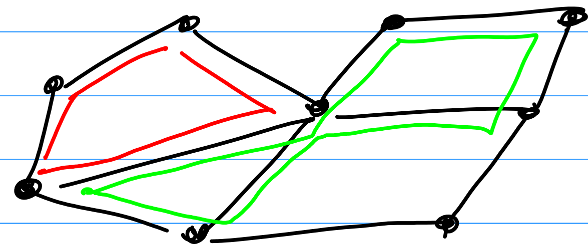
cycle: a subgraph in which every vertex has even degree



incident vectors form a vector space over GF(2), where vector addition corresponds the sym. difference

$$C' \oplus C'' = (C' \cup C'') \setminus (C' \cap C'')$$

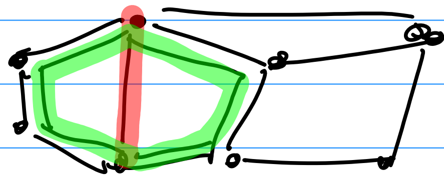
$\mathcal{C}(G)$: cycle space of dimension

$$\mu(G) = |E| - |V| + c(G)$$

nullity / cyclomatic number / first Betti number

cycle basis: A basis $\mathcal{B}$ of $\mathcal{C}(G)$

elementary cycle: connected + each vertex degree two

chord: 

simple cycle: elementary + no chord

length of a cycle: $|C|$ (number of edges)

length of a cycle basis:

$$\ell(\mathcal{B}) = \sum_{C \in \mathcal{B}} |C|$$

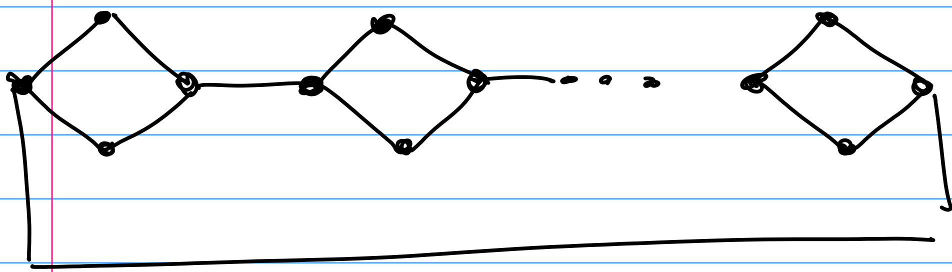
minimum cycle basis:
cycle basis of min length

relevant cycle: $R(G)$
cannot be written as $\oplus$ sum of shorter cycles

equiv:
contained in at least one minimum cycle basis

set of shortest cycles through edge e: $S(e)$

$$S(G) = \bigcup_{e \in E} S(e)$$



$R(G)$ grows exponentially!

Furthermore here: $S(G) = R(G)$

in general: $S(G) \subseteq R(G)$

Kirchoff-fundamental basis:

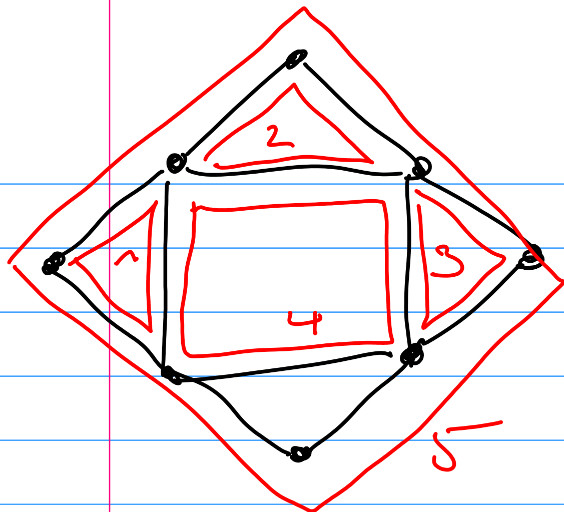
cycle basis wrt some spanning tree T (see slides)

A collection $\mathcal{S}$ of $\mu(G)$ cycles is fundamental, if there exists an ordering of these cycles, such that

$$C_j \setminus (C_1 \cup C_2 \cup \dots \cup C_{j-1}) \neq \emptyset$$

for $2 \le j \le \mu(G)$

strictly fundamental: for all orderings



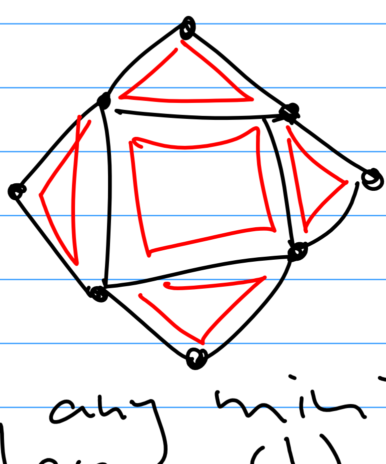
For a cycle basis it holds:
strictly fundamental
$\Longleftarrow$
Kirchhoff-fundamental

SSSR: Smallest set of Smallest Rings

was: minimum length Kirchhoff-fundamental basis
$\rightsquigarrow$ NP complete

is: minimum cycle basis

every minimum ~~cycle~~ basis is strictly fund.



- fundamental
- not strictly fundamental

Note: any minimum cycle basis of a planar (!) graph is fundamental

Minimum cycle basis:

general: $O(|E|^3|V|)$ [Horton 87]

$O(|E|^3 + |E| \cdot |V|^2 \log |V|)$ [de Pina 95]

$$O\left(\frac{|E|^2|V|}{\log|V|} + |E|\,|V|^2\right)$$

planar: $O(|V|^{\frac{3}{2}} \log |V|)$

many misconceptions and wrong implementations, even in CDK!

SSSR not unique

K-rings: union of all min. cycle bases

= set of relevant cycles

unique!

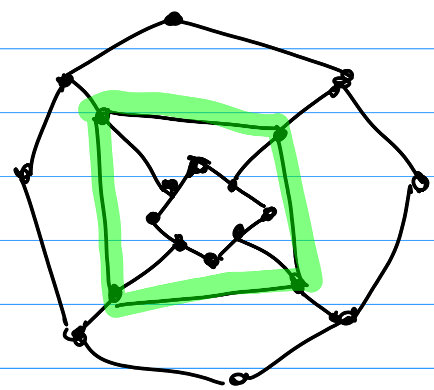
exp :(, however usually fast to compute :)

ESSR: Extended set of Smallest Rings (details: paper)

A cycle is in ESSR(G) if one of the following holds:

i) There is a planar embedding of G, such that C is a chord-less face.

ii) It's a class I cut face, i.e., it's smaller than at least one of it's adjacent simple faces

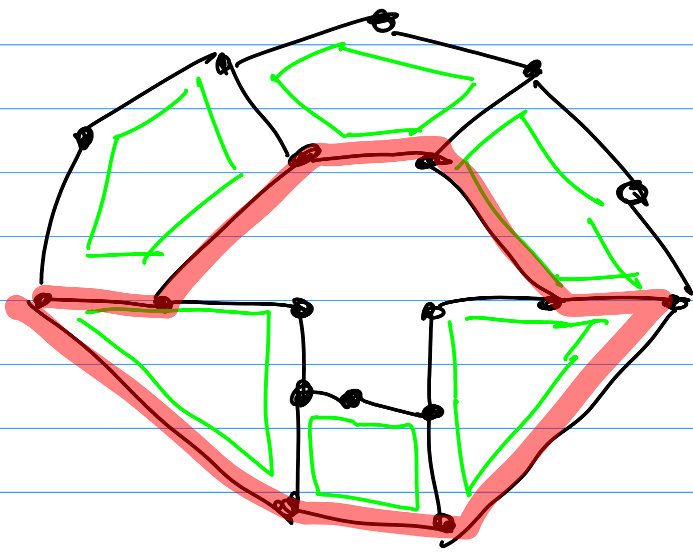


~ size 4, all adjacent faces size 5

iii) It's a class II cut face, i.e., same size as at least one adjacent face, never smaller [...]

~ Example DBR-28

In Downs et al: ~~$ESSR(G) = Faces(G) \cup R(G)$~~



$\mu(G) = |E| - |V| + 1$
$= 24 - 18 + 1$
$= 7$

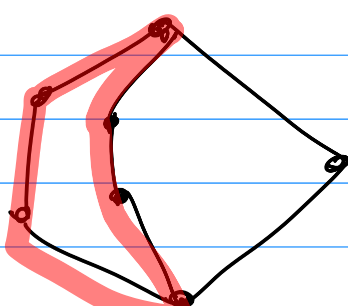
relevant:

6 pentagons

1 octagon

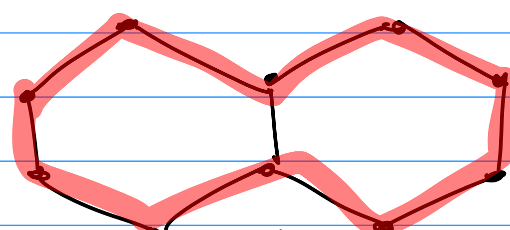
K-rings [1971]

want:



(norborane)

don't want:



(naphtalene)

SSSR fails.

HW:

length sorted chord-less faces of a plane embedding, greedily include cycles by increasing length, that are (lin. indep.)

Modifications:

i.) + all 3-cycles + all 4-cycles

ii) dependencies of max 3 shortest P-rings

↝ not unique since it depends on a particular planar embedding