Institut for Matematik & Datalogi
Syddansk Universitet

August 29, 2012
JBJ

Combinatorial Optimization I (DM208)) — Ugeseddel 2

Bring material to class: It is important that you bring the book and those sections of extra material that I have announced for the class with you. That way we can save time by referring to figures etc.

Stuff covered in week 35 I gave an overview of the course, including a brief recap of linear programming and duality as well as flows in networks – in particular the maximum flow minimum cut theorem. In the second lecture we covered basic stuff on matroids, including the greedy algorithm. In the third lecture we covered bipartite Matching, augmenting paths, direct solution of the maximum matching problem and solution via flows. Proofs of the theorems by König and Hall via the max-flow-min-cut theorem. Finally we also covered maximum weight matchings in bipartite graphs.

It is important that you have some basic knowledge of flows since we will use flows several times in the course. If you feel you need some extra information about this topic, please read Sections 3.1-3.5 in BJG. Especially the flow decomposition property and section 3.4 are very useful. You may also read SCH sections 4.2-4.5.

Handout material in week 35: Chapter 12 from the book Approximation algorithms by V.V.Vazirani, Springer Verlag 2001. If you did not get this, please drop by my office to pick up a copy.

Lecture September 4, 2012: This will be on connectivity of (di)graphs (Menger's theorem). We go on to show a method for determining the edge-connectivity of an undirected graph without using flows. We will also show how this method can be used to find a so-called small certificate for the edge connectivity of a graph. This is based on notes by a former masters student Mette Eskesen that are available for the homepage of the course (she finnished many years ago, but I dont have the final version, so that is why it says unfinished in the manuscript). The relevant material is

- SCH section 4.1
- BJG 7.3
- Notes by Mette Eskesen from the course page

Lecture September 5, 2012: We finish the stuff on Max-back orderings (from the notes by Eskesen) and continue with the primal-dual algorithm. PS Chapter 5.

Exercises for September 5, 2012:

- SCH Application 1.2 page 8 (read and understand so that you can explain it at the blackboard).
- SCH section 1.3 This describes the important extension of the shortest path problem where we can have negative weights on the arcs. (read and understand so that you can explain it at the blackboard).
- SCH application 1.3.
- Prove the following claims:
  - Let $D = (V, A)$ be a directed graph and let $\mathcal{F}_1$ denote those subsets A' of D with the property that no vertex v in D has more than one arc from A' whichs ends in v (an arc ends in v if itis of the form uv for some $u \in V$). Then $(A, \mathcal{F}_1)$ is a matroid on A.
  - Let $M = (S, \mathcal{F})$ be a matroid on S and let $X \subseteq S$. Define $\mathcal{F}_X$ as follows: $\mathcal{F}_X = \{Y \cap X | Y \in \mathcal{F}\}$. Then $(S, \mathcal{F}_X)$ is a matroid on S.
  - Let $M = (S, \mathcal{F})$ be a matroid on S and let $X \subseteq S$ with $X \in \mathcal{F}$ be given. Define $\mathcal{F}'_X$ as follows: $\mathcal{F}_X = \{Y \subset S - X | X \cup Y \in \mathcal{F}\}$. Then $(S, \mathcal{F}'_X)$ is a matroid on S.
  - Let $M = (S, \mathcal{F})$ be a matroid on S and let $\mathcal{F}^* = \{S - X | X \in \mathcal{F}\}$. Then $(S, \mathcal{F}^*)$ is a matroid on S (called the **dual matroid** of M.
  - Given a connected undirected graph $G = (V, E)$, a non-negative real-valued weight function ω on the edges and a subset $E' \subset E$ which forms a forrest. Then, using the greedy algorithm, one can find a cheapest spanning tree T (with respect to ω) which contains all edges of E'.

- SCH application 1.4 Project scheduling
- SCH application 1.7.
- Exam 2011 problem 3 (c)-(e).