Institut for Matematik og Datalogi
Syddansk Universitet

September 16, 2010
Daniel Merkle

MM524 / DM527: Weekly Notes 2010 (Week 38)

Lectures in week 38

Tuesday 21/09

- Rest of Section 1.6/1.7 (Proofs)
- Section 2.1 : Sets
- Section 2.2 : Set Operations
- Section 2.3 : Functions

Thursday 23/09

- Section 2.3 : Functions
- Section 2.4 : Sequences and Summations
- Section 3.4 : The Integers and Division

Exercises for discussion sections

S7: Wednesday 22.09. M1: Wednesday 22.09. S1: Friday 24.09.

1. Fallacy (direct proof):
   It is known that
   $$1 + 2 + 3 + \ldots + n = n(n+1)/2 \tag{1}$$
   is true.

   If (1) holds for all n, it works with n replaced with $n-1$:
   $$1 + 2 + 3 + ... + (n-1) = (n-1)n/2. \tag{2}$$
   Now, add 1 to both sides of (2):
   $$1 + 2 + 3 + ... + n = (n-1)n/2 + 1. \tag{3}$$
   Comparing (3) to (1) gives an equation:
   $$(n-1)n/2 + 1 = n(n+1)/2. \tag{4}$$

Multiplying through gives a simple equation:

$$n^2 - n + 2 = n^2 + n, \tag{5}$$

which is further simplified to $-n + 2 = n$, from which $n = 1$. But n was an arbitrary integer. So all such integers are equal to 1!

What went wrong?

2. Fallacy:
   Proposition: Either an implication or its converse must be true.
   Proof: Consider the following truth table for $(p \to q) \vee (q \to p)$, that shows the tautology:

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

   Based on that: Taking p and q appropriately, one has that

   If a number is prime, then it is odd, or
   if a number is odd, then it is prime.

   What went wrong?

3. Suppose N is the sum of squares of two integers. Prove or disprove that $2N$ is also the sum of squares of two integers.

4. Can you tile a 17×28 checkerboard using 4×7 tiles?

5. Section 2.1

   Exercise 3, 9, 16, 17, 18, 20

S7: Friday 24.09. M1: Friday 24.09. S1: Monday 27.09.

1. Section 2.2

   Exercise 12, 29a-c, 37, 44, 45

2. Section 2.3

   Exercise 12a-b, 15a, 18a, 29

3rd Mandatory Homework Assignment

The solutions must be handed in via Blackboard latest on

Friday, October 1st, 13:00.

Solutions to the problems given below need to be submitted **as one PDF file** (please ask your TA in case you do not know how to create a PDF file). Other formats are not accepted. The front page of a solution file should clearly specify

- your name,
- your discussion section name (M1, S1, or S7),
- a date and a time of when the PDF was created (such that the latest versions can be determined in case of resubmissions)

Submissions can be in Danish or English language.

Problem 1

The Knight's Tour is a mathematical problem involving a knight on a chessboard. (When a knight moves, it can move two squares horizontally and one square vertically, or two squares vertically and one square horizontally). The knight is placed on the empty board and, moving according to the rules of chess, must visit each square exactly once.

Proof or disprove: A Knight's Tour exists on a standard checkerboard when two corners that are diagonally opposite are removed from the chessboard.

Problem 2

Solve from Section 2.1 exercise 8.

Problem 3

Solve from Section 2.2 exercise 48(b).

Problem 4

Solve from Section 2.3 exercise 18(c).