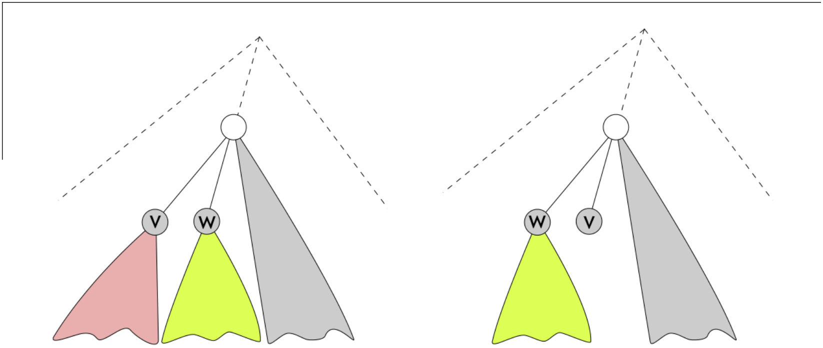
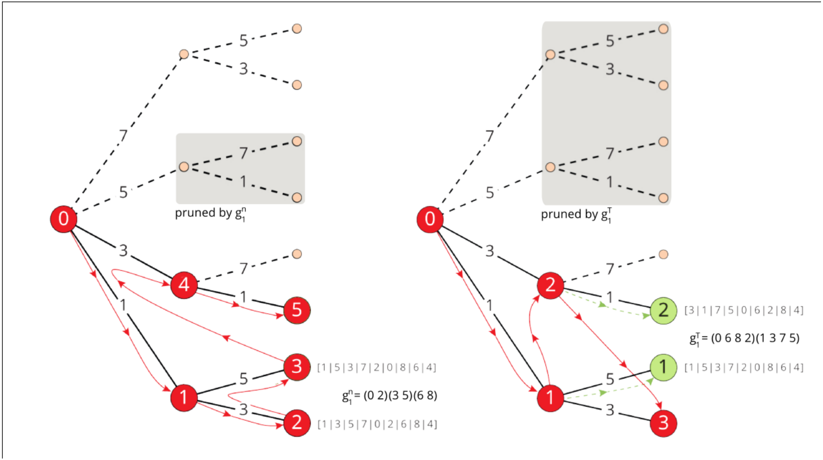
Traces

Traces Motivation



Assume that the canonical form is not associated to a leaf of the tree rooted at v. If v comes before w, then the whole subtree rooted at v is visited before it is realized that its construction could have been avoided.

Search Tree Nauty (left) / Traces (right)



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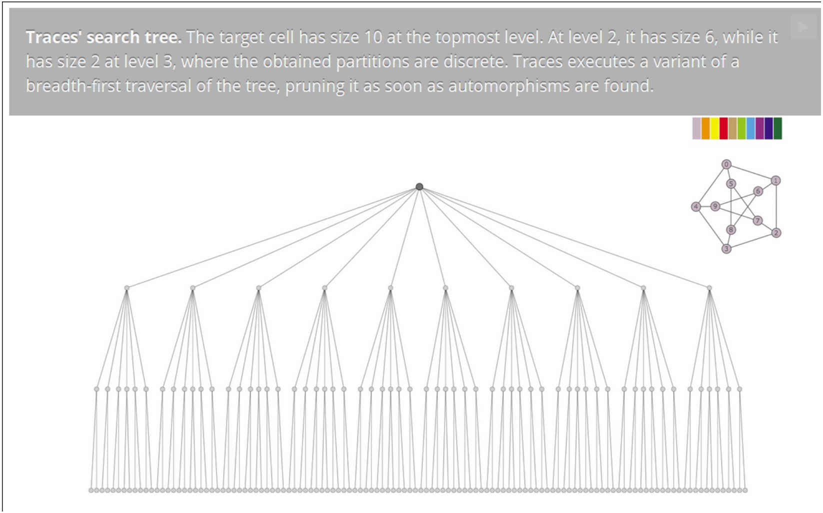
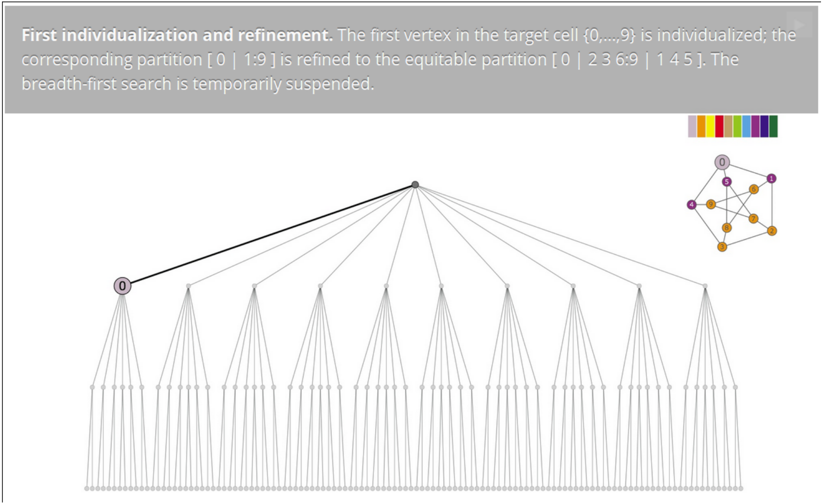
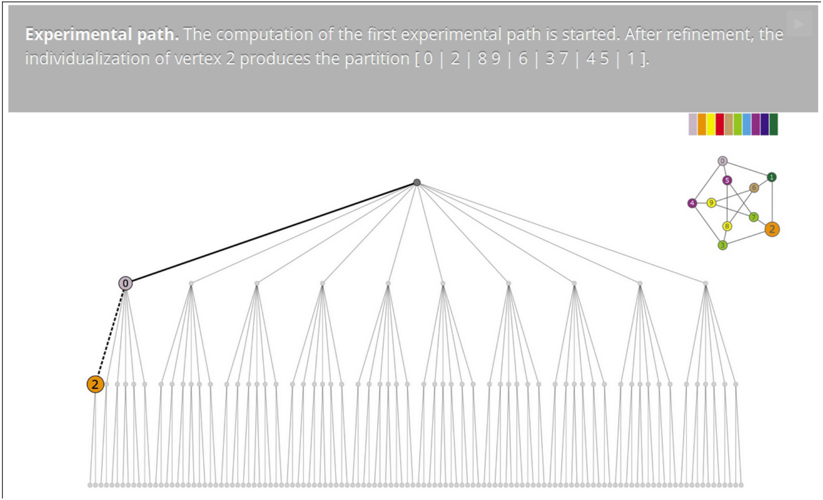

Traces' search tree. The target cell has size 10 at the topmost level. At level 2, it has size 6, while it has size 2 at level 3, where the obtained partitions are discrete. Traces executes a variant of a breadth-first traversal of the tree, pruning it as soon as automorphisms are found.

Traces


First individualization and refinement. The first vertex in the target cell {0,...,9} is individualized; the corresponding partition [0 | 1:9] is refined to the equitable partition [0 | 2 3 6:9 | 1 4 5]. The breadth-first search is temporarily suspended.
0
0
1
2
3
4
5
6
7
8
9

Traces


Experimental path. The computation of the first experimental path is started. After refinement, the individualization of vertex 2 produces the partition [0 | 2 | 8 9 | 6 | 3 7 | 4 5 | 1].
0
2

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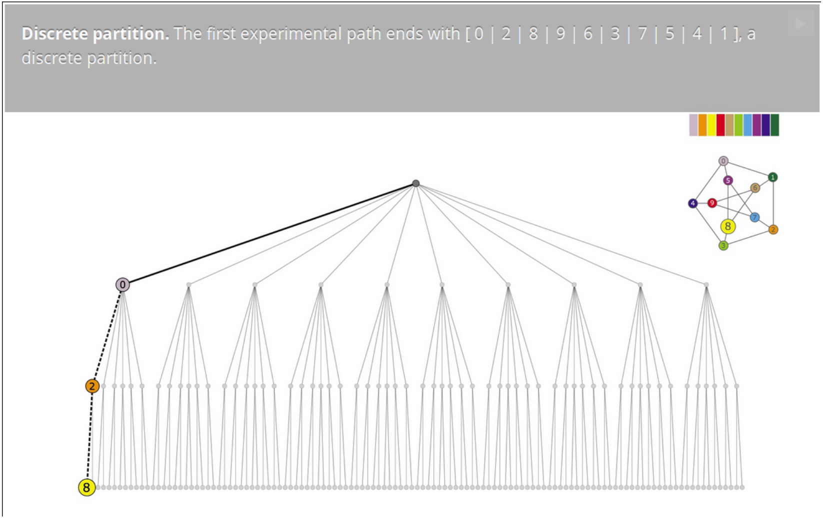
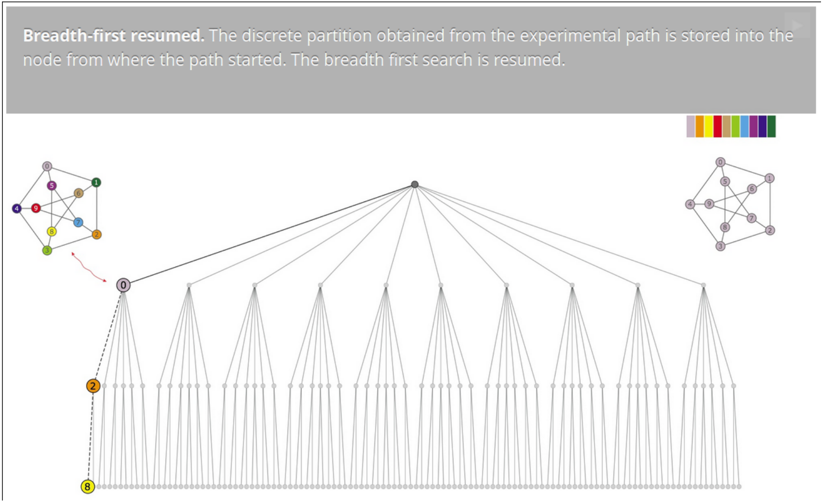


Discrete partition. The first experimental path ends with [0 | 2 | 8 | 9 | 6 | 3 | 7 | 5 | 4 | 1], a discrete partition.
0
2
8

Traces


Breadth-first resumed. The discrete partition obtained from the experimental path is stored into the node from where the path started. The breadth first search is resumed.

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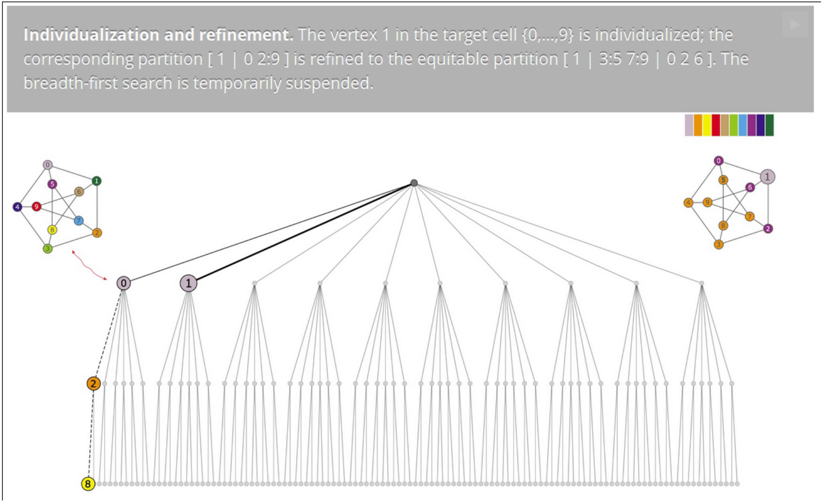
Individualization and refinement. The vertex 1 in the target cell {0,...,9} is individualized; the corresponding partition [1 | 0 2:9] is refined to the equitable partition [1 | 3:5 7:9 | 0 2 6]. The breadth-first search is temporarily suspended.

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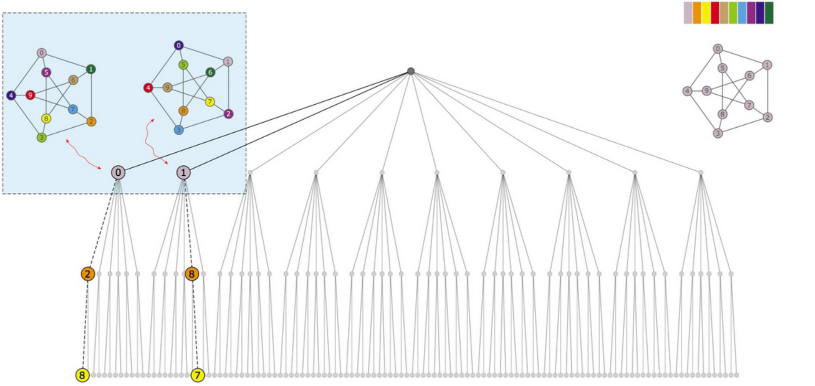
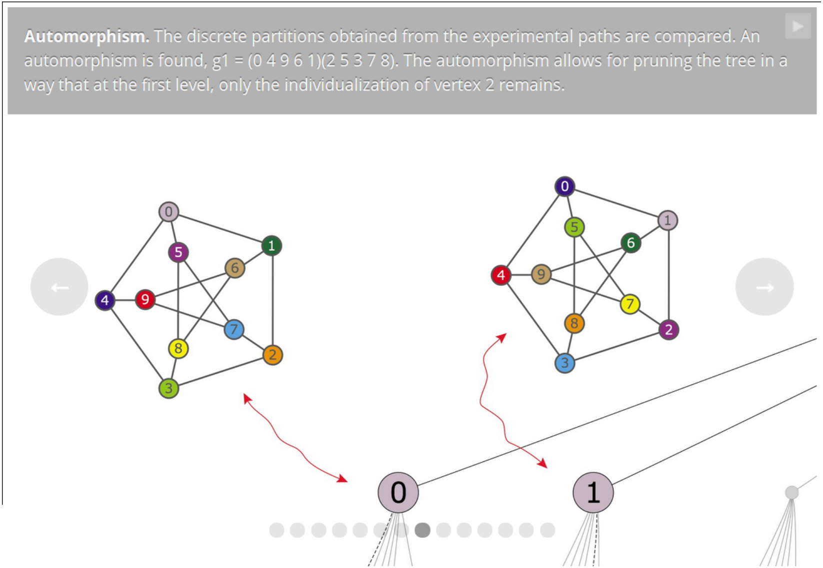
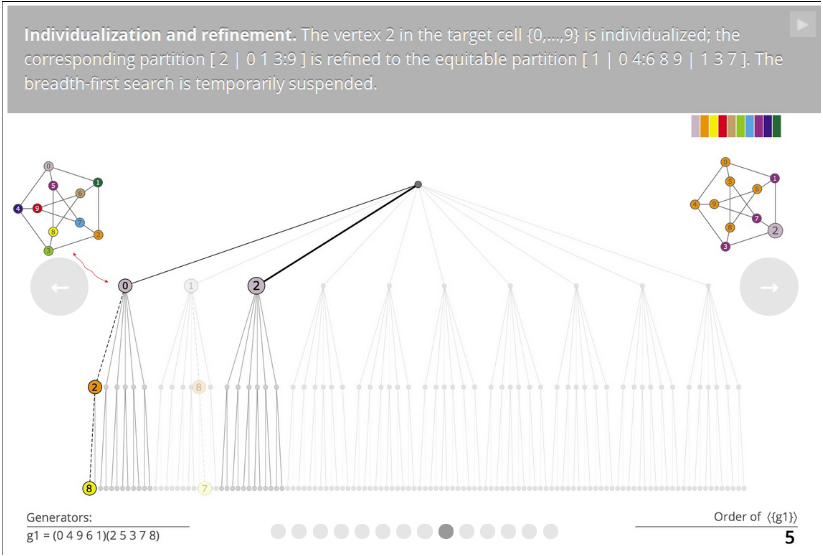
Experimental path. The experimental path from the current node is computed. It finally produces the discrete partition [1 | 8 | 7 | 4 | 9 | 5 | 3 | 2 | 0 | 6], which is stored into the current node itself.



Traces


Automorphism. The discrete partitions obtained from the experimental paths are compared. An automorphism is found, g1 = (0 4 9 6 1)(2 5 3 7 8). The automorphism allows for pruning the tree in a way that at the first level, only the individualization of vertex 2 remains.
0
1
2
3
4
5
6
7
8
9
0
1
2
3
4
5
6
7
8
9
0
1

Traces


Individualization and refinement. The vertex 2 in the target cell {0,...,9} is individualized; the corresponding partition [2 | 0 1 3:9] is refined to the equitable partition [1 | 0 4:6 8 9 | 1 3 7]. The breadth-first search is temporarily suspended.
Generators:
g1 = (0 4 9 6 1)(2 5 3 7 8)
Order of ⟨{g1}⟩
5

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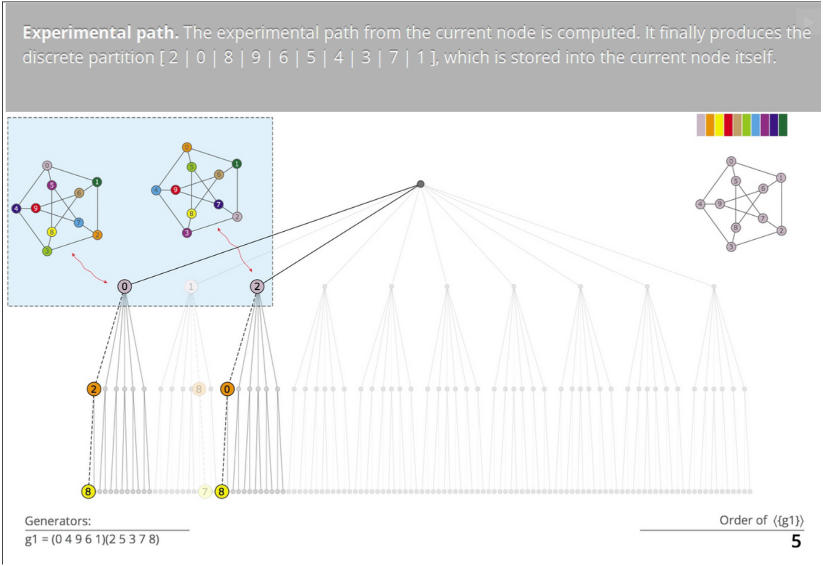
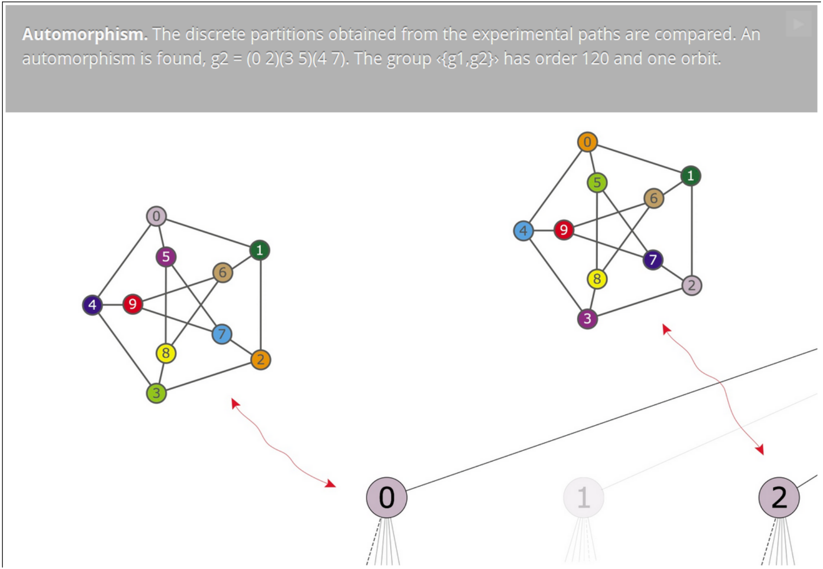
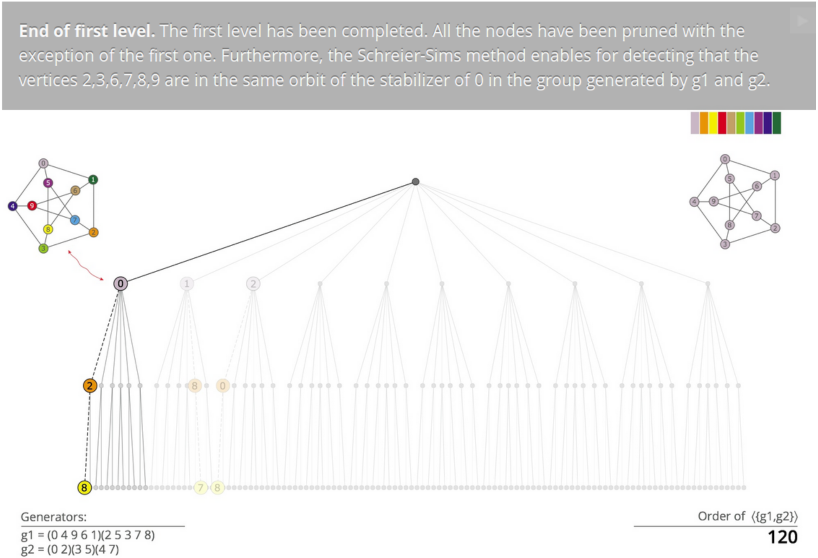


Experimental path. The experimental path from the current node is computed. It finally produces the discrete partition [2 | 0 | 8 | 9 | 6 | 5 | 4 | 3 | 7 | 1], which is stored into the current node itself.
Generators:
g1 = (0 4 9 6 1)(2 5 3 7 8)
Order of ⟨{g1}⟩
5

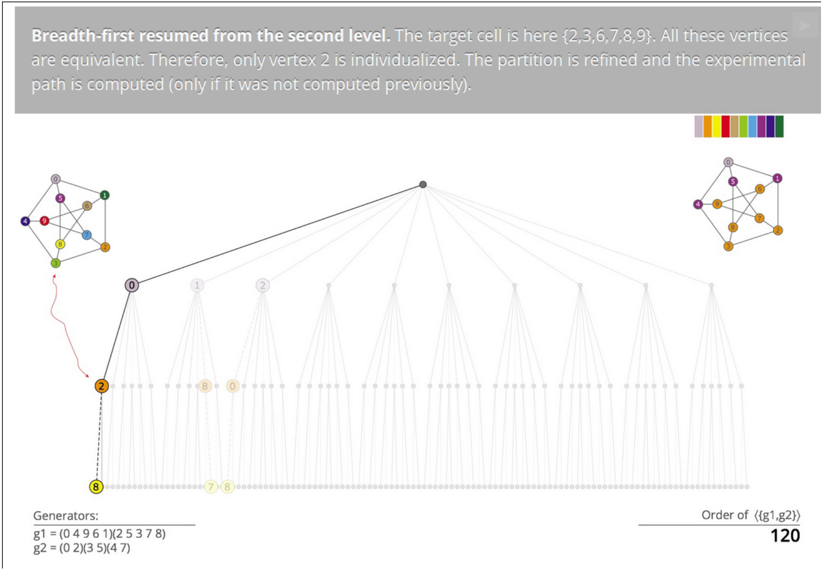
Traces


Automorphism. The discrete partitions obtained from the experimental paths are compared. An automorphism is found, g2 = (0 2)(3 5)(4 7). The group ‹{g1,g2}› has order 120 and one orbit.
0
1
2

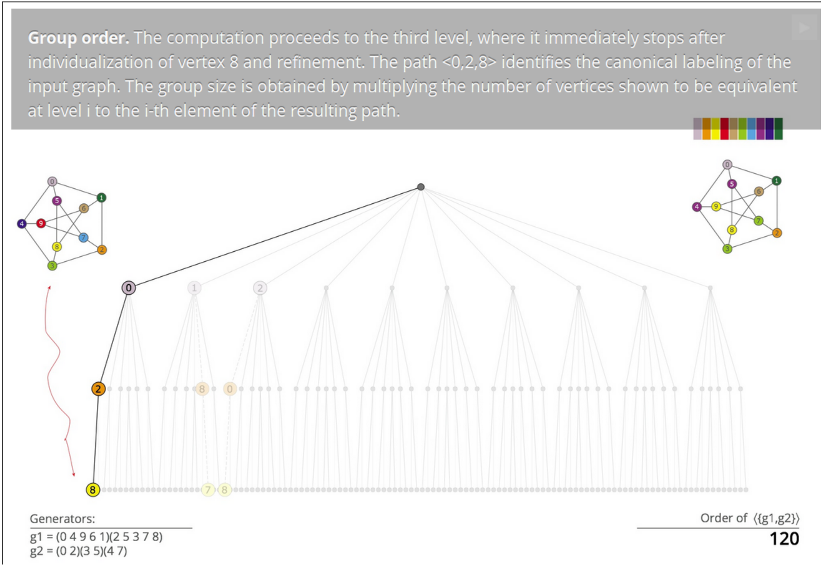
Traces


End of first level. The first level has been completed. All the nodes have been pruned with the exception of the first one. Furthermore, the Schreier-Sims method enables for detecting that the vertices 2,3,6,7,8,9 are in the same orbit of the stabilizer of 0 in the group generated by g1 and g2.
Generators:
g1 = (0 4 9 6 1)(2 5 3 7 8)
g2 = (0 2)(3 5)(4 7)
Order of ⟨{g1,g2}⟩
120

Traces


Breadth-first resumed from the second level. The target cell is here {2,3,6,7,8,9}. All these vertices are equivalent. Therefore, only vertex 2 is individualized. The partition is refined and the experimental path is computed (only if it was not computed previously).
Generators:
g1 = (0 4 9 6 1)(2 5 3 7 8)
g2 = (0 2)(3 5)(4 7)
Order of ⟨{g1,g2}⟩
120

Traces

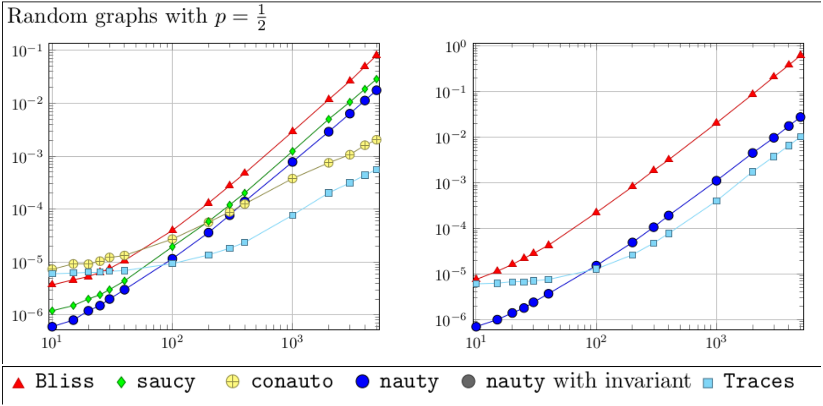


Group order. The computation proceeds to the third level, where it immediately stops after individualization of vertex 8 and refinement. The path <0,2,8> identifies the canonical labeling of the input graph. The group size is obtained by multiplying the number of vertices shown to be equivalent at level i to the i-th element of the resulting path.
Generators:
g1 = (0 4 9 6 1)(2 5 3 7 8)
g2 = (0 2)(3 5)(4 7)
Order of ⟨{g1,g2}⟩
120

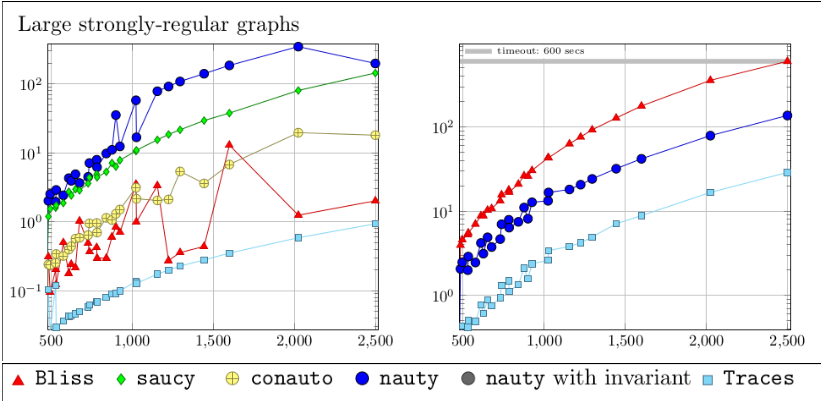
Performance Comparison

Runtime in Seconds

left: automorphism group right: canonical label



Performance Comparison



Performance Comparison