerable less potent



isomers

compounds have an identical empirical formula, but different molecular structures

Structural isomers different functional groups CHB Q-CHB Q					, , , , , , , , , , , , , , , , , , , ,	10.00 (10.00)		
different functional groups at different places CHB	constitutional	l isomers	stereoisomers					
	structural isomers different functional groups CH3 CH3 CH2 CH3 OH dimethyl ethanol	cositional isomers dentical functional groups at different places cooh cooh cooh chance cha cha	neighboring groups of double bonds can have two directions. H, COOH H, COOH C C C C C C C C C C C C C C C C C C C	chiral center if a molecule has one chiral atom, the isomers are like an image and mirror image (enantiomers); molecules with more than one chiral atom exist as enantiomers and diastereo- mers COOH COOH H-C-NH2 H2N-C-H CH3	chirality axial chirality (atropisomerism, helicene) if a molecule has four ligands which are placed pairwise along an axis and are not in one plane (atropisomer) 2,2'-dichloro-6,6'-dimethyl-1,1'-biphenyl a special case of axial chirality is the arrangement of the molecule as a right- or left-handed helix (helicene)	if the arrange- ment of a mole- cule can be distinguished into different sides	isomers different conformers by rotation around a single bond CH3 CH3 H H H H CH3 R-butane	

Figure 2-67. Classification of isomeric structures of organic compounds.

Stereochemistry in SMILES

Figure 2-78. The stereochemistry of (2R,3E,5E)-2-hydroxy-3,5-heptadiene nitrile can be expressed in the SMILES notation with @ or (back)slashes.

@@: when viewed from atom along the bond to the chiral center, the sequence of atoms (H), (O) and (C#N) appear clockwise

"/" and "\": "cis" and "trans" configuration

Stereochemistry in InChI

InChI=1/C5H7BrCl2O/c1-3(7)4(8)5(2,6)9/h9H,1-2H3/b4-3+/t5-/m1/s1

Stereochemical layer

Double bond stereo sub-layer

sp³ stereo sub-layer

File formats

Table 2-5. The most important file formats for exchange of chemical structure information.

File format	Suffix	Comments	Support	Ref.
MDL Molfile	*.mol	Molfile; the most widely used connection table format	www.mdli.com	50
SDfile	*.sdf	Structure-Data file; extension of the MDL Molfile containing one or more compounds	www.mdli.com	50
RDfile	*.rdf	Reaction-Data file; extension of the MDL Molfile containing one or more sets of reactions	www.mdli.com	50
SMILES	*.smi	SMILES; the most widely used linear code and file format	www.daylight.com	20, 21
PDB file	*.pdb	Protein Data Bank file; format for 3D structure information on proteins and polynucleotides	www.rcsb.org	53
CIF	*.cif	Crystallographic Information File format; for 3D structure infor- mation on organic molecules	www.iucr.org/iucr-top/cif/	55
JCAMP	*.jdx, *.dx, *.cs	Joint Committee on Atomic and Molecular Physical Data; structure and spectroscopic format	www.jcamp.org/	56
CML	*.cml	Chemical Markup Language; extension of XML with speciali- zation in chemistry	www.xml-cml.org	57–59

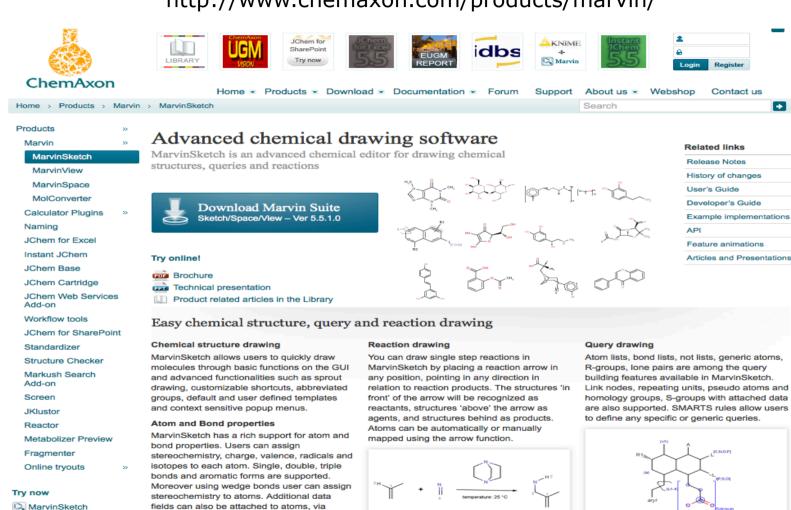
Molecular editors and viewers

"S-group" logic so that any user defined

structural information.

information can be stored directly with the

http://www.chemaxon.com/products/marvin/



MarvinSketch

MarvinView

Calculator Plugins

Link nodes, repeating units, pseudo atoms and homology groups, S-groups with attached data are also supported. SMARTS rules allow users

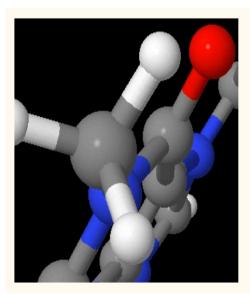
Molecular editors and viewers



http://jmol.sourceforge.net/

Jmol: an open-source Java viewer for chemical structures in 3D

with features for chemicals, crystals, materials and biomolecules



Jmol is an interactive web browser applet.

This is a still image, but you can get an animated display of Jmol abilities by clicking here.

(The applet may take some seconds to load. Please, wait and do not reload the page in the meantime.)

Structure models

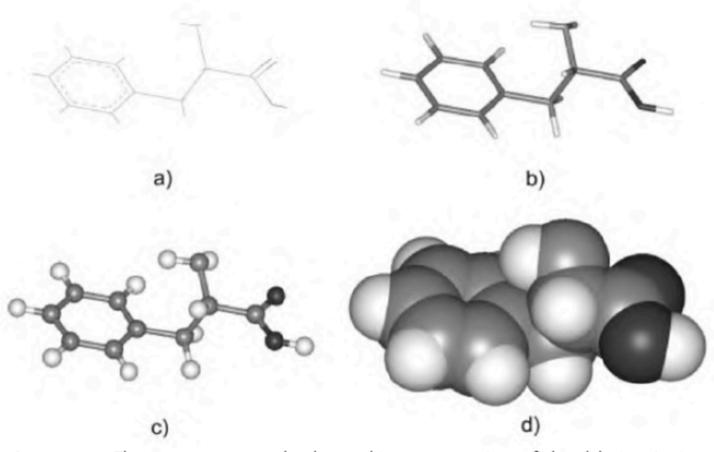


Figure 2-123. The most common molecular graphics representations of phenylalanine a) wire frame; b) capped sticks; c) balls and sticks; d) space-filling.

Format conversion

http://cactus.nci.nih.gov/translate/

Home | About | Contact | Disclaimer | Privacy



Online SMILES Translator and Structure File Generator

Form | News | Help | Acknowledgments

Input Format	Unique SMILES Output Format (Unique SMILES)		
C12C3C4C1C5C4C3C25 Start Structure Editor Please choose this field if you want to submit your own SMILES strings or create a SMILES string using the Structure Editor. A submitted file has precedence, so delete any entry below if you want to submit a new SMILES string.	 ○ Display on screen ○ SMILES TXT file ○ SDF ○ PDB ● MOL (only single structure generated) 	Use • Kekule or Aromatic SMILES representation (choose "Aromatic" for closer approximation to Daylight USMILES) SD, PDB or MOL files should contain • 2D 3D coordinates	
Please choose this field if you want to translate your own files. The service will automatically recognize SD files (single and multiple structure), text files with multiple SMILES fields, MOL files and PDB files (and in fact any other format CACTVS recognizes).	input formats will generate multiple structure out only the first structure will be used. SD files will c	tput will also be single structure. Multiple structure tput for those formats that support this. Otherwise, ontain a UNIQUE_SMILES field for unique SMILES and the user-supplied SMILES (if avaliable)	
Reset Translate			

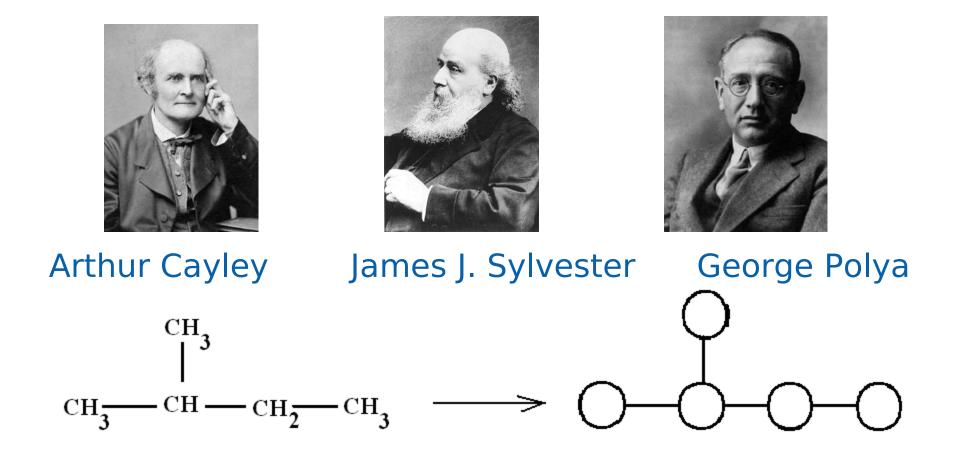
Examples of SMILES

CCN(CC)C(=0)[C@H]1CN(C)[C@@H]2Cc3c[nH]c4cccc(C2=C1)c34

C/C (=C/CO)/C=C/C=C(/C)/C=CC1=C(C)CCCC1(C)C

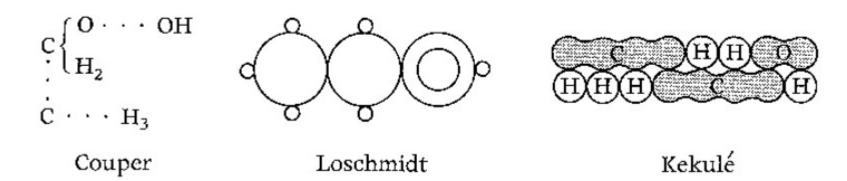
Graph Theory / Algebra / Chemistry

Enumeration of Chemical Isomers

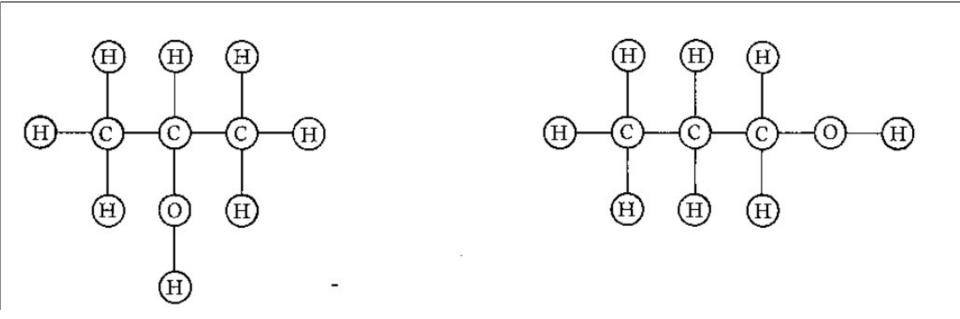


Couper (1858) / Loschmidt (1861) / Kekulé (1861)

CHEMICAL GRAPHS



Crum Brown (1864) and Frankland (1866)



8 cm, from the primary. Reverse the wires in the secondary circuit, reverse the wires in the primary circuit, how you please, the mercury always moves towards the point of the capillary.

8. Shouting or singing (excepting the above-mentioned note) produces no visible effect under the conditions

mentioned in Experiments 5, 6, and 7.

9. If the secondary coil be now moved close up, so as to cover as completely as possible the primary, talking to the telephone with the ordinary voice, i.e. with moderate strength and at any pitch, produces a definite movement of the mercury column for each word, some sounds of course giving more movement than others, but the movement is always towards the end of the capillary. Singing the note mentioned in Experiments 5, 6, and 7 loudly, produces a movement too large to be measured with the electrometer.

Reversing the poles of the magnet in the telephone does not alter the results of Experiments 5, 6, 7, and 9.

On mentioning the above results to Dr. Burdon Sanderson, he suggested that the apparently anomalous behaviour of the electrometer might be accounted for, by supposing that the mercury moved quicker when a current passed towards the point of the capillary than when it flowed in the opposite direction; so that if a succession of rapidly alternating currents be passed through the instrument, the mercury will always move towards the point of the capillary, the movement away from the point being masked by the sluggishness of the instrument in that direction. That this explanation is the correct one is proved by the following experiment:-The current from two Grove's cells is sent through a metal reed vibrating 100 times a second, the contact being made and broken at each vibration, the primary wire of a Du Bois Reymond's induction-coil is also included in the circuit; on connecting the electrometer with the secondary coil placed at an appropriate distance the mercury always moves to the point of the tube whatever be the direction of the F. J. M. PAGE

Physiological Laboratory, University College, London, February 2

those in Experiment 9 were observed.-F. J. M. P.

NOTE.—On February 4 Prof. Graham Bell kindly placed at my disposal a telephone much more powerful than any of those I had previously used. On speaking this instrument, the electrometer being in the circuit, movements of the mercury column as considerable as

CHEMISTRY AND ALGEBRA

TT may not be wholly without interest to some of the readers of NATURE to be made acquainted with an analogy that has recently forcibly impressed me between branches of human knowledge apparently so dissimilar as modern chemistry and modern algebra. I have found it of great utility in explaining to non-mathematicians the nature of the investigations which algebraists are at present busily at work upon to make out the so-called Grundformen or irreducible forms appurtenant to binary quantics taken singly or in systems, and I have also found that it may be used as an instrument of investigation in purely algebraical inquiries. So much is this the case that I hardly ever take up Dr. Frankland's exceedingly valuable "Notes for Chemical Students," which are drawn up exclusively on the basis of Kekulć's exquisite conception of valence, without deriving suggestions for new researches in the theory of algebraical forms. I will confine myself to a statement of the grounds of the analogy, referring those who may feel an interest in the subject and are desirous for further information about it to a memoir which I have written upon it for the new American Journal of Pure and Applied Mathematics, the first number of which will appear early in February.

The analogy is between atoms and binary quantics exclusively.

I compare every binary quantic with a chemical atom. The number of factors (or rays, as they may be regarded by an obvious geometrical interpretation) in a binary quantic is the analogue of the number of bonds, or the valence, as it is termed, of a chemical atom.

Thus a linear form may be regarded as a monad atom, a quadratic form as a duad, a cubic form as a triad, and so on.

An invariant of a system of binary quantics of various degrees is the analogue of a chemical substance composed of atoms of corresponding valences. The order of such invariant in each set of coefficients is the same as the number of atoms of the corresponding valence in the chemical compound.

A co-variant is the analogue of an (organic or inorganic) compound radical. The orders in the several sets of co-efficients corresponding, as for invariants, to the respective valences of the atoms, the free valence of the compound radical then becomes identical with the degree of the co-variant in the variables.

The weight of an invariant is identical with the number of the bonds in the chemicograph of the analogous chemical substance, and the weight of the leading term (or basic differentiant) of a co-variant is the same as the number of bonds in the chemicograph of the analogous compound radical. Every invariant and covariant thus becomes expressible by a graph precisely identical with a Kekuléan diagram or chemicograph. But not every chemicograph is an algebraical one. I show that by an application of the algebraical law of reciprocity every algebraical graph of a given invariant will represent the constitution in terms of the roots of a quantic of a type reciprocal to that of the given invariant of an invariant belonging to that reciprocal type. I give a rule for the geometrical multiplication of graphs, i.e. for constructing a graph to the product of in- or co-variants whose separate graphs are given. I have also ventured upon a hypothesis which, whilst in nowise interfering with existing chemicographical constructions, accounts for the seeming anomaly of the isolated existence as "monad molecules" of mercury, zinc, and arsenic—and gives a rational explanation of the " mutual saturation of bonds.

I have thus been led to see more clearly than ever I did before the existence of a common ground to the new mechanism, the new chemistry, and the new algebra. Underlying all these is the theory of pure colligation, which applies undistinguishably to the three great theories, all initiated within the last third of a century or thereabouts by Eisenstein, Kekulé, and Peaucellier.

Baltimore, January I J. J. SYLVESTER

PALMEN ON THE MORPHOLOGY OF THE TRACHEAL SYSTEM

DR. PALMEN, of Helsingfors, has recently published an interesting memoir on the tracheal system of insects. He observes that although the gills of certain aquatic larvæ are attached to the skin very near to the points at which the spiracles open in the mature insects, and though spiracles and gills do not co-exist in the same segment, yet the point of attachment of the gills never exactly coincides with the position of the future spiracle. Moreover, he shows that even during the larval condition, although the spiracles are not open, the structure of the stigmatic duct is present, and indeed that it opens temporarily at each moult, to permit the inner tracheal membrane to be cast, after which it closes again. In fact, then, he urges, the gills and spiracles do not correspond exactly, either in number or in position, and there can therefore be between them no genetic connection. He concludes that the insects with open tracheæ are not derived from ancestors provided with gills,

Some Polya Enumeration

Pólya Theory

Formally: Pólya Theory counts equivalence classes, where the equivalence classes are induced by group actions. Since groups describe symmetry, Pólya Theory is counting the number of distinct objects in the presence of symmetry.

Informally: Pólya Theory does "common sense" counting.

Chemical Isomers

Pólya's original objective was to determine the number of distinct compounds given a chemical formula.

There are three distinct compounds with the formula $C_6H_4Br_2$.

Example

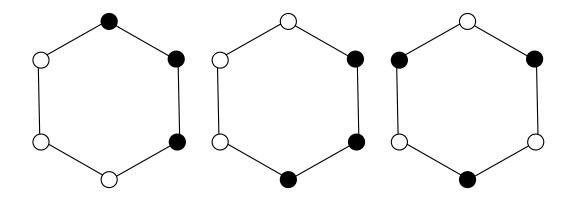


Figure 1: Common sense says that these the two bracelets on the left are the "same", the third bracelet is "different"

Another Example

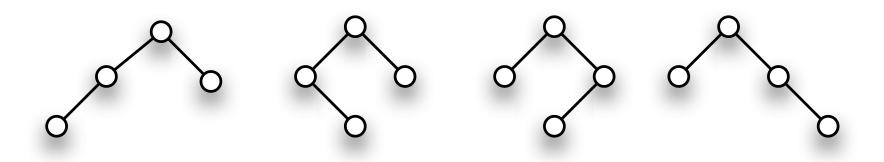


Figure 2: These 4 binary trees are equivalent if left and right are considered indistinguishable

Yet Another Example

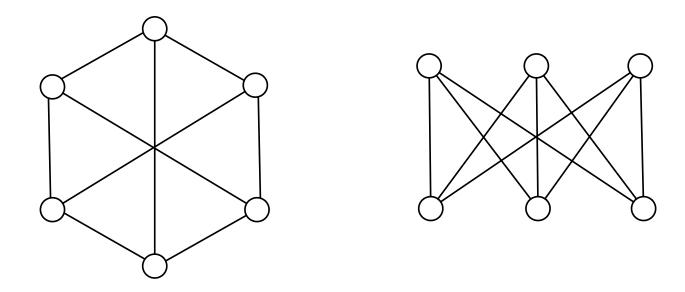


Figure 3: These two graphs are isomorphic, although it is not visually obvious

And Another

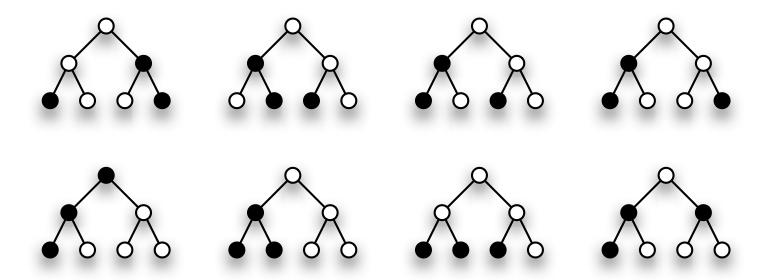
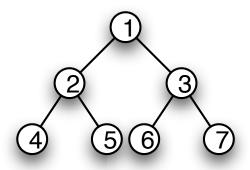


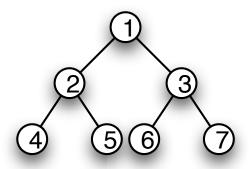
Figure 4: If left and right are indistinguishable then the top row of graph colorings are equivalent, and the bottom row are different.

Example Calculation

Problem: Count the number of distinct black and white colorings of the tree in Figure 7 if left and right are indistinguishable.



Step 1



Write down the permutations that leave the tree invariant if left and right are indistinguishable. This is a group G generated by:

$$\pi_1 = (1)(2 \ 3)(4 \ 6)(5 \ 7)$$

$$\pi_2 = (1)(2)(3)(4 \ 5)(6)(7)$$

$$\pi_3 = (1)(2)(3)(4)(5)(6 \ 7)$$

Step 2

List the group elements, calculate the monomials.

Element	Cycle	Monomial	
	Representation		
I	(1)(2)(3)(4)(5)(6)(7)	x_1^7	
π_1	$(1)(2\ 3)(4\ 6)(5\ 7)$	$x_1 x_2^3$	
π_2	$(1)(2)(3)(4\ 5)(6)(7)$	$x_1^5 x_2$	
π_3	(1)(2)(3)(4)(5)(67)	$x_1^5 x_2$	
$\pi_2\pi_3=\pi_3\pi_2$	$(1)(2)(3)(4\ 5)(6\ 7)$	$x_1^3 x_2^2$	
$\pi_1\pi_2=\pi_3\pi_1$	$(1)(2\ 3)(4\ 6\ 5\ 7)$	$x_1x_2x_4$	
$\pi_1\pi_3=\pi_2\pi_1$	$(1)(2\ 3)(4\ 7\ 5\ 6)$	$x_1 x_2 x_4$	
$\pi_1\pi_2\pi_3$	$(1)(2\ 3)(4\ 7)(5\ 6)$	$x_1 x_2^3$	

Table 1: The elements of G

Step 3

Add the monomials to get the cycle index for the group:

$$P_G(x_1, x_2, x_4) = \frac{1}{8}(x_1^7 + 2x_1^5x_2 + 2x_1x_2^3 + 2x_1x_2x_4 + x_1^3x_2^2)$$

Formally the cycle index is defined:

$$P_G(x_1, x_2, \dots, x_{|D|}) = \frac{1}{|G|} \sum_{\pi \in G} x_1^{l_1(\pi)} x_2^{l_2(\pi)} \dots x_{|D|}^{l_{|D|}(\pi)}$$

where D is the set acted on by elements of G, |D| is the size of the set, $l_k(\pi)$ is the number of cycles of length k in π .

Step 4

Pólya's Enumeration Theorem says that the number of distinct k-colorings is

$$P_G(k, k, k, k) = \frac{1}{8}(k^7 + 2k^6 + 2k^4 + 2k^3 + k^5)$$

$$= \frac{k^3}{8}(k^4 + 2k^3 + k^2 + 2k + 2)$$

$$= \frac{k^3}{8}(k+1)(k^3 + k^2 + 2)$$

This must be an integer for all integer values of k, so $k^3(k+1)(k^3+k^2+2)$ must be divisible by 8.

The Solution

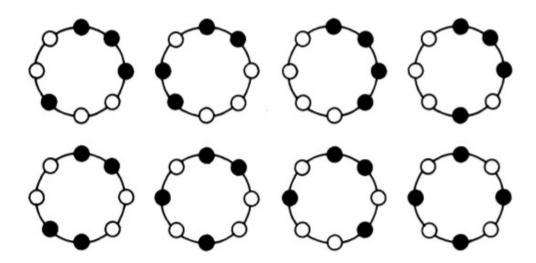
The number of 2-colorings of the binary tree, with left and right indistinguishable, is

$$P_G(2, 2, 2, 2) = \frac{1}{8}(2^7 + 2^5 + 2^7 + 2^4 + 2^5)$$

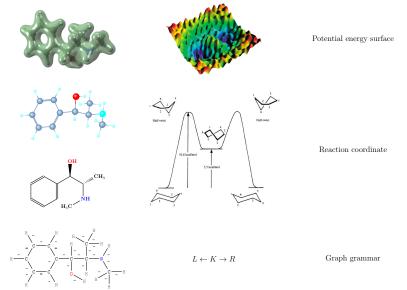
= 42

$$1 + x + 4x^2 + 5x^3 + 8x^4 + 5x^5 + 4x^6 + x^7 + x^8$$
.

Thus, for example, there are five inequivalent necklaces having three black beads, and eight with equal numbers of black and white beads. The latter are shown in FIGURE 1.

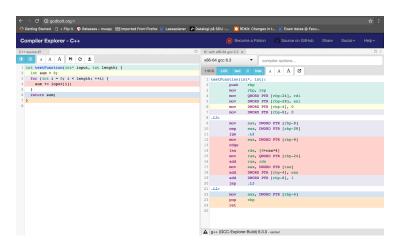


Levels of Abstraction in Computational Chemistry



[Andersen et al., Proceedings of the Royal Society A, 2017]

Levels of Abstraction in Programming



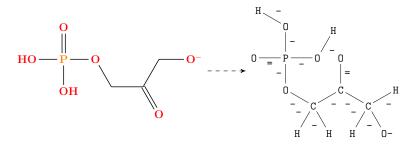
Declarative Description \leftrightarrow DSL \leftrightarrow C++ \leftrightarrow Assembler

Levels of Abstraction in Computer Science

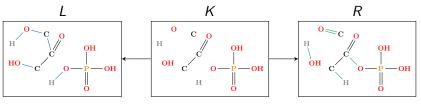


"The psychological profiling [of a Computer Scientist] is mostly the ability to shift levels of abstraction, from low level to high level. To see something in the small and to see something in the large."

- 1. Model molecules as labelled graphs.
 - ► An old idea: [J. J. Sylvester, *Chemistry and Algebra*, Nature 1878]
 - ▶ Molecule: simple, connected, labelled graph.
 - ▶ Vertex labels: atom type, charge.
 - ► Edge labels: bond type.



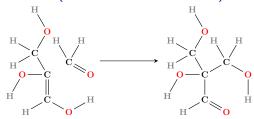
2. Model reaction types and graph transformation rules.



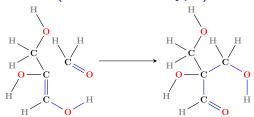
Example: Carbon rearrangement

- ightharpoonup Aldolase: ketone + aldehyde \longrightarrow ketone
- ightharpoonup Aldose-Ketose: aldehyde \longrightarrow ketone
- ightharpoonup Ketose-Aldose: ketone \longrightarrow aldehyde
- ▶ Phosphohydrolase: $H_2O+CnP \longrightarrow Cn+Pi$
- ▶ Phosphoketolase $Pi+ketone \longrightarrow carbonyl + CnP+water$
- ► Transaldolase: $Cn+Cm\longrightarrow C(n+3)+C(m-3)$
- ► Transketolase: $Cn+Cm\longrightarrow C(n+2)+C(m-2)$

Chemical Reactions (Educts → Products)

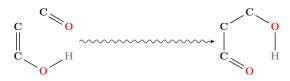


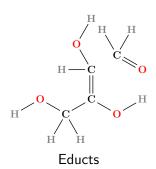
Chemical Reactions (of the Same Type)



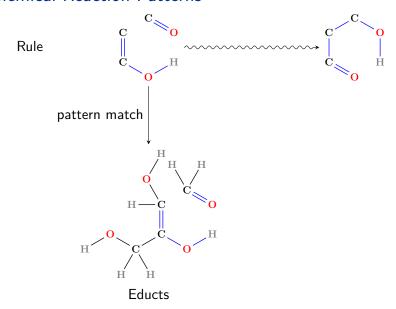
Chemical Reaction Patterns

Rule

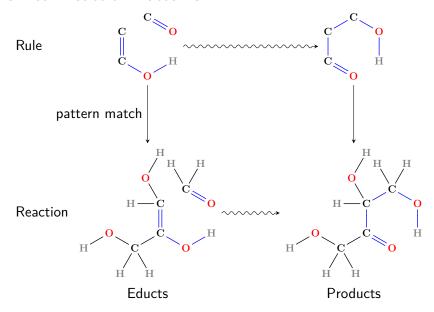




Chemical Reaction Patterns



Chemical Reaction Patterns



Grammar Example: The Formose Chemistry

Formaldehyde: Glycolaldehyde:

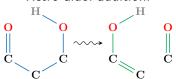
Keto-enol tautomerism:

$$\begin{array}{c|c}
H & H & H \\
\hline
C & H & C & H
\end{array}$$

$$\begin{array}{c|cccc}
C = 0 & C = 0 \\
 & & & & & & \\
C = 0 & C = 0 \\
 & & & & & \\
C = 0 & C = 0 \\
 & & & & & \\
C = 0 & C = 0
\end{array}$$

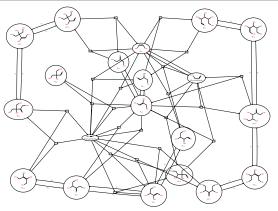


Retro aldol addition:

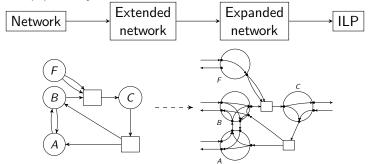


3. Generate a reaction network.

```
dg = dgRuleComp(inputGraphs,
    addSubset(inputGraphs) >> rightPredicate[
    lambda d: all(countCarbon(a) <= 5 for a in d.right)
    ](    repeat(inputRules) )
)
dg.calc()</pre>
```



4. Set up pathway model.

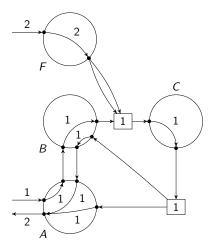


Conservation constraints:

$$\sum_{e \in \delta_{\widetilde{E}}^+(v)} m_v(e^+) f(e) - \sum_{e \in \delta_{\widetilde{E}}^-(v)} m_v(e^-) f(e) = 0 \qquad \forall v \in \widetilde{V}$$

5. Formulate pathway question.

Example: Given 2 formaldehyde and 1 glycolaldehyde, how can 2 glycolaldehyde be produced through autocatalysis.

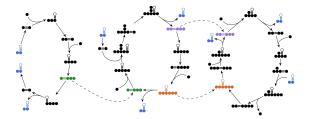


6. Enumerate many alternate pathways.

Example (Formose):

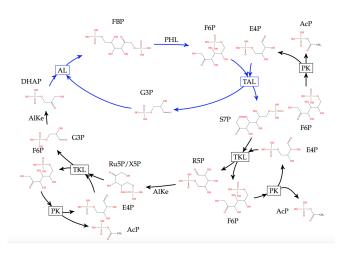
Network: all molecules with at most 9 carbon atoms.

	Maximum #C						
Reactions used	4	5	6	7	8	9	Sum
6	0	0	1	1	1	2	5
7	0	0	0	0	0	2	2
8	1	5	7	17	37	68	135
9	0	0	12	12	37	69	130
10	0	12	50	274	849	_	> 1185
11	0	5	41	190	738	_	_ ≥ 974
							> 243



Another Example: Non-oxidative Glycolysis

You specify: F6P + 2 P_i \rightarrow 3 AcP + 2H₂O You get (for example):



Many alternatives for suggestion in [Bogorad, Lin, and Liao, Nature, 2013]

Overview Labelled graphs Molecules, reaction patterns Graph transformation rules Stereochemistry Point groups Exploration strategies Rule composition Reaction networks | Directed hypergraphs **ILP** Tree search **Pathways** Integer hyperflows Pathway motifs Rule composition Petri-nets Pathway realisations Atom maps Atom traces

http://cheminf.imada.sdu.dk

Category theory Double Pushout Rule composition Monomorphisms Isomorphisms Canonicalisation Automorphisms

Software package: MØD

C++, Python, Bash, LATEX

Pentose phosphate pathway Glycolysis (EMP and ED) Non-oxidative glycolysis Citric acid cycle Enzyme mechanisms Formose Prebiotic chemistry (HCN) Eschenmoser's GLX scenario DNA templated computing